# Resource allocation for Hybrid ARQ based wireless networks

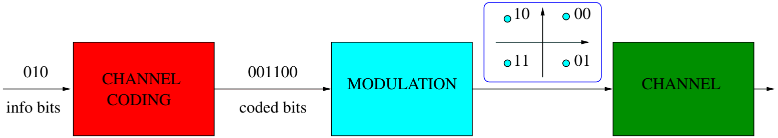
Philippe Ciblat

*Joint work with N. Ksairi, A. Le Duc, C. Le Martret, S. Marcille*

Télécom ParisTech
France

# Part 1 : Introduction to HARQ

# Standard communication scheme



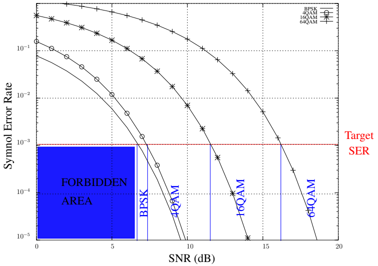

- No feedback about no-error or error at RX side
- Adaptive Modulation and Coding **if**
  - channel model
  - theoretical performance available for channel model
  - channel model parameters known at TX side

## Drawbacks

- Lack of robustness to channel mis-knowledge
- Limited number of Modulation and Coding Scheme (MCS) in practice
- More important, weak adaptability to the real propagation states (noise, etc)

# MCS design

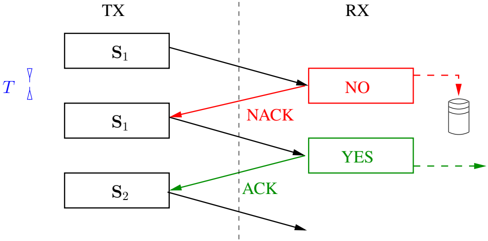
Example : QAM size (according to target Symbol Error Rate)




- AMC done on average performance, not on instantaneous one
- Idea : try/error principle
  - Send one (symbol) packet
  - if OK (ACK), ↗
  - if KO (NACK), ↘ send it again
- Need for one-bit feedback way (providing information on the instantaneous channel)

# From ARQ (*Automatic ReQuest*) ...

Let $\mathbf{S} = [s_0, \cdots, s_{N-1}]$ be a packet composed by $N$ uncoded symbols




Management for $T$ :

- Stop-and-Wait
- Parallel Stop-and-Wait
- Selective Repeat

**Assumption :** perfect feedback (neither error, nor delay $T = 0$)

# ... Towards Hybrid ARQ (HARQ) : Type-I HARQ

**Remark**

Retransmission does not contradict forward error coding (FEC)

Type-I HARQ : packet **S** is composed by coded symbols $s_n$

- first packet is more protected
- there is less retransmission
- transmission delay is reduced

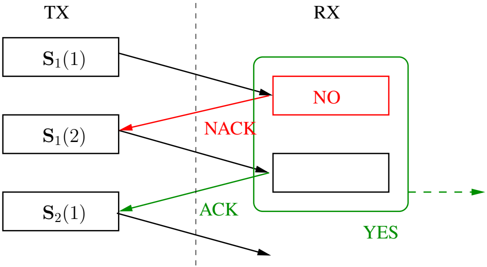
- Efficiency is upper-bounded by the code rate

**Drawbacks**

- Each received packet is treated independently
- Mis-decoded packet is thrown in the trash

# Type-II HARQ

Memory at RX side is considered ⇒ Type-II HARQ




Main examples :

- *Chase Combining* (CC)
- *Incremental Redundancy* (IR)

# Examples : CC-HARQ and IR-HARQ

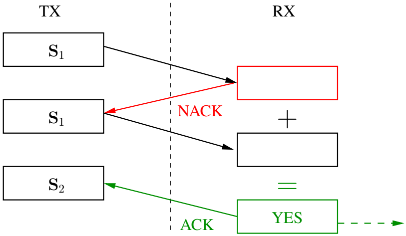
**CC**

$$\begin{aligned} Y_1 &= S_1 + N_1 \\ Y_2 &= S_1 + N_2 \end{aligned}$$

then detection on

$$Y = (Y_1 + Y_2)/2$$

SNR-Gain equal to 3dB



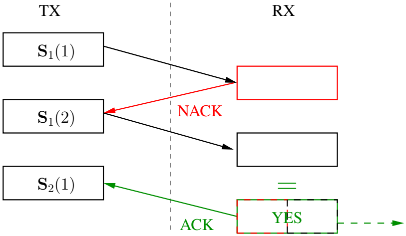

**IR**

$$\begin{aligned} Y_1 &= S_1(1) + N_1 \\ Y_2 &= S_1(2) + N_2 \end{aligned}$$

then detection on

$$Y = [Y_1, Y_2]$$

Coding gain



# Performance metrics

- **Packet Error Rate** (PER) :

$$\text{PER} = \text{Prob(information packet is not decoded)}$$

- **Efficiency** (*Throughput*/*Goodput*/etc) :

$$\eta = \frac{\text{information bits received without error}}{\text{transmitted bits}}$$

- **(Mean) delay** :

$$d = \#\ \text{transmitted packets when information packet is received}$$

- **Jitter** :

$$\sigma_d = \text{delay standard deviation}$$

## Quality of Service (QoS)

- Data : PER and efficiency
- Voice on IP : delay
- Vidéo Streaming : efficiency and jitter

# Closed-form expressions for metrics

$$\begin{aligned}
\text{PER} &= 1 - \sum_{k=1}^{L} p(k) \\
\eta &\propto \frac{\sum_{k=1}^{L} p(k)}{L(1 - \sum_{k=1}^{L} p(k)) + \sum_{k=1}^{L} kp(k)} \\
d &= \frac{\sum_{k=1}^{L} kp(k)}{\sum_{k=1}^{L} p(k)} \\
\sigma_d &= \sqrt{\frac{\sum_{k=1}^{L} k^2 p(k)}{\sum_{k=1}^{L} p(k)} - d^2}
\end{aligned}$$

with [Duc12]

- $p(k)$ probability to receive information packet in exactly $k$ transmissions
- $L$ maximum number of transmissions per information packet

# Example : Type-I HARQ

- Let $\pi_0$ be the probability the information packet is mis-decoded (in one transmission). Then

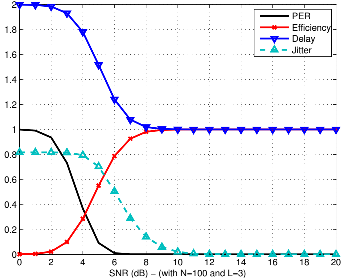
$$p(k) = (1 - \pi_0)\pi_0^{k-1}$$

- Let an information packet composed by $N$ BPSK uncoded symbols. Then

$$\pi_0 = 1 - \left(1 - Q\left(\sqrt{2\text{SNR}}\right)\right)^N$$

## Results

$$\begin{aligned} \text{PER} &= \pi_0^L \\ \eta &= 1 - \pi_0 \\ d &= L + \frac{1}{1-\pi_0} - \frac{L}{1-\pi_0^L} \\ \sigma_d &= \text{too long} \end{aligned}$$



# Part 2 : HARQ based resource allocation

# 2.1 - Trade-off between HARQ and physical layer

If PHY layer is designed in advance based on averaged channel performance

- Amount of retransmission is quite weak : only when channel realization is bad
- but resource is wasted : when channel realization is good

**What is the best trade-off between retransmission and physical layer ?**

If HARQ is employed

- what is the PHY layer performance leading to the maximum rate/efficiency
- where the PHY layer performance is described through $p(k)$

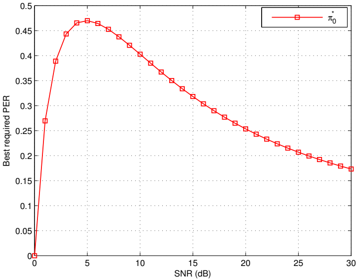
# Example : Type-I HARQ in Rayleigh channel

- Each packet is encoded with coding rate $R$

$$\pi_0 = \text{Prob}_h(\log_2(1+|h|^2 SNR) < R) \Leftrightarrow R_{\pi_0} = \log_2\left(1 + \frac{SNR}{\frac{1}{\log\left(\frac{1}{1-\pi_0}\right)}}\right)$$

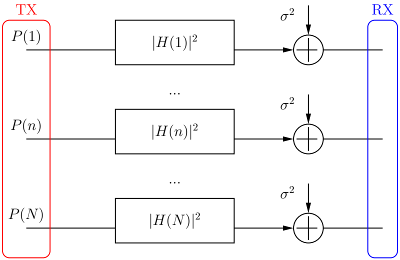
- Best PHY layer is given by [Jin09]

$$\pi_0^* = \arg\max_{\pi_0} \underbrace{R_{\pi_0}(1-\pi_0)}_{\eta_{\pi_0}\text{: efficiency}}$$




Reliable PHY layer is not required at all (thanks to retransmission) !

# 2.2 - Waterfilling-like algorithm




- Data rate maximization
- Power constraint :

$$\sum_{n=1}^{N} P(n) = P_{\max}$$

  with maximum power $P_{\max}$.
- Perfect CSIT

**Problem : which data rate criterion to be used ?**

- (Shannon) sum-capacity : $[P(1)^*, \cdots, P(N)^*] =$
  $$\arg\max_{P(1),\cdots,P(N)} \sum_{n=1}^{N} \log_2\left(1 + |H(n)|^2 \frac{P(n)}{\sigma^2}\right)$$
- (HARQ) sum-efficiency : $[P(1)^*, \cdots, P(N)^*] =$
  $$\arg\max_{P(1),\cdots,P(N)} \sum_{n=1}^{N} m(n) \cdot (1 - P_e(n))$$
  with $2^{m(n)}$-QAM and $P_e(n)$ SER (packet=symbol, here).

# Practical algorithms

Convex optimization problem ($\Rightarrow$ KKT conditions)

## Capacity : "Waterfilling" [Sha48]

$$P(n)^* = \left(\nu - \frac{\sigma^2}{|H(n)|^2}\right)^+$$

with

- $\nu$ chosen s.t. $\sum_{n=1}^N P(n)^* = P_{\max}$.
- $(\bullet)^+ = \max(0, \bullet)$.

## Efficiency : [unpublished]

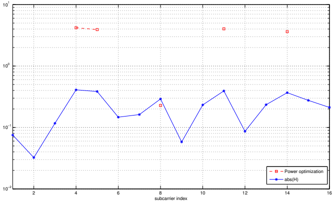
$$P(n)^* = \left(\frac{2\sigma^2}{\gamma(n)|H(n)|^2} \log\left(\frac{m(n)\gamma(n)|H(n)|^2}{\mu\sigma^2}\right)\right)^+$$

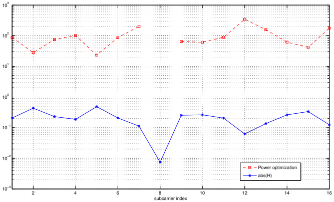
with

- $\mu$ chosen s.t. $\sum_{n=1}^N P(n)^* = P_{\max}$.
- $\gamma(n)$ modulation gain at channel $\#n$

# Numerical illustrations

$N = 16$, $SNR = 0dB$ (left), $SNR = 20dB$ (right)




## Remark

The best subcarriers do not have necessary the highest powers (at high SNR)

# 2.3 - Statistical CSIT based resource allocation

**Channel statistics known at the transmitter**

– fast-varying Rayleigh fading channel
– costly to report instantaneous channel realizations
– cheap to report statistics due to its coherence time

**HARQ to handle channel variation and mis-knowledge**

## Applications

- Mobile Ad Hoc networks (MANET)
- Cellular networks with high mobility

# Communication model

- **PHY layer**
  - OFDMA : cancel ISI due to multipath spread
  - Multiple access : cancel multiuser interference within a cell

- **Statistical channel model (for the $k$-th link)**
  - Let $h_k(m)$ be the $m$-th filter tap.
    *Independent but not identically distributed* $\sim \mathcal{CN}(0, \varsigma_{k,m}^2)$
  - Let $H_k(n)$ be the $n$-th Fourier component
    *non-independent wrt n but identically distributed* $\sim \mathcal{CN}(0, \varsigma_k^2)$ *with* $\varsigma_k^2 = \sum_m \varsigma_{k,m}^2$

$\Rightarrow$ Subcarriers are statistically equivalent
$\Rightarrow$ Rayleigh fading channel

**Consequence for the $k$-th link**

Bandwidth proportion and identical energy per subcarrier

# Resource allocation issue

- $Q_k$ : Energy of link $k$ in OFDM symbol
- $\gamma_k$ : Bandwidth proportion assigned to link $k$
- $E_k$ : Energy of link $k$ in entire bandwidth $\Rightarrow \quad Q_k = \gamma_k E_k$
- Modulation (order $2^{m_k}$) and coding scheme (rate $R_k$)

## Goal [Mar12,Ksa14]

$$\min \sum_{k=1}^{K} Q_k \quad \Leftrightarrow \min_{(\gamma, \boldsymbol{E})} \sum_{k=1}^{K} \gamma_k E_k$$

$$\text{s.t.} \qquad \mathbf{QoS}_k(\gamma_k, E_k) \geq \mathbf{QoS}_k^{(0)}, \ \forall k$$

$$\sum_{k=1}^{K} \gamma_k \leq 1$$

$$\gamma_k \geq 0, \ E_k \geq 0, \ \forall k$$

for various Quality of Service (QoS) : **eff., eff.+PER, ...**

# Type-II HARQ case

## Efficiency

$$\eta_k(\gamma_k, E_k) = m_k R_k \gamma_k \frac{1 - q_{k,L}(\mathrm{SNR}_k)}{1 + \sum_{\ell=1}^{L-1} q_{k,\ell}(\mathrm{SNR}_k)}$$

with

- $q_{k,\ell}$ probability the first $\ell$ transmissions of a HARQ all in error
- $\mathrm{SNR}_k \propto E_k$
- $q_{k,\ell}$ can be upper-bounded by $\pi_{k,l}$ with $\pi_{k,\ell}$ probability for not decoding information packet based on the first $\ell$ transmissions. [Duc12]

- Bit-Interleaved Coded Modulation (BICM)
- Random Subcarrier Assignment Scheme (SAS)

$$\pi_{k,\ell}(\mathrm{SNR}_k) \approx \frac{g_{k,\ell}(m_k, R_k)}{\mathrm{SNR}_k^{d_{k,\ell}(R_k)}}$$

where $d_{k,\ell}$ represents a minimal Hamming distance and the diversity.

# Optimization problem

$$\min_{\gamma, \boldsymbol{E}} \sum_{k=1}^{K} \gamma_k E_k$$

$$\text{s.t.} \quad \gamma_k \frac{1 - \pi_{k,L}(\mathrm{SNR}_k)}{1 + \sum_{\ell=1}^{L-1} \pi_{k,\ell}(\mathrm{SNR}_k)} \geq \frac{\eta_k^{(0)}}{m_k R_k}, \forall k$$

$$\sum_{k=1}^{K} \gamma_k \leq 1$$

$$\gamma_k > 0, E_k > 0, \forall k$$

## Results

- It is a *geometric program* [Ksa14]
- Transformation into a convex optimization problem by $\gamma_k = e^{x_k}$ and $E_k = e^{y_k}$
- KKT solved

# Algorithm

$\lambda \leftarrow 0$

REPEAT

FORALL $k \in \{1, 2, \ldots, K\}$

$\gamma_k \leftarrow \frac{\eta_k^{(0)}}{m_k R_k} F_k(\lambda)$

$E_k \leftarrow G_k(\lambda)$

ENDFOR

Increment $\lambda$

UNTIL $\sum_{k=1}^{K} \gamma_k \leq 1$

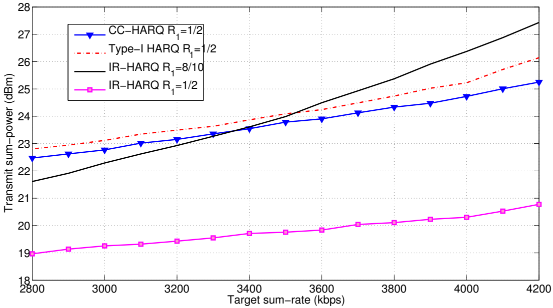
$\{\gamma_k^* = \gamma_k\}_{k=1\ldots K}$

$\{E_k^* = E_k\}_{k=1\ldots K}$

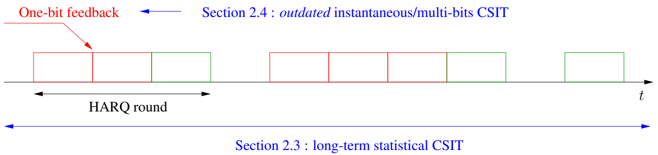
where $F_k$ and $G_k$ are well defined in closed-form.

# Numerical results

- $K = 10$ links, Bandwidth $W = 5$ MHz , QPSK
- Feasability condition satisfies for sum rates up to 5 Mbps.



# 2.4 - Outdated CSIT based resource allocation




- At each retransmission, instantaneous CSIT (via multi-bits feedback)
- Problem : outdated CSIT

**Goal [Taj13]**

How allocating power between retransmission packets based on outdated instantaneous CSIT under long-term power constraint

# Mathematical tools

- Generic expressions from Slide 9 can not be used since
  - long-term power constraints is not round per round,
  - and instantaneous CSIT can not be incorporated
- An other tool has to be used : *Markov Decision Process (MDP)*

**Go back to the original problem**

After the *n*-th transmission, if NACK is received, we have

- New action $A_{n+1}$ to do : here choosing $P_{n+1}$
- Available information : accumulated mutual information equal to $I_n = \sum_{\ell=1}^{n} \log_2(1 + G_\ell P_\ell)$ if IR-HARQ
- $K_n \in \{$ACK and new round, 1 attempt and NACK received, $\cdots$ , $L$ attempts and NACK received$\}$

$$P(K_{n+1}, I_{n+1} | K_n, I_n, \cdots) = P(K_{n+1}, I_{n+1} | K_n, I_n)$$

$\Rightarrow$ **We have a Markov Chain :** $S_n = (K_n, I_n)$

# Optimization problem

**A policy $\pi$ = how selecting $P_{n+1}$ given $S_n$**

- deterministic : $\pi(\bullet|S_n) = \delta(\bullet - f(S_n))$ with function $f$
- random : $\pi(\bullet|S_n) = p(\bullet|S_n)$ with pdf $p$

**Optimisation issue**

$$\pi^* = \arg\max_{\pi} \eta_\pi, \text{ with } \eta_\pi = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \underbrace{reward_\pi(S_n, A_n)}_{\#\text{information bits after ACK}}$$

s.t.

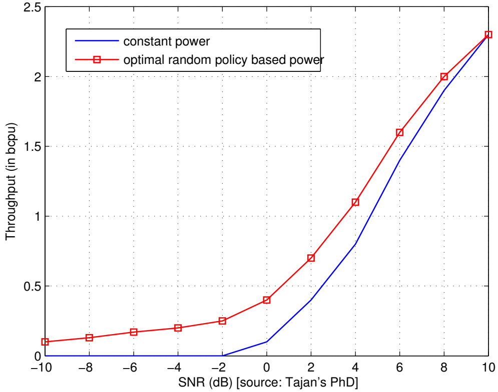
$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} P(S_n, A_n) \leq P_{\max}$$

## Solution

- Under mild conditions, optimal random policy exists
- Optimal pdf leads to linear programming

# Numerical results

Setup : IR-HARQ, $L = 5$, Rayleigh channel



# Open issues

## Single-user

Correlated channels : a mixture of outdated and known channels

## Multi-user

- Subcarrier allocation (number of states too huge)
- Performance on real MCS (not information-theoretic tools)

# References

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[Ksa14] N. Ksairi, P. Ciblat, and C. Le Martret, "Near-Optimal Resource Allocation for Type-II HARQ-based OFDMA Ad Hoc Networks under rate and power constraints," IEEE Trans. on Wireless Communications, Oct. 2014.

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