SEMI-BLIND ML CHANNEL ESTIMATION FOR MC-CDMA WITH CODE-MULTIPLEXED PILOTS

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Outline

- ✓ Motivation and Signal Model
- ✓ Semi-blind Channel Estimation for MC-CDMA
- ✓ (Asymptotic) analytical characterization
- ✓ Conclusions

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Motivation and Signal model

- The received signal in a MC-CDMA system with code-multiplexed training pilots over N existing carriers can be expressed as:

$$\mathbf{y} = \mathbf{\Lambda}_H \left(\mathbf{C}\mathbf{s} + \mathbf{p}\right) + \mathbf{n} = \left(\mathbf{\Lambda}_s + \mathbf{\Lambda}_p\right)\mathbf{G}^H\mathbf{h} + \mathbf{n} = \mathbf{\Lambda}\mathbf{G}^H\mathbf{h} + \mathbf{n}$$

M transmitted symbols

Flat-fading at each subcarrier

white noise

pilots

NxM-dim. code matrix

L first columns of an NxN Fourier matrix

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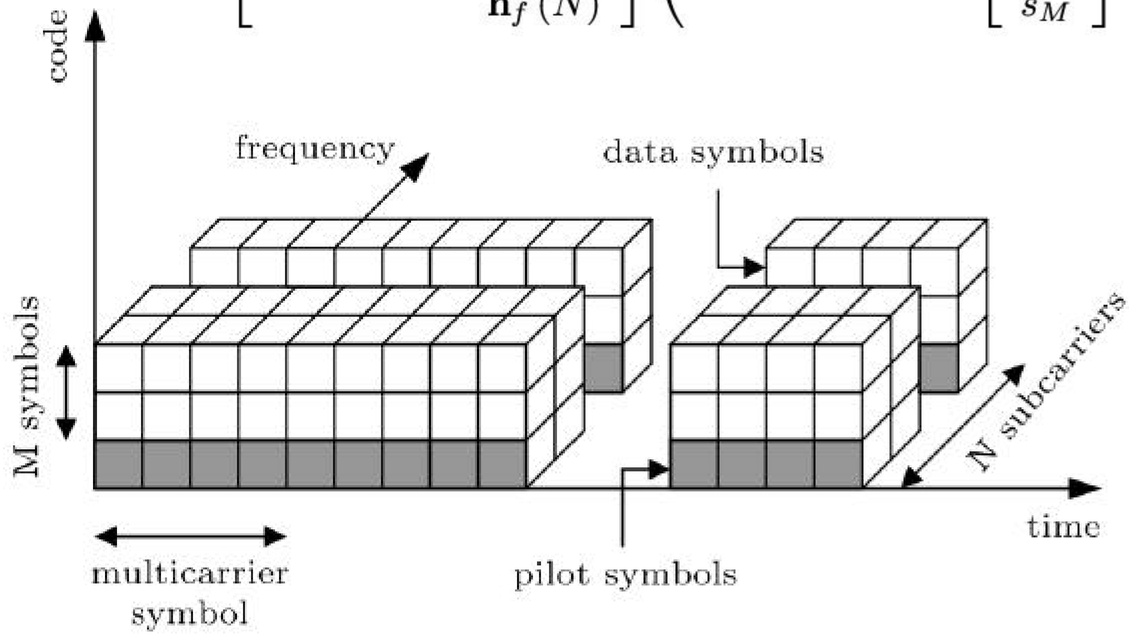
Motivation and Signal model

- Schematic representation of a MC-CDMA system with code-multiplexed training pilots and multicode capabilities:

$$\mathbf{y} = \begin{bmatrix} \mathbf{h}_f(1) & & \\ & \ddots & \\ & & \mathbf{h}_f(N) \end{bmatrix} \left(\begin{bmatrix} \mathbf{c}_1 & \cdots & \mathbf{c}_M \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_M \end{bmatrix} + \mathbf{p} \right) \quad \text{with} \quad \mathbf{h}_f = \mathbf{G}^H \mathbf{h}$$



Semi-blind CE for MC-CDMA (I)

- The unknown symbols are modeled as deterministic parameters and are then taken into account in the channel estimation procedure.

- The ML estimators for both data and channel impulse response can be obtained by minimizing the following negative log-likelihood function:

$$N \ln\left(\pi\sigma^2\right) + \frac{1}{\sigma^2}\left\|\mathbf{y} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{h}\right\|_2^2$$

- In particular, the channel estimate can be obtained as

$$\hat{\mathbf{h}} = \left(\mathbf{G}\hat{\mathbf{\Lambda}}^H\hat{\mathbf{\Lambda}}\mathbf{G}^H\right)^{-1}\mathbf{G}\hat{\mathbf{\Lambda}}^H\mathbf{y}$$

that introduced into the previous likelihood function gives us a new cost function with a reduced parameter space

$$N \ln\left(\pi\sigma^2\right) + \frac{1}{\sigma^2}\left\|\mathbf{P}_{\mathbf{\Omega}}^{\perp}\mathbf{y}\right\|_2^2$$

with $\mathbf{P}_{\mathbf{\Omega}} = \hat{\mathbf{\Lambda}}\mathbf{G}^H\left(\mathbf{G}\hat{\mathbf{\Lambda}}^H\hat{\mathbf{\Lambda}}\mathbf{G}^H\right)^{-1}\mathbf{G}\hat{\mathbf{\Lambda}}^H$

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Semi-blind CE for MC-CDMA(II)

- The semi-blind CML parameter estimates are then obtained from the following optimization problem

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} f(\mathbf{s}) = \arg\min_{\mathbf{s}} \left\| \mathbf{P}_{\mathbf{\Omega}}^{\perp} \mathbf{y} \right\|_2^2$$

- Since the objective function is quadratic separable in its two unknown variables both parameters can be iteratively estimated as

$$\hat{\mathbf{s}}_{k+1} = \left(\mathbf{C}^H \hat{\mathbf{\Lambda}}_{H(k)}^H \hat{\mathbf{\Lambda}}_{H(k)} \mathbf{C} \right)^{-1} \mathbf{C}^H \hat{\mathbf{\Lambda}}_{H(k)}^H \left(\mathbf{y} - \hat{\mathbf{\Lambda}}_{H(k)} \mathbf{p} \right)$$

$$\hat{\mathbf{\Lambda}}_{H(k)} = Diag\left(\mathbf{G}^H \hat{\mathbf{h}}_k \right)$$

and

$$\hat{\mathbf{h}}_k = \left(\mathbf{G} \hat{\mathbf{\Lambda}}_{(k)}^H \hat{\mathbf{\Lambda}}_{(k)} \mathbf{G}^H \right)^{-1} \mathbf{G} \hat{\mathbf{\Lambda}}_{(k)}^H \mathbf{y}$$

$$\hat{\mathbf{\Lambda}} = \hat{\mathbf{\Lambda}}_s + \mathbf{\Lambda}_p \qquad \hat{\mathbf{\Lambda}}_s = Diag(\hat{\mathbf{s}})$$

for a reliable enough first estimate obtained by fixing $\mathbf{s}_1$=0 (e.g. training-only approach!)

Asymptotic results

- <u>RESULT 1</u>. Let A be an NxN complex matrix with bounded spectral radius for all N. Let s =[s_1,…,s_N], whose entries are i.i.d. complex r.v. and $N^{1/2}\, s_i$ has zero mean, unit variance and finite eighth order moment. Let r be a similar vector independent of r, then

$$\mathbf{s}^H \mathbf{A}\mathbf{s} - \frac{1}{N}\operatorname{tr}[\mathbf{A}] \to 0 \qquad \mathbf{s}^H \mathbf{A}\mathbf{r} \to 0$$

almost surely as N → inf [Evans-Tse, IT-Trans.'00]

- <u>RESULT 2</u>. Consider the projection matrix $P_G = G^H(GG^H)^{-1}G$ and the following NxN complex diagonal deterministic matrix

$$\mathbf{D} = \operatorname*{diag}_{k=0\ldots K-1}\left[D\left(\frac{k}{K}\right)\right]$$

where D(f) is a bounded Riemann integrable function defined on [0,1]. Then, as N,L and K go to infinity at the same rate [Mestre, SP-Trans.'04]

$$\frac{1}{K}\operatorname{tr}[\mathbf{P}_G \mathbf{D}] \to \frac{L}{N}\int_0^1 D(f)\, df$$

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Algorithm characterization

- The structure of the MC signal model allows for an (asymptotic) analytical characterization of the performance of the algorithm by assuming random spreading and pilots, and letting N,L → inf with N/L → β fixed

- Under these assumptions the asymptotic expression for the symbol estimate is seen to be

$$\hat{\mathbf{s}}_{k+1} = f\left(\hat{\mathbf{s}}_k, SNR, \beta\right)$$

- Further, considering $\mathbf{s}_1$=0, it is found that

$$\hat{\mathbf{s}}_{k+1} = \eta_{k+1}\mathbf{s}$$

where η is a real positive scalar

$$\eta_{k+1} = f\left(\eta_k, SNR, \beta\right)$$

so that the analysis of the convergence of the algorithm can be approached by studying the evolution of η_k

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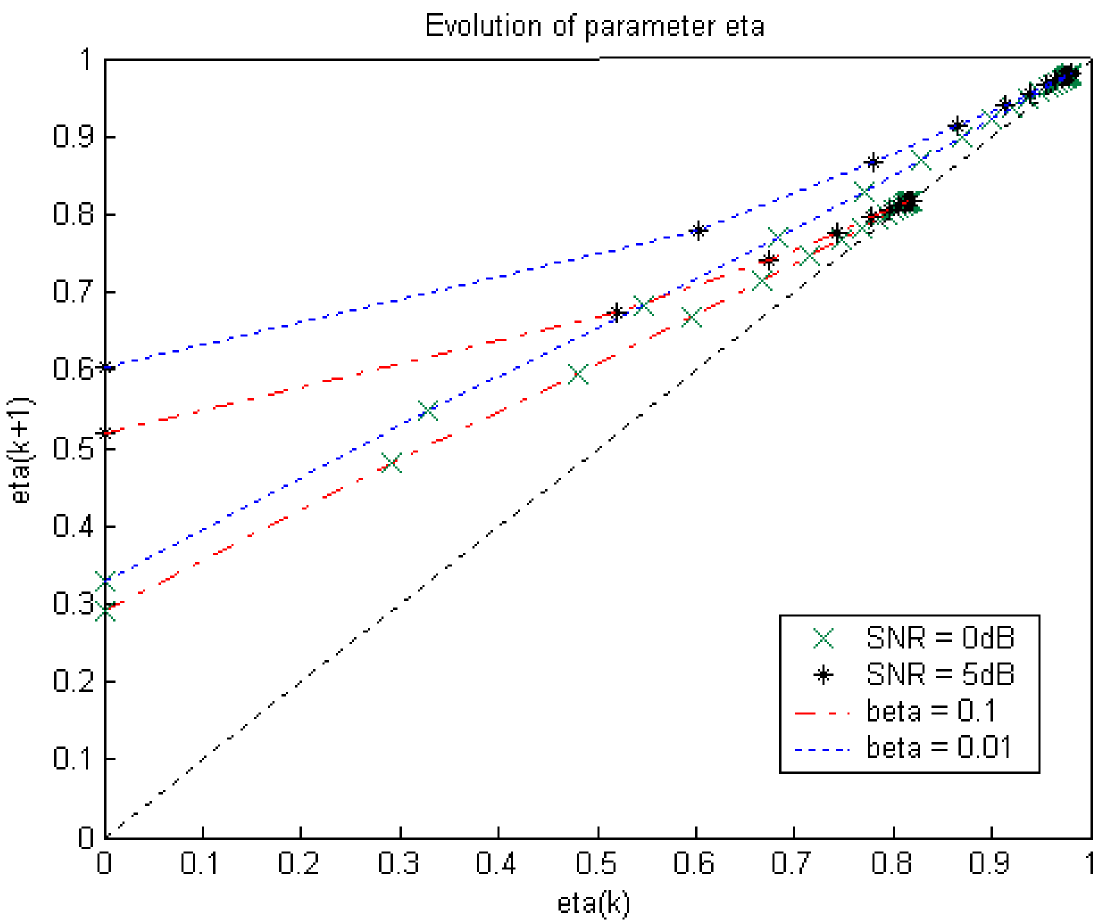
Estimator performance

- Evolution of parameter η for different values of SNR and β



$$SNR = (p_s+p_p)/\sigma^2$$

Regardless of the SNR, the algorithm converges always to the same point for a fixed degree of channel frequency selectivity. It is only the number of iterations until convergence that actually depends on the value of SNR.

Although it need not converge to 1 the analytical tracking of the value of this parameter can be an important aid in the detection task

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Summary

- We have shortly presented the performance of an iterative ML channel estimation algorithm for MC-CDMA that exploits the code-multiplexing of training pilots.
- We considered an asymptotic analysis that allows us to analytically characterize the ultimate symbol estimate quality in terms of a parameter related to its evolution through the iterations.
- The previous analytical characterization can further be employed for detection purposes, whereby it turns out to be of most benefit if applied to nonconstant-amplitude modulations like M-ary QAM.

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References

- Main reference:

F. Rubio and X. Mestre,

"Semi-blind ML Channel Estimation for MC-CDMA Systems with Code-multiplexed Pilots", SPAWC'05.

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