Resource allocation for Hybrid ARQ based Mobile Ad Hoc networks

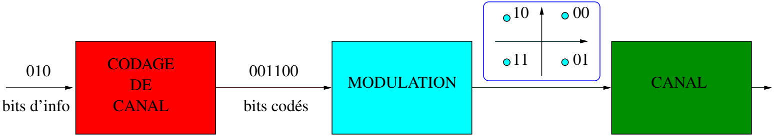
Philippe Ciblat

Joint work with N. Ksairi, A. Le Duc, C. Le Martret, S. Marcille

Télécom ParisTech, France

Part 1 : Introduction to HARQ

Standard communication scheme



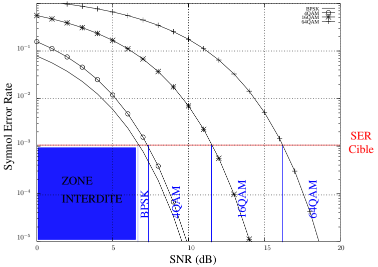
- No feedback about no-error or error at RX side
- Adaptive Modulation and Coding **if**
  - performance model available for considered channel
  - model parameters known at TX side

Drawbacks

- Lack of robustness to channel mis-knowledge
- Limited number of Modulation and Coding Scheme (MCS) in practice
- More important, weak adaptability to the real propagation states (noise, etc)

MCS design

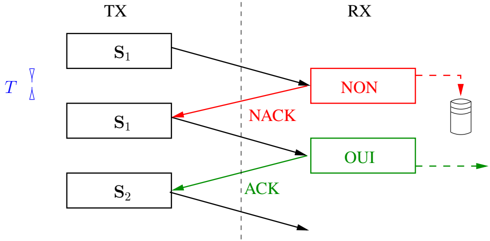
Example : QAM size (according to target Symbol Error Rate)



- AMC done on average performance, not on instantaneous one
- Idea : try/error principle
  - Send one (symbol) packet
  - if OK (ACK), ↗
  - if KO (NACK), ↘ send it again
- Need for one-bit feedback way (providing information on the instantaneous channel)

From ARQ (*Automatic ReQuest*) ...

Let $\mathbf{S} = [s_0, \cdots, s_{N-1}]$ be a packet composed by N uncoded symbols



Management for T :

- Stop-and-Wait
- Parallel Stop-and-Wait
- Selective Repeat

Assumption : perfect feedback (neither error, nor delay $T = 0$)

... Towards Hybrid ARQ (HARQ) : Type-I HARQ

Remark

Retransmission does not contradict forward error coding (FEC)

Type-I HARQ : packet **S** is composed by coded symboles s_n

- first packet is more protected
- there is less retransmission
- transmission delay is reduced

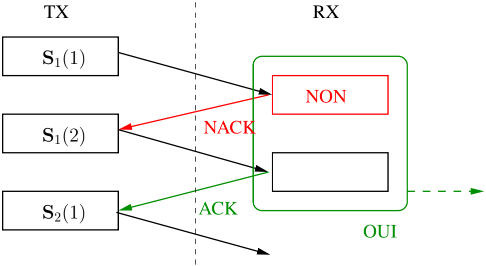
- Efficiency is upper-bounded by the code rate

Type-II HARQ

Comments on Type-I HARQ

- Each received packet is treated independently
- Mis-decoded packet is thrown in the trash

Memory at RX side is considered $\Rightarrow$ Type-II HARQ



Main examples :

- *Chase Combining* (CC)
- *Incremental Redundancy* (IR)

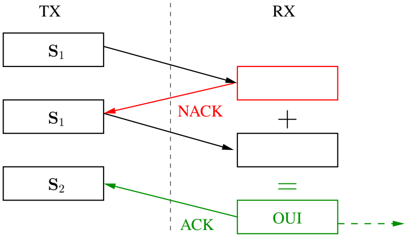
CC

$$\begin{aligned} Y_1 &= S_1 + N_1 \\ Y_2 &= S_1 + N_2 \end{aligned}$$

then detection on

$$Y = (Y_1 + Y_2)/2$$

SNR-Gain equal to 3dB



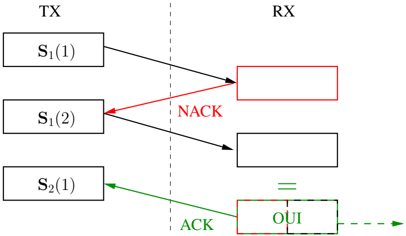
IR

$$\begin{aligned} Y_1 &= S_1(1) + N_1 \\ Y_2 &= S_1(2) + N_2 \end{aligned}$$

then detection on

$$Y = [Y_1, Y_2]$$

Coding gain



Performance metrics

- **Packet Error Rate** (PER) :

$$\text{PER} = \text{Prob(information packet is not decoded)}$$

- **Efficiency** (*Throughput*/*Goodput*/etc) :

$$\eta = \frac{\text{information bits received without error}}{\text{transmitted bits}}$$

- **(Mean) delay** :

$$d = \#\ \text{transmitted packets when information packet is received}$$

- **Jitter** :

$$\sigma_d = \text{delay standard deviation}$$

Quality of Service (QoS)

- Data : PER and efficiency
- Voice on IP : delay
- Vidéo Streaming : efficiency and jitter

Closed-form expressions for metrics

$$\text{PER} = 1 - \sum_{k=1}^{L} p(k)$$

$$\eta \propto \frac{\sum_{k=1}^{L} p(k)}{L(1 - \sum_{k=1}^{L} p(k)) + \sum_{k=1}^{L} kp(k)}$$

$$d = \frac{\sum_{k=1}^{L} kp(k)}{\sum_{k=1}^{L} p(k)}$$

$$\sigma_d = \sqrt{\frac{\sum_{k=1}^{L} k^2 p(k)}{\sum_{k=1}^{L} p(k)} - d^2}$$

with

- $p(k)$ probability to receive information pacjket in exactly k transmissions
- L maximum number of transmissions per information packet

Example : Type-I HARQ

- Let π_0 be the probability the information packet is mis-decoded (in one transmission). Then

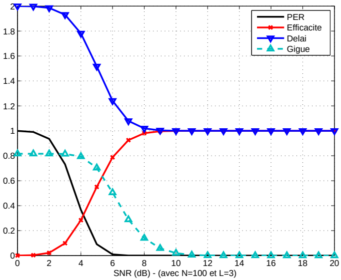
$$p(k) = (1 - \pi_0)\pi_0^{k-1}$$

- Let an information packet composed by N BPSK uncoded symbols. Then

$$\pi_0 = 1 - \left(1 - Q\left(\sqrt{2\mathrm{SNR}}\right)\right)^N$$

Results

$$\begin{aligned} \mathrm{PER} &= \pi_0^L \\ \eta &\propto 1 - \pi_0 \\ d &= L + \frac{1}{1-\pi_0} - \frac{L}{1-\pi_0^L} \end{aligned}$$



Part 2 : HARQ based resource allocation

2.1 Trade-off Retransmission-Physical Layer performance

2.2 Waterfilling-like algorithm

2.3 Application to Mobile Ad Hoc Networks (MANETs)

Trade-off Retransmission-Physical Layer performance

When only PHY layer is considered :

- Power minimization with target SNR
- Rate maximization with power constraint

$$\#bits/pcu = \left\lfloor \log_2 \left(1 + \frac{SNR}{\Gamma} \right) \right\rfloor$$

with Γ SNR-gap wrt. Shannon capacity for one FEC and target SER

Target PHY layer is required

When retransmission (HARQ) is considered :

- Rate is replaced with Efficiency
- Rate maximization leads to the optimal PHY layer performance through $p(k)$

PHY layer performance is not fixed in advance

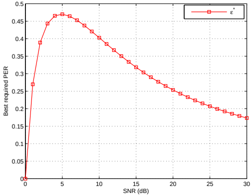
Example : Type-I HARQ

- Rayleigh channel :
- Each packet is encoded with coding rate R

$$\pi_0 = \mathrm{Prob}_h(\log_2(1+|h|^2 SNR) < R) \Leftrightarrow R_{\pi_0} = \log_2\left(1 + \frac{SNR}{\frac{1}{\log\left(\frac{1}{1-\pi_0}\right)}}\right)$$

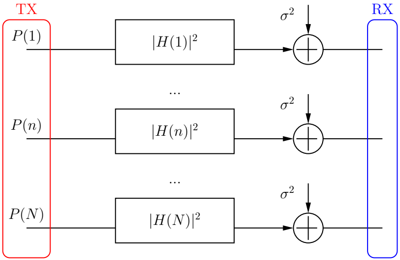
- Best PHY layer is given by [Jin09]

$$\pi_0^* = \arg\max_{\pi_0} \underbrace{R_{\pi_0}(1-\pi_0)}_{\eta_{\pi_0}\text{: efficiency}}$$



Reliable PHY layer is not required at all (thanks to retransmission) !

Waterfilling-like algorithm : data rate optimization



– Power constraint :

$$\sum_{n=1}^{N} P(n) = P_{\max}$$

with maximum power $P_{\max}$.

– Perfect CSIT

Problem : which data rate criterion to be used ?

- (Shannon) sum-capacity : $[P(1)^*, \cdots, P(N)^*] = \arg\max_{P(1),\cdots,P(N)} \sum_{n=1}^{N} \log_2 \left(1 + |H(n)|^2 \frac{P(n)}{\sigma^2}\right)$
- (HARQ) sum-efficiency : $[P(1)^*, \cdots, P(N)^*] = \arg\max_{P(1),\cdots,P(N)} \sum_{n=1}^{N} m(n) \cdot (1 - P_e(n))$

  with $2^{m(n)}$-QAM and $P_e(n)$ SER (packet=symb, here).

Practical algorithms

Convex optimization problem ($\Rightarrow$ KKT conditions)

Capacity : "Waterfilling" [Sha48]

$$P(n)^* = \left(\nu - \frac{\sigma^2}{|H(n)|^2}\right)^+$$

with

- ν chosen s.t. $\sum_{n=1}^{N} P(n)^* = P_{\text{max}}$.
- $(\bullet)^+ = \max(0, \bullet)$.

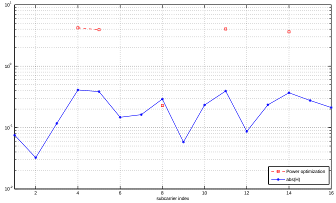
Efficiency : [unpublished]

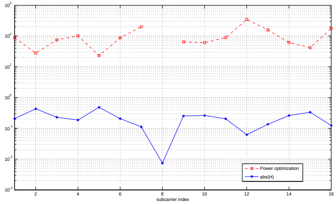
$$P(n)^* = \left(\frac{2\sigma^2}{\gamma(n)|H(n)|^2} \log\left(\frac{m(n)\gamma(n)|H(n)|^2}{\mu\sigma^2}\right)\right)^+$$

with

- μ chosen s.t. $\sum_{n=1}^{N} P(n)^* = P_{\text{max}}$.
- $\gamma(n)$ modulation gain at channel $\#n$

$N = 16$, $SNR = 0dB$ (left), $SNR = 20dB$ (right)



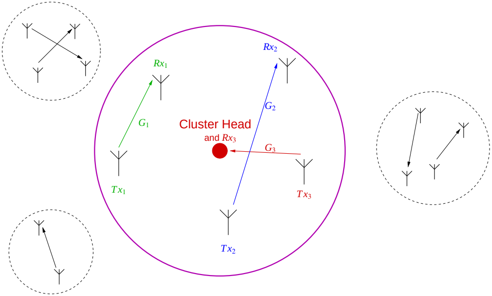


Remark

The best subcarriers do not have necessary the highest powers (at high SNR)

Mobile Ad Hoc Networks (MANETs)

- Infrastructure-free
- Highly flexible
- Fast and short-lived communications deployment



- Clusterization ⇒ **Centralized** coordination of the **pairwise** communications at CH
- Feedback latency ⇒ Channel **statistics** known at CH

Communication scheme

- **OFDMA**
  - PHY layer : cancel ISI due to multipath spread
  - Multiple access : cancel multiuser interference inside a cluster

- **HARQ**
  - manage fast channel variations

- **Statistical channel model :** let us consider the k-th link
  - Let $h_k(m)$ be the m-th filter tap. Ind. (but not i.d.) $\sim \mathcal{CN}(0, \varsigma_{k,m}^2)$
  - $H_k(n)$ non-ind. wrt n but i.d. $\sim \mathcal{CN}(0, \varsigma_k^2)$ with $\varsigma_k^2 = \sum_m \varsigma_{k,m}^2$
  - $\Rightarrow$ Rayleigh fading channel
  - $\Rightarrow$ Channel statistics (for $H_k(n)$) ind. of subcarrier n
  - $\Rightarrow$ Subcarriers are statistically equivalent

Consequence for the k-th link

Bandwidth proportion and identical energy per subcarrier

Resource allocation issue

- Q_k : Energy of link k in OFDM symbol
- γ_k : Bandwidth proportion assigned to link k
- E_k : Energy of link k in entire bandwidth $\Rightarrow \quad Q_k = \gamma_k E_k$
- Modulation (order 2^{m_k}) and coding scheme (rate R_k)

Goal [Mar13]

$$\min \sum_{k=1}^{K} Q_k \quad \Leftrightarrow \min_{(\gamma,\boldsymbol{E})} \sum_{k=1}^{K} \gamma_k E_k \quad \text{s.t.} \quad \begin{array}{l} \mathbf{QoS}_k(\gamma_k, E_k) \geq \mathbf{QoS}_k^{(0)}, \ \forall k \\ \sum_{k=1}^{K} \gamma_k \leq 1 \\ \gamma_k \geq 0, \ E_k \geq 0, \ \forall k \end{array}$$

for various Quality of Service (QoS) : **eff., eff.+PER, eff.+delay**

Type-I HARQ case

PER expression

$$P_k(\mathrm{SNR}) \propto \frac{1}{SNR^{d_{\min}}}$$

Valid for fast-fading channel model (BICM + FH)

Efficiency expression

$$\eta_k(\gamma_k, E_k) = \gamma_k m_k R_k (1 - P_k(G_k E_k))$$

Delay expression

$$d_k(\gamma_k, E_k) = \frac{1}{\gamma_k}\left(\frac{1}{1 - P_k(G_k E_k)} - \frac{L\, P_k(G_k E_k)^L}{1 - P_k(G_k E_k)^L}\right)$$

Optimization problem 1 : rate constrained

Write the problem in $(\gamma, \boldsymbol{Q})$ instead of $(\gamma, \boldsymbol{E})$

$$\min_{(\gamma,\boldsymbol{Q})} \sum_{k=1}^{K} Q_k \quad \text{s.t.} \quad \eta_k(\gamma_k, Q_k) \geq \eta_k^{(0)}, \ \forall k$$

$$\sum_{k=1}^{K} \gamma_k \leq 1$$

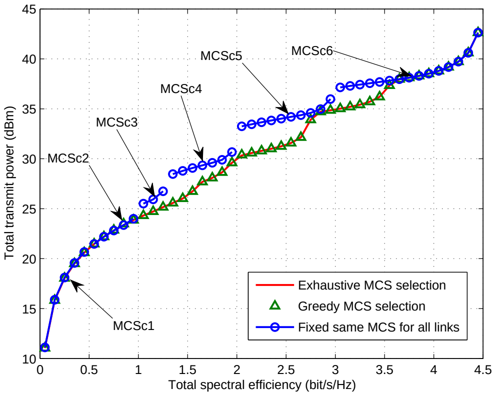
$$\gamma_k \geq 0, \ Q_k \geq 0, \ \forall k$$

Results

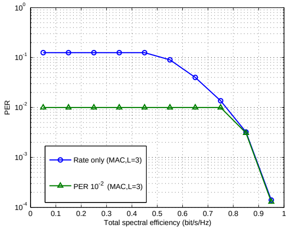
- Solution exists iff $\sum_{k=1}^{K} \eta_k^{(0)}/(m_k R_k) < 1$
- Problem is convex in $(\gamma, \boldsymbol{Q})$
- Optimal solutions exhibited in closed-form (from KKT) given $\text{mcs}_k = (m_k, R_k)$

Numerical results

- $K = 4$ links
- Free-space path loss
- Random distances in $[50, 1000]$ m
- $L = 3$



Optimization problem 2 : rate + PER constrained



$$\min_{(\gamma,\boldsymbol{Q})} \sum_{k=1}^{K} Q_k \quad \text{s.t.} \quad \eta_k(\gamma_k, Q_k) \geq \eta_k^{(0)}, \ \forall k$$

$$P_k(Q_k/\gamma_k) \leq P_k^{(0)}, \ \forall k$$

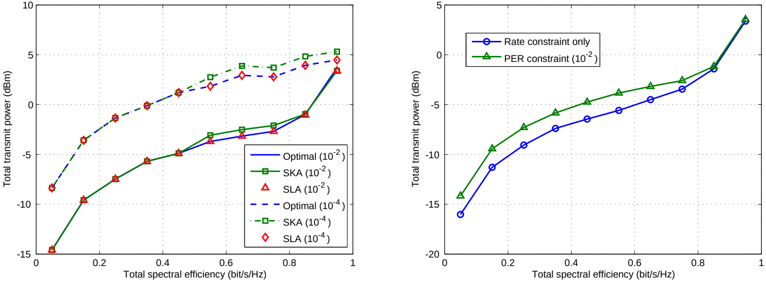
$$\sum_{k=1}^{K} \gamma_k \leq 1$$

$$\gamma_k \geq 0, \ Q_k \geq 0, \ \forall k$$

Results

- P_k is a quasi-convex function of (γ_k, Q_k)
- KKT are optimal [Las10] <u>but</u> it is $O(2^{K-1})\ldots$
- …Suboptimal KKT resolution (SKA)
- …Suboptimal alternate directional descent wrt. (γ_k, E_k) (SLA)

Numerical Results



Remarks

- SLA offers almost the same performance as KKT
- Constraining the PER to 10^{-2} adds an energy cost of about 2 dB

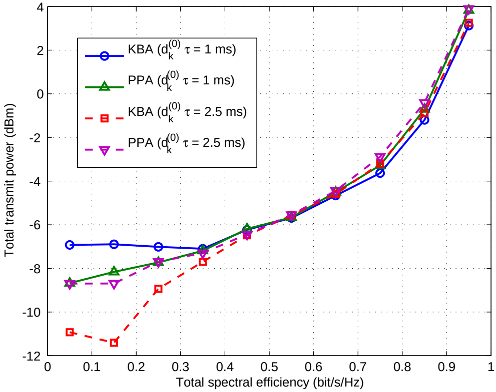
Optimization problem 3 : rate + delay constrained

$$\min_{(\gamma,\boldsymbol{E})} \sum_{k=1}^{K} \gamma_k E_k \quad \text{s.t.} \quad \begin{aligned} & \eta_k(\gamma_k, E_k) \geq \eta_k^{(0)}, \ \forall k \\ & d_k(\gamma_k, E_k) \leq d_k^{(0)}, \ \forall k \\ & \sum_{k=1}^{K} \gamma_k \leq 1 \\ & \gamma_k \geq 0, \ E_k \geq 0, \ \forall k \end{aligned}$$

Results

- Solution exists iff $\sum_{k=1}^{K} \max\left(\eta_k^{(0)}/(m_k R_k), 1/d_k^{(0)}\right) < 1$
- d_k quasi-convex in E_k, convex in γ_k, no information in joint directions
- KKT-based algo. (KBA) : KKT solved, but no optimality theorem
- Ping-Pong algo. (PPA) : optimization alternately in both directions

Numerical Results



Remark

- KBA is optimal when the delay constraint is strictly satisfied

Type-II HARQ case

Much more complicated : see efficiency expression in Slide 9

Actually, the efficiency for any Type-II HARQ writes as follows

$$\eta_k(\gamma_k, E_k) = m_k R_k \gamma_k \frac{1 - q_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} q_{k,l}(G_k E_k)}$$

where $q_{k,l}(\mathrm{SNR}_k)$ is the probability that the first l transmissions of a HARQ round are all received in error.
Let $\pi_{k,l}$ be the probability for not decoding the information packet based on the first l transmissions.

$$\pi_{k,l} \approx \frac{g_{k,l}(m_k, R_k)}{\mathrm{SNR}_k^{d_{k,l}(R_k)}}$$

where $d_{k,l}$ represents a minimal Hamming distance.

Remark

$q_{k,l}$ can be upper-bounded by $\pi_{k,l}$ [Duc10]

Optimization problem : rate

$$\min_{\gamma, \boldsymbol{E}} \sum_{k=1}^{K} \gamma_k E_k \quad \text{s.t.} \qquad \gamma_k \frac{1 - \pi_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(G_k E_k)} \geq \frac{\eta_k^{(0)}}{m_k R_k}, \forall k$$

$$\sum_{k=1}^{K} \gamma_k \leq 1$$

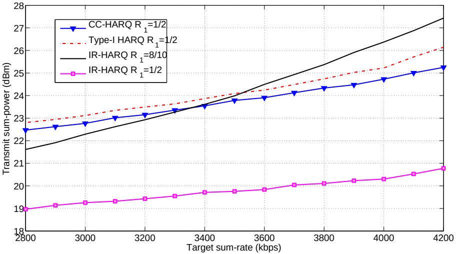
$$\gamma_k > 0, E_k > 0, \forall k$$

Results

- It is a *geometric program* [Ksa13]
- Transformation into a convex optimization problem by $\gamma_k = e^{x_k}$ and $E_k = e^{y_k}$
- KKT solved

Numerical results

- $K = 10$ links, Bandwidth $W = 5$ MHz , QPSK
- Feasability condition satisfies for sum rates up to 5 Mbps.



Remarks

- Nested-codes IR-HARQ outperforms RCP-codes IR-HARQ
- $4/5$-rate IR-HARQ outperforms $1/2$-rate Type-I and CC-HARQ up to 3.3 Mbps. For higher target rates, its worse pre-HARQ performance ($\pi_{k,1}$) plays a role.

Conclusion

- Extension done to per-link power constraint
- Applications : HSPA, Wimax, LTE
- New metrics for resource allocation

References :

[Jin09] P. Wu and N. Jindal, "Coding Versus ARQ in Fading Channels : How reliable should the PHY be ?," IEEE Globecom Conference, Nov. 2009.

[Las10] J. B. Lasserre, "On representations of the feasible set in convex optimization," *Optim. Lett.*, 2010.

[Duc10] A. Le Duc, "Derivations and analysis of HARQ schemes in a cross-layer context," Telecom ParisTech PhD thesis, 2010.

[Mar13], S. Marcille, "Resource allocation for HARQ based mobile ad hoc network," Telecom ParisTech PhD thesis, 2013.

[Ksa13], N. Ksairi, P. Ciblat, and C. Le Martret, "Optimal Resource Allocation for Type-II HARQ-based OFDMA Ad Hoc Networks," submitted to IEEE GlobaSIP Conference, Dec. 2013.