Performance analysis of IR-UWB in multi-user environment

Philippe Ciblat

Dpt Comelec, Télécom ParisTech

Joint work with C. Le Martret (Thales) and F. Kharrat (currently, Alcatel-Lucent)

Ultra-wide band context

History

- Concept introduced in 50's (for military purpose)
- Originally, very short impulse (< ns) $\Rightarrow$ bandwidth > GHz
- Originally, multiple access via Time Hopping
- Low power spectral density $\Rightarrow$ LPI / LPD
- Term *Impulse Radio - Ultra Wide Band* used since 90's

Normalization

- IEEE 802.15.3a : Impulse Radio UWB / MB-OFDM
- IEEE 802.15.4a : Impulse Radio UWB

Assumptions on UWB based system

- Impulse Radio UWB (for sake of simplicity PAM-modulated)
- Asynchronous multi-user context : *Time-Hopping* or *Direct Sequence* ?
- Multipath propagation channel
- Rake (single-user) receiver : coherent detection

Objectives

- Multi-user interference impact on the theoretical performance
- Design and Counting of optimal multiple access codes
- What is the best multiple access scheme : TH or DS ?

Transmit PAM signal

Transmit signal from user n is expressed as follows

$$s_n(t) = \sum_{i=-\infty}^{\infty} d_n(i) \sum_{j=0}^{N_c-1} c_n(j) w(t - iT_s - jT_c - \theta_n)$$

with

- T_s the symbol period where each symbol consists of N_c chips of duration T_c,
- $w(t)$ is the normalized impulse of duration $T_w \ll T_c$,
- $d_n(i)$ are the i.i.d. PAM information symbols of user n,
- $\{c_n(j)\}_{j=0}^{N_c-1}$ is the multiple access code of user n, with either $c_n(j) \in \{-1, 1\}$ for DS code or $c_n(j) \in \{0, 1\}$ for TH code,
- θ_n is the time asynchronism uniformly distributed within $[0, T_s]$.
- N_c is the number of chips per symbol ($N_c = N_s N_h$ with N_s the number of frames and N_h the number of chips per frame in TH scheme).

Multipath propagation channel

$$h_n(t) = \sum_{k=1}^{N_p} A_n^k \cdot \delta(t - \tau_n^k)$$

with A_n^k and τ_n^k the magnitude and delay of the k-th path respectively.

In addition, we assume that

$$A_n^k = a_n^k \cdot f(\tau_n^k)$$

with

- $f(\bullet)$ decreasing function
- a_n^k *i.i.d.* zero-mean process with variance σ_a^2, independent of τ_n^k
- τ_n^k standard clustered model developed in Molish.

Rake receiver

Received signal is as follows

$$y(t) = \sum_{n=1}^{N_u} \sqrt{P_n} \left(\sum_{k=1}^{N_p} A_n^k s_n(t - \tau_n^k) \right) + n(t)$$

with P_n the captured power and $n(t)$ an AWGN.

Receiver description : the Rake one

- User of interest 1
- $L_r \leq N_p$ fingers picked in the selected paths subset $\mathcal{L}$
- *Maximum Ratio Combining*
- $\{A_1^\ell, \tau_1^\ell\}_{\ell \in \mathcal{L}}$ available at the receiver side

Decision variable :

$$z = \sum_{\ell \in \mathcal{L}} A_1^\ell \int_0^{T_s} y(t + \tau_1^\ell) \times \underbrace{\sum_{j=0}^{N_c - 1} c_1(j) w(t - jT_c)}_{\text{template signal}} dt + \eta,$$

Received signal decomposition

$$z = z_U + z_I + z_M + \eta$$

where

- z_U is the **U**seful part of user 1 signal,
- z_I is the **I**nter-Symbol Interference from user 1,
- z_M is the **M**ulti-User Interference (MUI),

In the following, we neglect z_I (time guard interval well designed),

First objective

Performance (Error Probability - P_e) of the Rake receiver (threshold detector on z) with respect to the multiple access codes

Remark : only $\sigma_M^2 := \mathbb{E}[z_M^2]$ available in the literature [LeMartret-Ciblat2006]

Distribution of z_M

Remark : Generalized Gaussian Distribution (GGD) well described the MUI distribution in IR-UWB context (for TH in AWGN [Fiorina2007], for TH/DS in multipath).

GGD

$$p(x) = \frac{\sqrt{\Gamma_c(3/\alpha)}}{2\sigma\sqrt{\Gamma_c(1/\alpha)}\Gamma_c(1+1/\alpha)} e^{-\left|\frac{\sqrt{\Gamma_c(3/\alpha)}}{\sigma\sqrt{\Gamma_c(1/\alpha)}}x\right|^{\alpha}},$$

with

- σ^2 the variance,
- $\alpha > 0$ the so-called shape parameter,
- $\Gamma_c(.)$ the (complete) Gamma function.

Error Probability

$$P_e = \mathrm{Prob}(\nu > \sqrt{P_1} N_s) = \int_{\sqrt{P_1} N_s}^{+\infty} p_\nu(x) dx,$$

with $\nu = z_M + \eta$ and $p_\nu(x)$ its associated distribution.

Remark : z_M is GGD, η is Gaussian (thus GGD) $\Rightarrow \nu$ is GGD.

Consequence

$$P_e = \frac{1}{2\alpha \Gamma_c(1+1/\alpha)} \Gamma_i \left[\frac{1}{\alpha}, \left(\frac{N_s \sqrt{P_1} \sqrt{\Gamma_c(3/\alpha)}}{\sigma \sqrt{\Gamma_c(1/\alpha)}} \right)^{\alpha} \right]$$

with

- $\Gamma_i[a,x] = \int_x^{+\infty} t^{a-1} e^{-t} \mathrm{d}t$ the incomplete Gamma function.

GGD parameters : the variance

$$\sigma^2 = \sigma_M^2 + \sigma_\eta^2$$

where

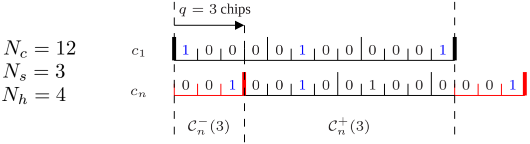
- σ_η^2 is independent of the multiple access codes
- σ_M^2 is the MUI variance already given in [LeMartret-Ciblat2006].

As a reminder, we have

$$\sigma_M^2 \propto \sum_{n=2}^{N_u} P_n \psi_n \sum_{q=0}^{N_c-1} \left[\mathcal{C}_n^{-2}(q) + \mathcal{C}_n^{+2}(q) \right]$$

with

- ψ_n constant depending on the channel statistics of user n
- $\mathcal{C}_n^-(q) = \sum_{k=0}^{q-1} c_1(k) c_n(k-q)$, $\mathcal{C}_n^+(q) = \sum_{k=q}^{N_c-1} c_1(k) c_n(k-q)$



GGD parameters : the shape

$$\alpha = F^{(-1)}\left(\frac{D^4}{\sigma^4}\right)$$

with

- $F^{(-1)}$ the inverse of $x \mapsto F(x) = \frac{\Gamma_c(5/x)\Gamma_c(1/x)}{\Gamma_c^2(3/x)}$
- $D^4 = \mathbb{E}[\nu^4]$ equal to $D^4 = D_M^4 + 3\sigma_\eta^4 + 6\sigma_M^2\sigma_\eta^2$ with $D_M^4 = \mathbb{E}[z_M^4]$

Result [Kharrat-LeMartret-Ciblat2009]

$$D_M^4 \propto \sum_{n=2}^{N_u} P_n^2\phi_n \sum_{q=0}^{N_c-1}\left[\mathcal{C}_n^{+4}(q)+\mathcal{C}_n^{-4}(q)+6\mathcal{C}_n^{+2}(q)\mathcal{C}_n^{-2}(q)\right] + \frac{3(N_u-2)}{N_u-1}\sigma_M^4$$

with ϕ_n constant depending on the channel statistics of user n

Second objective

Optimal codes characterization and counting

Optimal codes (TH) : characterization

Optimal codes

In order to minimize the Error Probability of user 1, we need to select the following pairs of TH codes $\{(c_1, c_n),\ n = 2, \ldots, N_u\}$ satisfiying

$$\sum_{q=0}^{N_c-1} \left(\mathcal{C}_n^+(q) + \mathcal{C}_n^-(q)\right)^2 = N_s^2.$$

- Codes minimizing the MUI variance only satisfy

$$\sum_{q=0}^{N_c-1} \mathcal{C}_n^{+2}(q) + \mathcal{C}_n^{-2}(q) = N_s^2$$

- Codes minimizing the Error probability is only a subset of codes minimizing the MUI variance.

Optimal codes (TH) : counting

Result

The probability π, that a pair of TH code (c_1, c_n) is optimal, is lower bounded by

$$\pi \geq 1 - \frac{N_s(N_s - 1)^2}{N_h}\left[N_s - \frac{4}{3} + \frac{1}{3}\left(\frac{1}{N_h}\right)^2\right].$$

- π converges to 1 when N_h goes to ∞ for a given value N_s.
- The probability that two codes picked at random form an optimal pair goes to 1 when N_h is large for a fixed N_s.
- For a given N_s, increasing the number of optimal pairs can be done by increasing N_h.

Optimal codes (DS)

Optimal codes

In order to minimize the Error Probability of user 1, we need to select the following pairs of DS codes $\{(c_1, c_n),\ n = 2, \ldots, N_u\}$ satisfiying

$$\sum_{q=0}^{N_c-1} \mathcal{C}_n^{+2}(q) + \mathcal{C}_n^{-2}(q) = N_c.$$

- Minimizing the Error Probability $\Leftrightarrow$ minimizing the MUI variance
- The probability π that a pair of DS code (c_1, c_n) is optimal, is upper-bounded by

$$\pi \leq 2^{\frac{1-N_c}{2}}$$

  - π converges to 0 when N_c goes to ∞.
  - Finding optimal codes is hard for DS whereas easy task for TH.

Numerical illustrations : MUI distribution

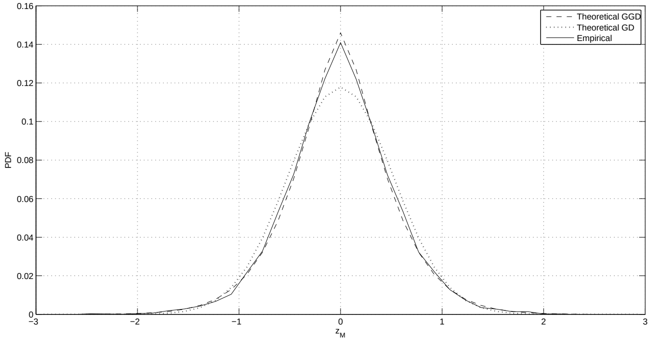


FIG.: Comparison of the empirical MUI PDF to those based on the GGD and the Gaussian distribution (GD) for TH-UWB ($N_c = 32$, $N_s = 2$, $T_c = 5$ ns, $N_u = 32$, $L_r = 3$ and $N_p = 20$).

Generalized gaussian distribution is accurate

Error Probability

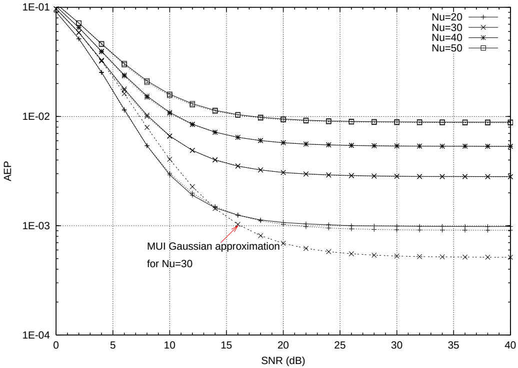


FIG.: Theoretical AEP and empirical BER for TH-UWB ($N_c = 16$, $N_s = 4$, $T_c = 3$ ns).

P_e well approximate by our new expressions

Codes optimization

case 1 corresponds to random codes,
case 2 corresponds to the codes minimizing the MUI variance σ_M^2,
case 3 corresponds to the codes minimizing the P_e

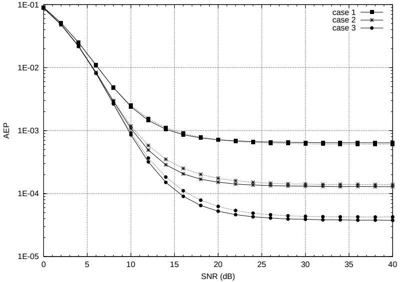


FIG.: Performance wrt the codes properties for TH-UWB ($N_c = 24$, $N_s = 4$, $T_c = 3$ ns, $N_u = 30$).

The exhibited optimal codes really improve the performance

TH and DS comparison

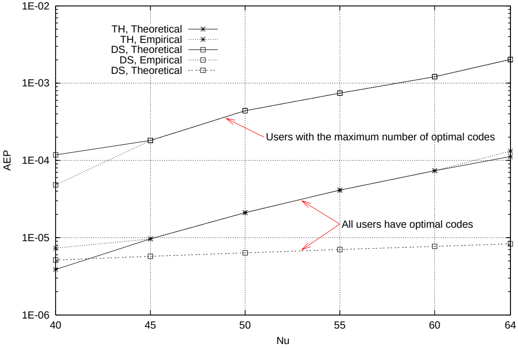


FIG.: Comparison of TH-UWB and DS-UWB ($N_s = 6$, $T_s = 108$ ns, SNR $= 30$ dB).

TH better than DS if codes property taken into account