Fully-distributed spectrum sensing: application to cognitive radio

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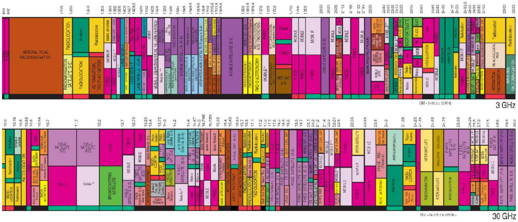
Joint work with F. Iutzeler (PhD student funded by DGA grant)

Cognitive radio principle

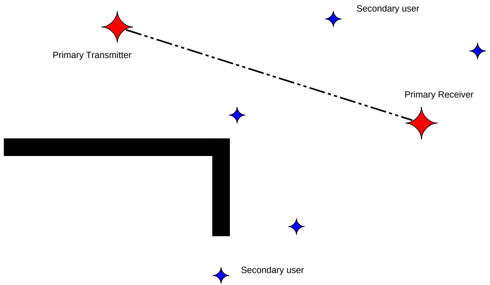
- Spectrum is, at a first glance, entirely used



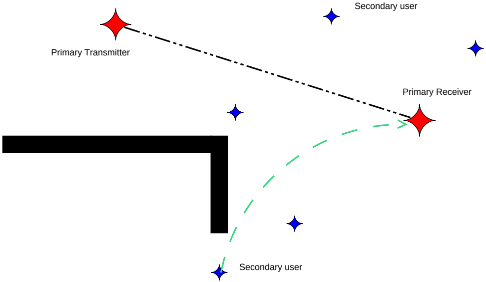
- However, at given time, an assigned subband can be free
  $\Rightarrow$ **white space**
- Two kinds of users
  - **Primary :** have paid for using an pre-assigned subband
  - **Secondary :** are allowed to use a white space
- Insert secondary users into white spaces

How detecting the presence of a primary user ?

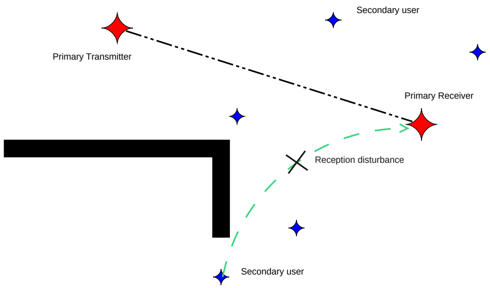
Hidden terminal issue



Hidden terminal issue



Hidden terminal issue

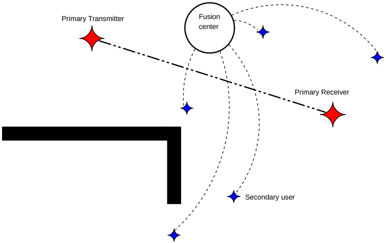


Problem : a secondary user is disturbing the primary receiver

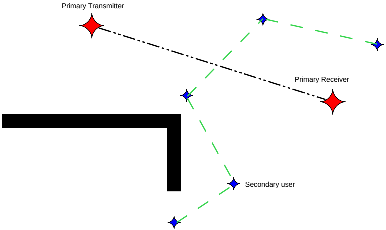
Solution : secondary users have to cooperate to detect the primary user

Two ways for cooperating

Centralized detection



(Fully)-Distributed detection



Detection with more than one sensors

- If fusion center is available, centralized (also called distributed) detection
- If fusion center is not available, **(fully)-distributed** detection
  - robust against nodes attack
  - simple network management

System model

$$\begin{cases} \mathcal{H}_0 & \text{(absence of primary user)} & : y_k(n) = b_k(n) & k = 1, \cdots, K \\ \mathcal{H}_1 & \text{(presence of primary user)} & : y_k(n) = x_k(n) + b_k(n) & n = 1, \cdots, N_s \end{cases}$$

with

- secondary user index k and time index n
- $b_k(n)$ Gaussian with variance σ_k^2 known at secondary user k
- $\{x_k(n)\}_n$ coming from primary user known at secondary user k

Performance metric

Detection probability : $P_D = P(\mathcal{H}_1|\mathcal{H}_1)$

False alarm probability : $P_{FA} = P(\mathcal{H}_1|\mathcal{H}_0)$

Goal : minimizing P_{FA} such that $P_D \geq P_D^{\text{target}}$

Remarks :

- If $\{x_k(n)\}_n$ unknown but Gaussian $\Rightarrow$ Energy detector
- Hard detection (local decision and then voting) not considered

Reminder on (soft) centralized detection

Optimal test : Log-Likelihood Ratio (LLR)

$$\Lambda(\mathbf{y}) = \log\left(\frac{p(\mathbf{y}|\mathcal{H}_1)}{p(\mathbf{y}|\mathcal{H}_0)}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \mu, \quad \text{with } \mu \text{ chosen for ensuring } P_D^{\text{target}}$$

Application to our practical case :

$$T(\mathbf{y}) = \frac{1}{K}\sum_{k=1}^{K} t_k(\mathbf{y}_k) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta, \quad \text{with} \quad t_k(\mathbf{y}_k) = \frac{\mathbf{y}_k^{\mathrm{T}}\mathbf{x}_k}{\sigma_k^2}$$

and $\mathbf{y}_k = [y_k(1), \cdots, y_k(N_s)]^{\mathrm{T}}$, $\mathbf{x}_k = [x_k(1), \cdots, x_k(N_s)]^{\mathrm{T}}$, $(.)^{\mathrm{T}}$ = transpose.

Threshold computation

$$\eta = \sqrt{\varsigma_T}\, Q^{(-1)}\left(P_D^{\text{target}}\right) + m_T$$

where $Q^{(-1)}$ is the inverse of the Gaussian tail function, and

$$m_T = N_s\left(\frac{1}{K}\sum_{k=1}^{K}\mathrm{SNR}_k\right) \text{ and } \varsigma_T = \frac{N_s}{K}\left(\frac{1}{K}\sum_{k=1}^{K}\mathrm{SNR}_k\right).$$

Fully-distributed detection (I)

Sensing step of duration N_s	Gossiping step of duration N_g
$t_\ell = \mathbf{y}_\ell^{\mathrm{T}} \mathbf{x}_\ell / \sigma_\ell^2$ for each node ℓ	$T_k \approx average_\ell(t_\ell)$

Question : How computing the average of t_ℓ in a distributed way

$\Rightarrow$ Gossiping (also called consensus) algorithms

Gossiping algorithm description : an example (Pairwise Gossip)

$\mathbf{x}(0) = [x_1(0), \cdots, x_K(0)]^{\mathrm{T}}$: initial values

At time t, a node i wakes up and calls one of its neighbor j. Then

$$\begin{aligned} x_i(t+1) &= (x_i(t) + x_j(t))/2 \\ x_j(t+1) &= (x_i(t) + x_j(t))/2 \end{aligned}$$

$$\Rightarrow \mathbf{x}(t+1) = \mathbf{W}(t)\mathbf{x}(t) \overset{t\to\infty}{\to} x_{average}\mathbf{1}$$

$$\begin{bmatrix} T_1(\mathbf{y}) \\ \vdots \\ T_K(\mathbf{y}) \end{bmatrix} = \mathbf{P} \begin{bmatrix} t_1(\mathbf{y}_1) \\ \vdots \\ t_K(\mathbf{y}_K) \end{bmatrix}$$

with $\mathbf{P} = (p_{k\ell})_{k,\ell=1,\cdots,K}$ the gossiping algorithm matrix after N_g iterations.

Fully-distributed detection (II)

The final test function at node k is

$$T_k(\mathbf{y}) = \sum_{\ell=1}^{K} p_{k\ell} \frac{\mathbf{y}_\ell^{\mathrm{T}} \mathbf{x}_\ell}{\sigma_\ell^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_k,$$

where the threshold (for pre-defined P_D^{target}) is given by

$$\eta_k = \sqrt{\varsigma_k} Q^{(-1)} \left(P_D^{\text{target}}\right) + m_k$$

with

$$m_k = N_s \sum_{\ell=1}^{K} p_{k\ell} \mathrm{SNR}_\ell \quad \text{and} \quad \varsigma_k = N_s \sum_{\ell=1}^{K} p_{k\ell}^2 \mathrm{SNR}_\ell .$$

Problem

Threshold not computable in a distributed way due to the terms $p_{k\ell}^2$.

Approach 1 : distributed with knowledge of K

$$\eta_k^{(1)} = \sqrt{\varsigma_k^{(1)}} Q^{(-1)}\left(P_D^{\text{target}}\right) + m_k \quad \text{with} \quad \varsigma_k^{(1)} = \frac{N_s}{K} \sum_{\ell=1}^{K} p_{k\ell} \mathrm{SNR}_\ell.$$

Approach 2 : fully-distributed

Using Sum-Weight-like gossip in order to perform the average and the sum.

$$\mathbf{z} := \mathbf{Qt}, \ \mathbf{w}^{(1)} := \mathbf{Q1}, \ \mathbf{w}^{(e)} := \mathbf{Qe}$$

where

- $\mathbf{Q}$ the gossip algorithm matrix after N_g iterations,
- $\mathbf{e}$ the K-sized vector whose first component is 1 and the others 0.

Each node k calculates the k-th component of

$$\mathbf{z}_p = \mathbf{z} \oslash \mathbf{w}^{(1)} = \mathbf{Pt} \to t_{average}\mathbf{1} \quad \text{and} \quad \mathbf{z}_s = \mathbf{z} \oslash \mathbf{w}^{(e)} = \mathbf{St} \to t_{sum}\mathbf{1}$$

where

- $\oslash$ the elementwise division.
- $\mathbf{P} = \mathrm{diag}\left(\mathbf{1} \oslash \mathbf{Q1}\right)\mathbf{Q}$ and $\mathbf{S} = \mathrm{diag}\left(\mathbf{1} \oslash \mathbf{Qe}\right)\mathbf{Q}$.

Two approaches for threshold computation (II)

The threshold is then as follows

$$\eta_k^{(2)} = \sqrt{\varsigma_k^{(2)}} Q^{(-1)}\left(P_D^{\text{target}}\right) + m_k \quad \text{with} \quad \varsigma_k^{(2)} = N_s \frac{\left(\sum_{\ell=1}^K p_{k\ell}\mathrm{SNR}_\ell\right)^2}{\sum_{\ell=1}^K s_{k\ell}\mathrm{SNR}_\ell}.$$

Remarks

- Algorithm fully distributed since the number of nodes not required.
- New threshold does not ensure the target probability of detection

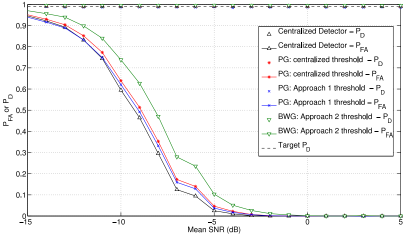
$$P_D(k) = Q\left(\sqrt{\frac{\varsigma_k^{(2)}}{\varsigma_k}} Q^{(-1)}\left(P_D^{\text{target}}\right)\right).$$

Setup :

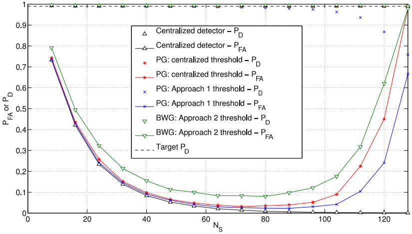
- $T = N_s + K = 128$ with $N_s = N_g = 64$.
- $P_D^{\text{target}} = 0.99$
- $K = 10$

Considered algorithms : energy-based algorithm

- centralized detection
- pairwise gossip (PG) with centralized threshold
- pairwise gossip with Approach 1 based distributed threshold
- broadcast sum-weight gossip (BWG) with Approach 2 based distributed threshold



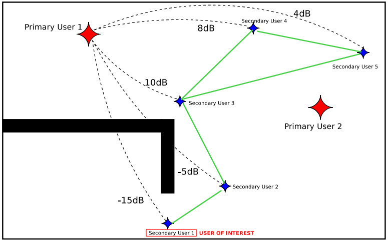
P_{FA} and P_D versus mean SNR



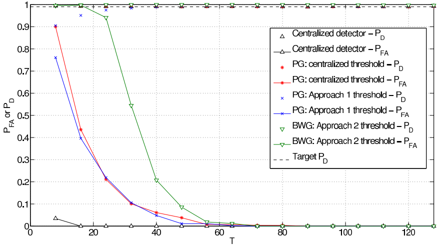
P_{FA} and P_D versus N_s

- Fully-distributed algorithm performs well
- Sensing time equivalent to gossiping time

Hidden terminal context



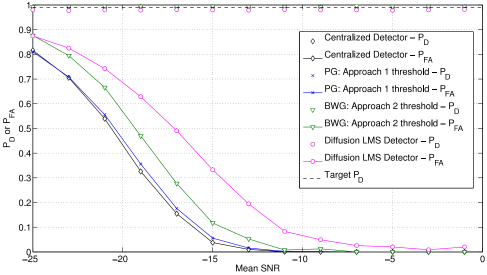
Hidden terminal configuration



P_{FA} and P_D versus T

- Fast convergence for fully-distributed algorithm
- Hidden terminal issue is fixed

- Training based algorithm
- Comparison with "A. Sayed, *Distributed detection over adaptive networks using diffusion adaptation*, IEEE Trans. Signal Processing, May 2011".
- In this paper, sensing and gossiping steps are interleaved



- Outperforming existing approach
- More relevant choice of threshold

Conclusion

- New fully-distributed detection algorithm
- Outperforms existing approaches

Other works :

- Max-consensus : asynchronous algorithm
- Average-consensus : fast algorithm (BWG) and theoretical analysis

- ... distributed optimization (to be done)

References :
F. Iutzeler and P. Ciblat, "*Fully-distributed spectrum sensing : application to cognitive radio*", submitted for publication to Eusipco, 2013.
F. Iutzeler, P. Ciblat, and W. Hachem, "*Analysis of Sum-Weight-like algorithms for averaging in Wireless Sensor Networks*", accepted for publication to IEEE Trans. Signal Processing.
F. Iutzeler, P. Ciblat, and J. Jakubowicz, "*Analysis of max-consensus algorithms in wireless channels*", IEEE Trans. Signal Processing, vol. 60, no. 11, pp. 6103-6107, November 2012.