Near-Optimal Resource Allocation for Type-II HARQ based OFDMA Networks under Rate and Power Constraints

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***Abstract*—We address the problem of multiuser power and bandwidth allocation for a general class of OFDMA-based wireless networks that employ a Type-II Hybrid Automatic Repeat reQuest (HARQ) mechanism along with practical Modulation and Coding Schemes (MCSs). This problem is formulated as minimizing the sum transmit power required to satisfy individual goodput constraints without exceeding maximum allowable per-link or per-node transmit power levels. We assume that the resource manager has only statistical knowledge of the Channel State Information (CSI) of the Rayleigh-distributed fast-fading links of the network. Using a tight approximation of the goodput, we propose an algorithm allowing to compute the corresponding optimal resource allocation. We finally provide an efficient selection of the different MCSs that can be coupled with the proposed resource allocation algorithm to significantly boost its performance.**

I. Introduction

To cope with the growing demands in terms of spectral efficiency and quality of Service (QoS), modern wireless network standards include a combination of advanced physical and link layer techniques such as Orthogonal Frequency Division Multiple Access (OFDMA), adaptive MCS, Bit-Interleaved Coded Modulation (BICM), and HARQ. Both HARQ and adaptive MCS are powerful mechanisms that allow reliable communications over time-varying channels. Among the different HARQ schemes, the so-called Type-II, which includes Chase Combining (CC-HARQ) and Incremental Redundancy (IR-HARQ) [1], is the most promising in terms of performance. In addition, a random Subcarrier Assignment Scheme (SAS) along with BICM allows to harvest the inherent diversity in wireless links while OFDMA allows to properly handle the multi-path and the multi-user interference. The benefits of the above-mentioned techniques are not limited to infrastructure-based networks but extend to ad hoc configurations too.

This article deals with sum transmit power minimization for wireless networks using OFDMA, BICM and practical MCSs at the physical layer and Type-II HARQ at the link layer. This problem is of great interest for two reasons. First, transmit power minimization is crucial to reduce energy consumption and to minimize the impact produced by the network on other systems through interference. Second, Type-II HARQ (as opposed to the simpler Type-I) is a very promising link-layer mechanism as we already mentioned. Although the above problem arises in a wide class of wireless systems, we hereafter rather focus on ad hoc networks. In such networks, generally a node called "resource manager" is elected to perform the resource allocation even if pairwise communications are allowed. The delay between the initiation of a specific link and the reception by the resource manager of the CSI feedback associated with that link may span several frame periods. Consequently, the resource manager has only outdated CSI whereas it can have accurate *statistical* CSI due to the much larger coherence time of the latter. We therefore assume that the resource allocation should be done with only statistical CSI about the different wireless links.

In order for our system model to be compatible with any network topology, we assume that each node can potentially establish more than one outgoing communication link. Either Time-Division Multiplexing (TDM) or Orthogonal Frequency-Division Multiplexing (OFDM) can be used to enable simultaneous operation of the links coming out of the same node. Moreover, real-world wireless communication systems place practical limitations on the operation of the transmitter of each node. In this article, we consider two of these limitations [2], namely, either *i)* a constraint on *the long-term average transmit power* due to limited battery life or *ii)* a constraint on *the short-term average transmit power* imposed either by regulations or to avoid nonlinearities of the power amplifiers. We show in Section III that the first limitation gives rise to a *per-node transmit power constraint* while the second can be equivalent either to a per-node or to a *per-link transmit power constraint* depending on the applied multiplexing technique.

In the literature, only few works *e.g.,* [3]-[11], have addressed the problem of multiuser resource allocation for communication systems utilizing HARQ, and none of them considered individual power constraints, to the best of our knowledge. In [3], a *rate-adaptive* optimization problem is addressed where the objective is to determine user scheduling and resource allocation that maximize the sum of the individual information-theoretic data rates in an IR-HARQ based network under the assumption of perfect CSI. The problem is simplified by separately performing power control and bandwidth assignment leading thus to a suboptimal solution. The authors of [4] address the problem of average sum transmit power minimization in a network using IR-HARQ under maximum average delay constraints. Again, perfect CSI is assumed and practical MCSs are not taken into account. In

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[5] and [6], the system goodput is maximized with respect to user selection and power and rate allocation for a system employing Type-II HARQ and outdated CSIT. However, due to the user selection, no more than one user can be scheduled at any given time. Moreover, the system goodput is computed using an information-theoretical approach that fails to account for practical MCSs. This last limitation also applies to [7]. In [8], perfect CSIT and Type-I HARQ are considered along with practical MCSs. Nevertheless, the proposed resource allocation is suboptimal since subchannel assignment and power allocation are not jointly optimized. A heuristic suboptimal resource allocation scheme is proposed in [9] for multiuser HARQ-based uplink communications in Single Carrier (SC)-FDMA systems. In the context of cognitive radio, some works have been devoted to resource allocation for secondary users when HARQ is employed [10]. Finally, in [11], transmit power minimization is done in presence of statistical CSIT and practical MCS but only for Type-I HARQ and without individual transmit power constraints. *In this paper, our main contribution is to address the previous problem in the context of Type-II HARQ while also adding per-node or per-link transmit power constraints*. It is worth mentioning that the extension from Type-I to Type-II HARQ is not straightforward since the closed-form expressions for the performance metrics of the latter are much more complicated. The same difficulty arises from adding the individual power constraints.

It is worth noting that several works in the literature (*e.g.,* [12]-[15]) have addressed power and rate allocation for HARQ schemes but in a single-user context where the objective is to adapt transmit power and coding rate over the different retransmissions of the same HARQ process. This problem, while important, is not within the scope of our paper since the gain is very small when no perfect CSIT is available [12].

The rest of the article is organized as follows. The system model is depicted in Section II. In Section III, the resource allocation problem is mathematically formulated for both the cases of per-node or per-link transmit power constraints. The solution to these two problems is then analytically derived in Sections IV and V, respectively. The issue of MCS selection is next addressed in Section VI. Numerical results are presented in Section VII, while conclusions are finally provided in Section VIII.

II. SYSTEM MODEL

A. Channel model

We focus on a single-cluster network with K active nodes. One of these nodes is the resource manager which performs the proposed resource allocation algorithm. Each node ($k \in \{1, 2, \ldots, K\}$) has $I_k \geq 1$ outgoing active links. From now on, we use the couple (k, i) to designate the ith ($i \in \{1, \ldots, I_k\}$) link of node k. Each of these $\sum_{k=1}^{K} I_k$ links is considered as a time-varying frequency-selective channel whose M time-domain taps are Rayleigh distributed. It is assumed that OFDM (with N subcarriers covering a total bandwidth of W Hz) is employed and that channels remain constant over one OFDM symbol but change independently between consecutive OFDM symbols.

Let $\mathbf{h}_{k,i}(j) = [h_{k,i}(j, 0), \ldots, h_{k,i}(j, M-1)]^{\mathrm{T}}$ be the M-long channel impulse response of link (k, i) associated with OFDM symbol j where the superscript $(.)^{\mathrm{T}}$ stands for the transposition operator. The multi-variate complex-valued circular symmetric Gaussian distribution with mean $\mathbf{a}$ and covariance matrix $\mathbf{\Sigma}$ is hereafter denoted by $\mathcal{CN}(\mathbf{a}, \mathbf{\Sigma})$. Let $\mathbf{H}_{k,i}(j) = [H_{k,i}(j, 0), \ldots, H_{k,i}(j, N-1)]^{\mathrm{T}}$ be the discrete Fourier transform of $\mathbf{h}_{k,i}(j)$ *i.e.,* $H_{k,i}(j, n) = \sum_{m=0}^{M-1} h_{k,i}(j, m) e^{-2\pi\imath\frac{nm}{N}}$ for $n = 0, 1, \ldots, N-1$. The received signal associated with OFDM symbol j for link (k, i) at subcarrier n is

$$Y_{k,i}(j, n) = H_{k,i}(j, n) X_{k,i}(j, n) + Z_{k,i}(j, n)\,, \qquad (1)$$

where $X_{k,i}(j, n)$ is the symbol transmitted at subcarrier n of the jth OFDM symbol on link (k, i), and where $Z_{k,i}(j, n) \sim \mathcal{CN}(0, N_0 W/N)$ is an additive noise and N_0 is the noise level in the power spectral density. It is assumed that the time-domain channel taps $\{h_{k,i}(j, m)\}_{j, 0 \leq m \leq M-1}$ are independent random variables with variances $\varsigma^2_{k,i,m}$ that are constant w.r.t. the OFDM symbol index j but which possibly vary from tap to tap, *i.e.,* $\mathbf{h}_{k,i}(j) \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_{k,i})$ with $\mathbf{\Sigma}_{k,i} \stackrel{\text{def}}{=} \mathrm{diag}_{M\times M}(\varsigma^2_{k,i,0}, \ldots, \varsigma^2_{k,i,M-1})$. The subcarriers of a single link are thus identically distributed as $H_{k,i}(j, n) \sim \mathcal{CN}(0, \varsigma^2_{k,i})$ where $\varsigma^2_{k,i} = \mathrm{Tr}(\mathbf{\Sigma}_{k,i})$. We define the average gain-to-noise ratio for link (k, i) as

$$G_{k,i} \stackrel{\text{def}}{=} \frac{\mathbb{E}\left[|H_{k,i}(j, n)|^2\right]}{N_0} = \frac{\varsigma^2_{k,i}}{N_0}\,. \qquad (2)$$

B. The HARQ mechanism and other link-layer assumptions

At the Medium Access Layer (MAC), each active link (k, i) receives an infinite stream of information bits coming from the upper layer while arranged in packets of n_b bits each. A Type-II HARQ scheme is then used to transmit each information packet in at most L transmissions. The content of each one of these L transmissions is called a MAC Packet (MP). Notice that the power used by the system does not depend on the number of retransmissions really used by the HARQ mechanism since there is always a MP to be sent. We assume that the MPs produced by the different I_k links coming out of a node k are multiplexed either *i)* by assigning them non-overlapping subsets of subcarriers of each OFDM symbol, or *ii)* by letting each link use all the subcarriers available to its node till the end of transmission of its current MP before passing them over to the next link of the node in a *Round-Robin* TDM manner.

We examine two possible Type-II HARQ schemes: *i) CC-HARQ:* The MP is obtained by encoding the information packet with a Forward Error Correcting (FEC) code of rate $R_{k,i}$. At the end of each transmission $1 \leq l \leq L$, the receiver combines the so-far received l MPs according to the *maximum ratio combining* principle [16]. *ii) IR-HARQ:* The information packet is firstly encoded by a FEC code of rate $R_{k,i}/L$ (known as the mother code). The resulting codeword is then split into L MPs by following the *rate compatible coding* principle [17]. We assume identical lengths for the MPs. After the reception of the lth MP, the receiver tries to decode the information packet by concatenating the l received MPs.

At the physical layer, we assume that the symbols transmitted on any link (k,i) are chosen from a $2^{m_{k,i}}$-QAM constellation. The MCS associated with link (k,i) can thus be represented by the couple $(m_{k,i}, R_{k,i})$.

Let $\mathcal{E}_{k,i,l}$ be the event that decoding the information packet based on the first l MPs results in an error and define $\pi_{k,i,l} \stackrel{\text{def}}{=} \mathbb{P}\{\mathcal{E}_{k,i,l}\}$. We assume that BICM along with a random SAS are utilized by all the wireless links. If these two techniques are well tuned to the coherence time of the channels, the links can be considered as fast fading. Consequently, if Gray mapping and convolutional codes are used, then we have

$$\pi_{k,i,l} \leq \tilde{\pi}_{k,i,l} \stackrel{\text{def}}{=} \frac{g_{k,i,l}}{\mathrm{SNR}_{k,i}^{d_{k,i,l}}} , \tag{3}$$

where $g_{k,i,l}$ is the smallest value that ensures the upper-bound property. It can be determined using simulations and curve fitting. Obviously, $g_{k,i,l}$ depends on the modulation order $m_{k,i}$, on the coding rate $R_{k,i}$ and on the particular HARQ scheme in use. Moreover the upper-bound in Eq. (3) is tight for medium-to-high SNRs. The term $d_{k,i,l}$ corresponds to the diversity order and is either *i)* the minimal Hamming distances of the codes associated with transmissions $l = 1, \ldots, L$ in IR-HARQ case or *ii)* equal to $l d_{k,i,1}$ in CC-HARQ case [19, Section V]. Eq. (3) comes from either a straightforward adaptation of [18, Eq.(65)] to frame error rate instead of bit error rate or a direct application of [19, Eq.(21)]. A table that maps each MCS $(m_{k,i}, R_{k,i})$ to the set $\{(d_{k,i,l}, g_{k,i,l})\}_{l=1\ldots L}$ is assumed to be available to the resource manager.

C. Power and bandwidth parameters

The resource manager is assumed to only know the average gains $G_{k,i}$. Since each of these gains is subcarrier-independent, we cannot decide which subset of subcarriers a link or a node should use, but only how many. In the following, we denote by n_k ($k \in \{1, \ldots, K\}$) the number of subcarriers assigned to node k. For a node that has only one outgoing link ($I_k = 1$), the occupied portion of the bandwidth is $\gamma_k \stackrel{\text{def}}{=} \frac{n_k}{N}$. As for a node k that has more than one link ($I_k > 1$), either OFDM or TDM should be used to schedule transmission on these links, as we mentioned in Section I. Denote by $n_{k,i} \leq n_k$ the number of subcarriers assigned to link (k,i) out of the available n_k subcarriers and by $\gamma_{k,i}$ the bandwidth proportion occupied by the link. In the case where OFDM is used for link scheduling, $\gamma_{k,i}$ can be expressed as $\frac{n_{k,i}}{N}$. As for the case where node k uses Round-Robin TDM instead of OFDM, the n_k subcarriers of the node are available to each link (k,i) for $1/I_k$ of the time. In other words, the proportion $\gamma_{k,i}$ occupied by link (k,i) of the available time-frequency units can be expressed as $\frac{n_k}{I_k N}$. Irrespective of the multiplexing method, $\gamma_{k,i}$ can thus be seen as the bandwidth parameter of link (k,i) to be optimized. For the sake of tractability, we consider from now on that $\gamma_{k,i}$ can take any value in $(0,1)$.

Since $G_{k,i}$ is subcarrier-independent, it is natural to transmit with the same average power $P_{k,i} \stackrel{\text{def}}{=} \mathbb{E}\left[|X_{k,i}(j,n)|^2\right]$ on all the $n_{k,i}$ subcarriers. Let $E_{k,i} \stackrel{\text{def}}{=} \frac{P_{k,i}}{W/N}$ be the energy consumed to transmit one symbol on one subcarrier of (k,i) and define $\sigma_{k,i}^2 \stackrel{\text{def}}{=} N_0 W/N$. Note that the energy consumed on link (k,i) to send its part of the OFDM symbol is $N\gamma_{k,i}E_{k,i}$. As energy is proportional to power, we refer to $\{E_{k,i}\}_{1\leq k\leq K, 1\leq i\leq I_k}$ as the transmit power parameters or shortly as the "transmit powers" by abuse of terminology. Finally, each subcarrier of (k,i) has an average signal-to-noise ratio (SNR) equal to

$$\mathrm{SNR}_{k,i} \stackrel{\text{def}}{=} \frac{\varsigma_{k,i}^2 P_{k,i}}{\sigma_{k,i}^2} = G_{k,i}E_{k,i} . \tag{4}$$

III. Resource allocation optimization problems with fixed MCS

We consider for the moment that the MCSs of the different links are fixed in advance. The selection of these MCSs is dealt with in Section VI. Our goal is to minimize the total average transmit power proportional to $\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}$ by jointly optimizing the power and the bandwidth parameters $\{E_{k,i}, \gamma_{k,i}\}_{1\leq k\leq K, 1\leq i\leq I_k}$, while an individual maximum allowable transmit power is respected and while a minimum goodput $\eta_{k,i}^{(0)}$ is guaranteed for each link (k,i). The goodput corresponds to the average number per channel use of information bits in successfully-decoded packets, and is denoted for link (k,i) by $\eta_{k,i}$. Mathematically speaking, we have

$$\begin{aligned}\eta_{k,i} &= \lim_{l\to\infty} \frac{\text{number of successfully decoded bits up to } l}{\text{number of channel uses up to } l} \\ &= \lim_{l\to\infty} \frac{b(l)}{l\frac{n_b}{m_{k,i}R_{k,i}}} ,\end{aligned} \tag{5}$$

where l designates with a slight abuse of notation the number of MAC transmissions carried out so far, and $b(l)$ is the total number of information bits in the correctly decoded packets up to the lth transmission. As for the transmit power constraint, it should be made explicit depending on the relevant situation among the following possible situations:

1) The main concern is to limit the *long-term* average power of each node in order to maximize its battery life. In case OFDM is used for link scheduling, we define the long-term average power of a node as the mean of its transmit power on a frame of N_f OFDM symbols given by $\frac{1}{N_f NT} N_f \sum_{i=1}^{I_k} n_{k,i}E_{k,i} = \frac{1}{T}\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}$, where $T \stackrel{\text{def}}{=} 1/W$ and where $N_f \sum_{i=1}^{I_k} n_{k,i}E_{k,i}$ is the energy spent by node k during the transmission of the $N_f NT$-seconds long frame. As for the TDM case, we take into account Round-Robin scheduling by defining the long-term average power of a node as its mean transmit power on a frame of $I_k N_f$ OFDM symbols equal to $\frac{1}{N_f I_k NT} N_f \sum_{i=1}^{I_k} n_k E_{k,i} = \frac{1}{T}\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}$, where $n_k E_{k,i}$ is the energy spent by node k during one OFDM symbol assigned to link (k,i). Therefore, the relevant long-term average power constraint is always a *per-node constraint* given by
$$\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i} \leq Q_k^{(0)} . \tag{6}$$
2) The main concern is that the *short-term* (per OFDM symbol) average power of each node does not exceed a certain level imposed by regulations and/or to avoid

nonlinearities in the power amplifier. If a node k uses OFDM to schedule its own links, the short-term average power is given by $\frac{1}{T}\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}$ and the corresponding constraint is the same as in Eq. (6). If TDM is assumed instead, the short-term average power depends on the current scheduled link and is given for link (k,i) by $\frac{1}{T}\gamma_{k,i}E_{k,i}$. This leads to the following I_k *per-link constraints*:

$$\gamma_{k,i}E_{k,i} \le Q_k^{(0)}\ , \forall i \in \{1,\dots,I_k\}\ . \tag{7}$$

Therefore, the optimization issue of interest writes as follows.

Problem 1. *The general optimization problem is:*

$$\min_{\gamma_{1,1}\dots\gamma_{1,I_1},\gamma_{K,1}\dots\gamma_{K,I_K},E_{1,1}\dots E_{1,I_1},E_{K,1}\dots E_{K,I_K}} \sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i} \tag{8a}$$

subject to

Goodput constraints: $\eta_{k,i} \ge \eta_{k,i}^{(0)}$, (8b)
$\forall k \in \{1,\dots,K\}, i\in\{1,\dots,I_k\}$,

Bandwidth constraint: $\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i} \le 1$, (8c)

Power constraints: *either Eq.*(6) *or Eq.*(7), $\forall k \in \{1,\dots,K\}$, (8d)

Positivity constraints: $\gamma_{k,i} > 0, E_{k,i} > 0$, (8e)
$\forall k \in \{1,\dots,K\}, i\in\{1,\dots,I_k\}$.

According to [16], we know that the goodput for any Type-II HARQ writes as:

$$\eta_{k,i} = m_{k,i}R_{k,i}\gamma_{k,i}\frac{1-q_{k,i,L}}{1+\sum_{l=1}^{L-1}q_{k,i,l}}\ , \tag{9}$$

where $q_{k,i,l} \overset{\text{def}}{=} \mathbb{P}\{\mathcal{E}_{k,i,1},\mathcal{E}_{k,i,2},\dots,\mathcal{E}_{k,i,l}\}$ is the probability that the first l transmissions of a HARQ process are all received in error. The factor $\gamma_{k,i}$ in the Right-Hand Side (RHS) of Eq. (9) reflects the fact that the goodput is proportional to the number of the assigned subcarriers. It is difficult to get $q_{k,i,l}$ in closed-form. However, $q_{k,i,l} = \mathbb{P}\{\mathcal{E}_{k,i,1},\mathcal{E}_{k,i,2},\dots,\mathcal{E}_{k,i,l}\} \le \mathbb{P}\{\mathcal{E}_{k,i,l}\} = \pi_{k,i,l}$ and $\pi_{k,i,l}$ can furthermore be upper-bounded by $\tilde{\pi}_{k,i,l}$ which has an analytical expression given in Eq. (3). Therefore, $q_{k,i,l} \le \tilde{\pi}_{k,i,l}$. We have checked that this upper-bound is tight for practical values of L and for medium-to-high SNR values. The goodput $\eta_{k,i}$ can thus be lower-bounded as follows:

$$\eta_{k,i} \ge m_{k,i}R_{k,i}\gamma_{k,i}\frac{1-\tilde{\pi}_{k,i,L}}{1+\sum_{l=1}^{L-1}\tilde{\pi}_{k,i,l}}\ . \tag{10}$$

We thus slightly modify Problem 1 by replacing the Left-Hand Side (LHS) of Eq. (8b) with the RHS of Eq. (10) and by using the closed-form expression of $\tilde{\pi}_{k,i,l}$. First, we focus on the case with per-link transmit power constraints.

Problem 2. *The optimization problem with* **per-link** *power constraints becomes:*

$$\min_{\gamma_{1,1}\dots\gamma_{1,I_1},\gamma_{K,1}\dots\gamma_{K,I_K},E_{1,1}\dots E_{1,I_1},E_{K,1}\dots E_{K,I_K}} \sum_{k=1}^{K}\gamma_{k,i}E_{k,i} \tag{11a}$$

subject to

$$\gamma_{k,i}\frac{1-g_{k,i,L}/(G_{k,i}E_{k,i})^{d_{k,i,L}}}{1+\sum_{l=1}^{L-1}g_{k,i,l}/(G_{k,i}E_{k,i})^{d_{k,i,l}}} \ge \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}}, \tag{11b}$$
$$\forall k \in \{1,\dots,K\}, i\in\{1,\dots,I_k\}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i} \le 1\ , \tag{11c}$$

$$\gamma_{k,i}E_{k,i} \le Q_{k,i}^{(0)}, \forall k \in \{1,\dots,K\}, i\in\{1,\dots,I_k\}\ , \tag{11d}$$

$$\gamma_{k,i} > 0, E_{k,i} > 0, \forall k \in \{1,\dots,K\}, i\in\{1,\dots,I_k\}\ . \tag{11e}$$

It is quite obvious that Problem 2 is feasible if and only if forcing each link (k,i) to consume a power $\gamma_{k,i}E_{k,i} = Q_{k,i}^{(0)}$ results in a feasible problem. Setting $E_{k,i} = Q_{k,i}^{(0)}/\gamma_{k,i}$, constraint (11b) writes as $\mathcal{G}_{k,i,Q_{k,i}^{(0)}}(\gamma_{k,i}) \ge 0$ where

$$\gamma \mapsto \mathcal{G}_{k,i,Q_{k,i}^{(0)}}(\gamma) \overset{\text{def}}{=} m_{k,i}R_{k,i}\gamma\frac{1-g_{k,i,L}/\left(G_{k,i}Q_{k,i}^{(0)}/\gamma\right)^{d_{k,i,L}}}{1+\sum_{l=1}^{L-1}g_{k,i,l}/\left(G_{k,i}Q_{k,i}^{(0)}/\gamma\right)^{d_{k,i,l}}} - \eta_{k,i}^{(0)}\ . \tag{12}$$

It is easy to verify that $\mathcal{G}_{k,i,Q_{k,i}^{(0)}}(0) < 0$. Two cases are thus possible. If $\mathcal{G}_{k,i,Q_{k,i}^{(0)}}$ has no zeros on $(0,1]$, then $\mathcal{G}_{k,i,Q_{k,i}^{(0)}}(\gamma) < 0, \forall\gamma \in (0,1]$, meaning that constraint (11b) cannot be satisfied. Otherwise if $\mathcal{G}_{k,i,Q_{k,i}^{(0)}}$ has a zero or more on $(0,1]$, we should use the smallest of them to test the feasibility of Problem 2. This leads to Lemma 1 where we defined:

$$\gamma_{k,i,Q_{k,i}^{(0)}}^{(0)} \overset{\text{def}}{=} \begin{cases} +\infty\ , \text{if } \mathcal{G}_{k,i,Q_{k,i}^{(0)}} \text{ has no zeros on } (0,1]\ , \\ \text{the smallest zero of } \mathcal{G}_{k,i,Q_{k,i}^{(0)}}\ , \text{otherwise.} \end{cases} \tag{13}$$

Lemma 1. *Problem 2 is feasible if and only if* $\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i,Q_{k,i}^{(0)}}^{(0)} \le 1$*. Moreover, this inequality implies that Slater's condition holds.*

Now, we turn our attention to the case with per-node transmit power constraints.

Problem 3. *The optimization problem with* **per-node** *power* *constraints becomes:*

$$\min_{\gamma_{1,1}\cdots\gamma_{1,I_1},\gamma_{K,1}\cdots\gamma_{K,I_K},E_{1,1}\cdots E_{1,I_1},E_{K,1}\cdots E_{K,I_K}} \sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i} \tag{14a}$$

subject to

$$\gamma_{k,i}\frac{1-g_{k,i,L}/(G_{k,i}E_{k,i})^{d_{k,i,L}}}{1+\sum_{l=1}^{L-1}g_{k,i,l}/(G_{k,i}E_{k,i})^{d_{k,i,l}}} \geq \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}}, \tag{14b}$$
$$\forall k\in\{1,\ldots,K\}, i\in\{1,\ldots,I_k\}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i}\leq 1\,, \tag{14c}$$

$$\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}\leq Q_k^{(0)}, \forall k\in\{1,\ldots,K\}\,, \tag{14d}$$

$$\gamma_{k,i}>0, E_{k,i}>0, \forall k\in\{1,\ldots,K\}, i\in\{1,\ldots,I_k\}\,. \tag{14e}$$

Next Lemma states that Problem 3 is feasible if and only if we get a feasible problem by *i)* forcing each node k to consume the maximum allowable power $\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}=Q_k^{(0)}$ and by *ii)* finding a combination $(Q_{k,i}\stackrel{\text{def}}{=}\gamma_{k,i}E_{k,i})_{1\leq i\leq I_k}$ of powers on the outgoing links of each node k that sum up to $Q_k^{(0)}$ and that result in a feasible resource allocation.

Lemma 2. *Problem 3 is feasible* iff $\sum_{k=1}^{K}\min_{\sum_{i=1}^{I_k}Q_{k,i}=Q_k^{(0)}}\sum_{i=1}^{I_k}\gamma_{k,i,Q_{k,i}}^{(0)}\leq 1$, *where* $\gamma_{k,i,Q_{k,i}}^{(0)}$ *is defined as in Eq.* (13). *Moreover, this inequality implies Slater's condition.*

Hereafter, we start with Problem 3 (related to per-node constraints) since the solution of Problem 2 (related to per-link constraints) is a special case of that of Problem 3.

IV. Solving the per-node power-constrained problem

One can remark that Problem 3 is a *geometric program* [20] which means that all the involved functions (except Eq. (14e) considered as an implicit constraint) are posynomials w.r.t. $\{\gamma_{k,i}\}_{k=1\cdots K,i=1\cdots I_k}$ and $\{E_{k,i}\}_{k=1\cdots,K,i=1\cdots I_k}$. Eq. (14a) and Eq. (14c) are straightforwardly posynomials. Rewriting Eq. (14b) leads to the following new expression of the constraint related to the goodput

$$\frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}}\gamma_{k,i}^{-1}+\sum_{l=1}^{L-1}\frac{\eta_{k,i}^{(0)}g_{k,i,l}}{m_{k,i}R_{k,i}G_{k,i}^{d_{k,i,l}}}\gamma_{k,i}^{-1}E_{k,i}^{-d_{k,i,l}}+ \frac{g_{k,i,L}}{G_{k,i}^{d_{k,i,L}}}E_{k,i}^{-d_{k,i,L}}\leq 1, \tag{15}$$

$\forall k\in\{1,\ldots,K\}, i\in\{1,\ldots,I_k\}$. Clearly, the previous constraint (which replaced Eq. (14b)) is also posynomial. It is well-known that a geometric program can be transformed into a convex optimization problem [20]. In our case, this can be accomplished thanks to the change of variables $\gamma_{k,i}=e^{x_{k,i}}$ and $E_{k,i}=e^{y_{k,i}}$ for $k\in\{1,\ldots,K\}, i\in\{1,\ldots,I_k\}$. Consequently, Problem 3 can be viewed as a convex optimization problem whose Karush–Kuhn–Tucker (KKT) conditions provide a globally-optimal solution since Slater's condition holds if the condition in Lemma 2 is satisfied. Let $\mu_{k,i},\lambda,\nu_k$ be the non-negative Lagrangian multipliers associated with constraints (14b), (14c), (14d) respectively and define functions $x\mapsto f_{k,i}(x)$ for any value $x\in\mathbb{R}_+^*$ of the SNR as

$$f_{k,i}(x)\stackrel{\text{def}}{=}\frac{1+\sum_{l=1}^{L-1}g_{k,i,l}/x^{d_{k,i,l}}}{1-g_{k,i,L}/x^{d_{k,i,L}}}\,. \tag{16}$$

Note that the LHS of Eq. (14b) is equal to $\gamma_{k,i}/f_{k,i}\left(G_{k,i}E_{k,i}\right)$ and that $f_{k,i}$ is decreasing on $\left(g_{k,i,L}^{1/d_{k,i,L}},+\infty\right)$. Here, $g_{k,i,L}^{1/d_{k,i,L}}$ is the smallest value that the SNR $G_{k,i}E_{k,i}$ can take while the approximate goodput (the RHS of Eq. (10)) is non-negative.

Since the optimization problem at hand is convex in variables $\{x_{k,i},y_{k,i}\}_{k=1\ldots K,i=1\ldots I_k}$, the associated KKT conditions should be first derived in these variables. In a second step, they can be rewritten in the original variables $\{\gamma_{k,i},E_{k,i}\}_{k,i}$ giving hence rise to:

$$(1+\nu_k)\gamma_{k,i}E_{k,i} -\mu_{k,i}\frac{\eta_{k,i}^{(0)}\gamma_{k,i}^{-1}}{m_{k,i}R_{k,i}}\left(1+\sum_{l=1}^{L-1}\frac{g_{k,i,l}}{(G_{k,i}E_{k,i})^{d_{k,i,l}}}\right)+\lambda\gamma_{k,i}=0\,, \tag{17}$$

$$(1+\nu_k)\gamma_{k,i}E_{k,i}= \mu_{k,i}\left(\frac{\eta_{k,i}^{(0)}\gamma_{k,i}^{-1}}{m_{k,i}R_{k,i}}\sum_{l=1}^{L-1}\frac{g_{k,i,l}d_{k,i,l}}{(G_{k,i}E_{k,i})^{d_{k,i,l}}}+\frac{g_{k,i,L}d_{k,i,L}}{(G_{k,i}E_{k,i})^{d_{k,i,L}}}\right), \tag{18}$$

$$\mu_{k,i}\left(\frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}}-\frac{\gamma_{k,i}}{f_{k,i}\left(G_{k,i}E_{k,i}\right)}\right)=0\,, \tag{19}$$

$$\lambda\left(\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i}-1\right)=0\,,\quad \nu_k\left(\sum_{i=1}^{I_k}\gamma_{k,i}E_{k,i}-Q_k^{(0)}\right)=0\,. \tag{20}$$

In the following, we manipulate the above equations to the end of writing the resource allocation parameters $E_{k,i}$ and $\gamma_{k,i}$ as functions of one multiplier, namely the multiplier λ associated with the bandwidth constraint Eq. (14c), by eliminating $\mu_{k,i}$ and ν_k. By referring to Eq. (14e), we note that $\gamma_{k,i}E_{k,i}>0$. We also have $1+\nu_k>0$ because ν_k is a Lagrange multiplier. We thus get from Eq. (18) that $\mu_{k,i}\neq 0$, which means that the constraint associated with the goodput (Eq. 14b) is always active. Eq. (19) thus yields

$$\gamma_{k,i}=\frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}}f_{k,i}\left(G_{k,i}E_{k,i}\right)\,. \tag{21}$$

We can thus eliminate $\mu_{k,i}$ by plugging Eqs. (18) and (21) into Eq. (17) to get

$$\lambda=\frac{1}{G_{k,i}}F_{k,i}\left(G_{k,i}E_{k,i}\right)+\frac{\nu_k}{G_{k,i}}\left(F_{k,i}\left(G_{k,i}E_{k,i}\right)+G_{k,i}E_{k,i}\right), \tag{22}$$

where we defined for any value $x\in\left(g_{k,i,L}^{1/d_{k,i,L}},+\infty\right)$ of the SNR $G_{k,i}E_{k,i}$:

$$F_{k,i}(x)\stackrel{\text{def}}{=}\frac{x}{\frac{\sum_{l=1}^{L-1}d_{k,i,l}g_{k,i,l}/x^{d_{k,i,l}}}{1+\sum_{l=1}^{L-1}g_{k,i,l}/x^{d_{k,i,l}}}+\frac{d_{k,i,L}g_{k,i,L}/x^{d_{k,i,L}}}{1-g_{k,i,L}/x^{d_{k,i,L}}}}-x\,. \tag{23}$$

The following lemma summarizes the properties of function

$F_{k,i}$ that will be useful in the remaining derivations. Its proof is provided in Appendix A.

Lemma 3. *For any link (k,i) there exists a unique $s_{k,i} > g_{k,i,L}^{1/d_{k,i,L}} > 0$ such that i) $F_{k,i}(s_{k,i}) = 0$, ii) $F_{k,i}(x) < 0$ for any $x < s_{k,i}$, and iii) $F_{k,i}$ is increasing from 0 to $+\infty$ on $[s_{k,i}, +\infty)$ so that its inverse $F_{k,i}^{-1}$ exists on $[0, +\infty)$ and is increasing on its domain.*

For the moment, we assume that a genie tells us the value of the Lagrange multiplier $\boldsymbol{\lambda}$. As we have done with the multipliers $\mu_{k,i}$, we proceed to eliminating ν_k. We first note by referring to Eq. (23) that $\forall k \in \{1,\ldots,K\}, i \in \{1,\ldots,I_k\}$, $G_{k,i}E_{k,i} + F_{k,i}\left(G_{k,i}E_{k,i}\right) > 0$ provided that $G_{k,i}E_{k,i} > g_{k,i,L}^{1/d_{k,i,L}}$. Eq. (22) can thus be rewritten as:

$$\begin{aligned} \nu_k = \frac{G_{k,1}\lambda - F_{k,1}\left(G_{k,1}E_{k,1}\right)}{G_{k,1}E_{k,1} + F_{k,1}\left(G_{k,1}E_{k,1}\right)} = \cdots \\ \cdots = \frac{G_{k,I_k}\lambda - F_{k,I_k}\left(G_{k,I_k}E_{k,I_k}\right)}{G_{k,I_k}E_{k,I_k} + F_{k,I_k}\left(G_{k,I_k}E_{k,I_k}\right)} \,. \end{aligned} \tag{24}$$

Now since the Lagrange multiplier ν_k in the LHS and all the denominators in the RHS of Eq. (24) are non-negative, the numerators should be non-negative too. We thus obtain the following upper bound on the transmit power of any link (k,i):

$$G_{k,i}E_{k,i} \leq F_{k,i}^{-1}\left(G_{k,i}\lambda\right) \,, \forall k \in \{1,\ldots,K\}, i \in \{1,\ldots,I_k\} \,. \tag{25}$$

Moreover, Eq. (24) allows us to establish $I_k - 1$ equalities involving $\{E_{k,i}\}_{1\leq i\leq I_k}$. These equalities suggest that the I_k power parameters of any node k could be represented by one variable, namely the transmit power of *one* of the I_k links. We will show that this reference link should be chosen as the link $i_k \in \{1,\ldots,I_k\}$ satisfying

$$i_k = \arg\max_{i\in\{1,\ldots,I_k\}} \frac{s_{k,i}}{G_{k,i}} \,. \tag{26}$$

In order to compute the transmit powers $E_{k,i}$ of a link $i \in \{1,\ldots,I_k\}\setminus\{i_k\}$ as a function of λ and the reference transmit power E_{k,i_k}, we refer to Eq. (24) to write:

$$\begin{aligned} \frac{G_{k,i_k}\lambda + G_{k,i_k}E_{k,i_k}}{F_{k,i_k}(G_{k,i_k}E_{k,i_k}) + G_{k,i_k}E_{k,i_k}} F_{k,i}(G_{k,i}E_{k,i}) + \\ \frac{G_{k,i_k}\lambda - F_{k,i_k}(G_{k,i_k}E_{k,i_k})}{F_{k,i_k}(G_{k,i_k}E_{k,i_k}) + G_{k,i_k}E_{k,i_k}} G_{k,i}E_{k,i} = G_{k,i}\lambda \,. \end{aligned} \tag{27}$$

Define $\mathcal{X}_{k,i,\lambda}(G_{k,i_k}E_{k,i_k})$ as the solution (which is unique, *cf.* Lemma 4) in variable $\mathcal{X} \stackrel{\text{def}}{=} G_{k,i}E_{k,i}$ to Eq. (27). Note that $\mathcal{X}_{k,i_k,\lambda}(G_{k,i_k}E_{k,i_k}) = G_{k,i_k}E_{k,i_k}$. We thus can write:

$$E_{k,i} = \frac{1}{G_{k,i}} \mathcal{X}_{k,i,\lambda}(G_{k,i_k}E_{k,i_k}) \,, \forall i \in \{1,\ldots,I_k\} \,. \tag{28}$$

In the same way, parameters $(\gamma_{k,i})_{i=1\ldots I_k}$ can be obtained thanks to Eq. (21) as

$$\gamma_{k,i} = \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} f_{k,i}\left(\mathcal{X}_{k,i,\lambda}(G_{k,i_k}E_{k,i_k})\right) \,, \forall i \in \{1,\ldots,I_k\} \,. \tag{29}$$

Denote by $\mathcal{C}(\lambda)$ the subset of nodes **with an active transmit power constraint**. Due to Eqs. (28) and (29), we can rewrite $\sum_{i=1}^{I_k} \gamma_{k,i}E_{k,i} = Q_k^{(0)}$ for any node $k \in \mathcal{C}(\lambda)$ as:

$$\begin{aligned} \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\lambda}(G_{k,i_k}E_{k,i_k}) \\ \times f_{k,i}\left(\mathcal{X}_{k,i,\lambda}(G_{k,i_k}E_{k,i_k})\right) = Q_k^{(0)} \,, \forall k \in \mathcal{C}(\lambda) \,. \end{aligned} \tag{30}$$

Note that a sufficient and necessary condition for a node k to be in $\mathcal{C}(\lambda)$ is that the above equation has a solution in variable E_{k,i_k} on $\left[s_{k,i_k}/G_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\lambda\right)/G_{k,i_k}\right)$ that we denote as $E_{k,i_k}^{(0)}$. Here, we require $G_{k,i_k}E_{k,i_k}^{(0)} < F_{k,i_k}^{-1}\left(G_{k,i_k}\lambda\right)$ so that the upper bound in Eq. (25) is respected. In other words, $E_{k,i_k}^{(0)}$ is the optimal transmit power on the reference link (k,i_k) for nodes $k \in \mathcal{C}(\lambda)$. The parameters $E_{k,i}$ and $\gamma_{k,i}$ on the other links $i \in \{1,\ldots,I_k\} \setminus \{i_k\}$ of these nodes can be simply computed by setting $E_{k,i_k} = E_{k,i_k}^{(0)}$ in Eqs. (28) and (29).

As for nodes $k \in \overline{\mathcal{C}(\lambda)}$ **with inactive transmit power constraints**, it is straightforward to compute their resource allocation parameters by plugging $\nu_k = 0$ into Eq. (24) and by referring to Eq. (21) to obtain:

$$\begin{aligned} E_{k,i} &= \frac{1}{G_{k,i}} F_{k,i}^{-1}(G_{k,i}\lambda) \,, \\ \gamma_{k,i} &= \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} f_{k,i}(F_{k,i}^{-1}(G_{k,i}\lambda)) \,, \forall k \in \overline{\mathcal{C}(\lambda)} \,. \end{aligned} \tag{31}$$

Putting all pieces together, the optimal transmit power, denoted $E_{k,i_k}(\lambda)$, on the reference link of a general node k is given by

$$G_{k,i_k}E_{k,i_k}(\lambda) \stackrel{\text{def}}{=} \begin{cases} G_{k,i_k}E_{k,i_k}^{(0)}, \text{if Eq.(30) has a solution on} \\ \left[\frac{s_{k,i_k}}{G_{k,i_k}}, \frac{1}{G_{k,i_k}}F_{k,i_k}^{-1}\left(G_{k,i_k}\lambda\right)\right) \,, \\ \\ F_{k,i_k}^{-1}\left(G_{k,i_k}\lambda\right), \text{otherwise.} \end{cases} \tag{32}$$

The above equation allows us to write $\mathcal{C}(\lambda)$ in the following compact form:

$$\mathcal{C}(\lambda) = \left\{ k \,\middle|\, \lambda > \frac{1}{G_{k,i_k}} F_{k,i_k}\left(G_{k,i_k}E_{k,i_k}(\lambda)\right) \right\} \,. \tag{33}$$

So far we managed to characterize the optimal solution to the resource allocation problem in terms of one Lagrange multiplier (λ). **We now turn our attention to the determination of $\boldsymbol{\lambda}$** in order to obtain a practical resource allocation algorithm. Such an algorithm would involve a line search to find the value of λ. We therefore should extend the definition of quantities that were so far only defined assuming λ is known to any arbitrary value Λ. **The first step** is to make sure that $\mathcal{X}_{k,i,\Lambda}$ (defined by Eq. (27) for $\Lambda = \lambda$) is still well defined for any $\Lambda \geq 0$. The following lemma (proven in Appendix B) states that this is true thanks to the way we choose the reference link i_k (*cf* Eq. (26)). First let $x = G_{k,i_k}E_{k,i_k}$ and $\mathcal{X} = G_{k,i}E_{k,i}$ in Eq. (27) and note that we are only interested in values $s_{k,i} \leq \mathcal{X} \leq F_{k,i}^{-1}\left(G_{k,i}\Lambda\right)$ and $s_{k,i_k} \leq x \leq F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)$ due to the upper bound in Eq. (25). Also note that $\left[s_{k,i}, F_{k,i}^{-1}\left(G_{k,i}\Lambda\right)\right]$ and $\left[s_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right]$ are not empty by direct application of Lemma 3.

Lemma 4. *For any $\Lambda \geq 0$, any link (k, i) and any $s_{k,i_k} \leq x \leq F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)$, the following equation in $\mathcal{X}$ has a unique solution, denoted as $\mathcal{X}_{k,i,\Lambda}(x)$, on $\left[s_{k,i}, F_{k,i}^{-1}(G_{k,i}\Lambda)\right)$:*

$$\frac{G_{k,i_k}\Lambda + x}{F_{k,i_k}(x) + x} F_{k,i}(\mathcal{X}) + \frac{G_{k,i_k}\Lambda - F_{k,i_k}(x)}{F_{k,i_k}(x) + x}\mathcal{X} = G_{k,i}\Lambda . \quad (34)$$

The second step is to verify that for any node k, Eq. (30) still has a solution if we replace λ with an arbitrary value Λ. First, we define $\mathcal{X}_{k,i,\infty}(x)$ for any $x \in [s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\Lambda))$ as the unique solution to $\frac{G_{k,i_k}/G_{k,i}}{F_{k,i_k}(x)+x}F_{k,i}(\mathcal{X}) + \frac{G_{k,i_k}/G_{k,i}}{F_{k,i_k}(x)+x}\mathcal{X} = 1$ on $[s_{k,i}, +\infty)$ (obtained by letting $\Lambda \to +\infty$ in Eq. (34)). Second, we define the following function on $[s_{k,i_k}, +\infty)$ for any given $\Lambda \geq 0$:

$$\mathcal{Q}_{k,\Lambda}(x) \stackrel{\text{def}}{=} \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\Lambda}(x) f_{k,i}\left(\mathcal{X}_{k,i,\Lambda}(x)\right) . \quad (35)$$

Finally, we get the following lemma proven in Appendix C.

Lemma 5. *For any node $k \in \{1, \ldots, K\}$, if*

$$\mathcal{Q}_{k,\infty}(s_{k,i_k}) \stackrel{def}{=} \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\infty}(s_{k,i_k}) f_{k,i}\left(\mathcal{X}_{k,i,\infty}(s_{k,i_k})\right) \leq Q_k^{(0)} , \quad (36)$$

then there exists a unique $\Lambda_k^{(0)} \geq 0$ such that $\forall \Lambda > \Lambda_k^{(0)}$, the following equation:

$$\mathcal{Q}_{k,\Lambda}(x) = Q_k^{(0)} \quad (37)$$

has a solution in variable x on $\left[s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)\right)$ that is unique and bounded with respect to Λ, while no solution exists on the latter interval $\forall \Lambda < \Lambda_k^{(0)}$.

The above lemma implies that in settings satisfying inequality (36), a *necessary* condition for node k to have active transmit power constraint is $\lambda > \Lambda_k^{(0)}$. This condition is also *sufficient*. To see why, note that if we set $\Lambda = \lambda > \Lambda_k^{(0)}$, then due to the above lemma Eq. (37) will have a unique solution x that belongs to $\left[s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\lambda)\right)$. Plugging $E_{k,i_k} = x/G_{k,i_k}$ in Eq. (24) hence leads to a strictly-positive value for the multiplier ν_k. Therefore, the determination of whether a node should or should not have an active power constraint reduces to the search for the value of the multiplier λ which is addressed later on. During this search, we define a provisional transmit power $E_{k,i_k}(\Lambda)$ that generalizes $E_{k,i_k}(\lambda)$ defined in Eq. (32) to any value of the search variable Λ as:

$$G_{k,i_k}E_{k,i_k}(\Lambda) \stackrel{\text{def}}{=} \begin{cases} \text{the solution on } \left[s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)\right) \text{ to Eq. (37) ,} \\ \text{if } \Lambda > \Lambda_k^{(0)} , \\ \\ F_{k,i_k}^{-1}(G_{k,i_k}\Lambda) , \\ \text{if } \Lambda \leq \Lambda_k^{(0)} . \end{cases} \quad (38)$$

When $\Lambda = \lambda$, it is clear that $E_{k,i_k}(\lambda)$ is equal to the optimal transmit power on the reference link (k, i_k) and that the resource allocation parameters of the other links of node k can be computed by setting $E_{k,i_k} = E_{k,i_k}(\lambda)$ in Eqs. (28) and (29).

The final step is to extend the definition given by Eq. (33) of the subset $\mathcal{C}(\lambda)$ to any value $\Lambda \geq 0$ to get $\mathcal{C}(\Lambda) \stackrel{\text{def}}{=} \left\{k \left| G_{k,i_k}\Lambda > F_{k,i_k}\left(G_{k,i_k}E_{k,i_k}(\Lambda)\right)\right.\right\}$. Note that $\mathcal{C}(\Lambda) = \left\{k \left| \Lambda > \Lambda_k^{(0)}\right.\right\}$ due to Eq. (38). Also note that $\mathcal{C}(\Lambda)$ has a "physical" meaning (as the subset of nodes whose transmit power constraint should be active) *only when* $\Lambda = \lambda$. For any other arbitrary value of Λ, $\mathcal{C}(\Lambda)$ serves as an intermediate tool in the search for the optimal resource allocation. In particular, $\mathcal{C}(\Lambda) = \{1, \ldots, K\}$ for Λ large enough since $F_{k,i_k}\left(G_{k,i_k}E_{k,i_k}(\Lambda)\right)$ is bounded $\forall \Lambda \geq 0$ due to Lemma 5. This means that increasing Λ has the effect of forcing more nodes to consume all their maximum allowable transmit power $Q_k^{(0)}$. Now define the following function on $\mathbb{R}_+$:

$$\Gamma(\Lambda) \stackrel{\text{def}}{=} \sum_{k \in \mathcal{C}(\Lambda)} \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} f_{k,i}\left(\mathcal{X}_{k,i,\Lambda}(G_{k,i_k}E_{k,i_k}(\Lambda))\right) + \sum_{k \in \overline{\mathcal{C}(\Lambda)}} \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} f_{k,i}\left(F_{k,i}^{-1}(G_{k,i}\Lambda)\right) . \quad (39)$$

Note that if condition (36) holds for all nodes k, then $\Gamma(\Lambda)$ is well defined $\forall \Lambda \geq 0$ thanks to Lemma 5. In particular, when $\Lambda = \lambda$, $\Gamma(\lambda)$ is the sum of the optimal bandwidth sharing factors. In Appendix D, we prove that Γ is continuous on its domain.

Lemma 5 inspires us to replace the feasibility condition given by Lemma 2, which is difficult to test, with the following more restrictive (but easier to verify) condition.

Assumption 1. *The following two conditions are satisfied:*

1) *The inequality in Eq.* (36) *holds* $\forall k \in \{1, \ldots, K\}, i \in \{1, \ldots, I_k\}$.
2) $$\Gamma_\infty \stackrel{def}{=} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} \times f_{k,i}\left(\mathcal{X}_{k,i,\infty}\left(G_{k,i_k}E_{k,i_k}(+\infty)\right)\right) \leq 1 .$$

In Appendix D we prove that $\Gamma_\infty = \lim_{\Lambda \to +\infty} \Gamma(\Lambda)$. The sub-optimality of this feasibility condition is due to the requirement that Eq. (37) has a solution for any $\Lambda \geq 0$ and that $\sum_{k=1}^{K}\sum_{i=1}^{I_k}\gamma_{k,i}(\Lambda) \leq 1$ for $\Lambda \to \infty$, instead of requiring that these conditions hold only for $\Lambda = \lambda$. Nonetheless, Assumption 1 was satisfied in all the feasible settings encountered in the simulations, and violated in all the infeasible ones. If Assumption 1 was not held, The KKT based algorithm failed to work but any standard numerical algorithm for convex optimization can be used.

Under Assumption 1, we know that function Γ (which is continuous) tends to a value smaller than one as $\Lambda \to +\infty$. Only one of the following two cases is thus possible. Either $\Gamma(0) \leq 1$, then the Lagrange multiplier λ is equal to zero. Or $\Gamma(0) > 1$, meaning that the equation $\Gamma(\Lambda) = 1$ has at least one solution. In this case, λ can be considered to be any such solution. We thus get the following theorem proven in Appendix D.

Theorem 1. *Let Assumption 1 hold true. The optimal solution to Problem 3 is as follows.*

1) *If* $\mathit{\Gamma}(0) \leq 1$*, then for each* $k \in \mathcal{C}(0)$*,* $E_{k,i} = \frac{1}{G_{k,i}} \mathcal{X}_{k,i,0}\left(G_{k,i_k} E_{k,i_k}(0)\right)$*, and for each* $k \in \overline{\mathcal{C}(0)}$*,* $E_{k,i} = \frac{1}{G_{k,i}} F_{k,i}^{-1}(0)$*.*
2) *Else, for each* $k \in \mathcal{C}(\lambda)$*,* $E_{k,i} = \frac{1}{G_{k,i}} \mathcal{X}_{k,i,\lambda}\left(G_{k,i_k} E_{k,i_k}(\lambda)\right)$*, and for each* $k \in \overline{\mathcal{C}(\lambda)}$*,* $E_{k,i} = \frac{1}{G_{k,i}} F_{k,i}^{-1}(G_{k,i}\lambda)$*, with* $\lambda > 0$ *any solution in* $\mathbb{R}_+^*$ *to* $\mathit{\Gamma}(\mathit{\Lambda}) = 1$*.*

In both cases, $\gamma_{k,i} = \frac{\eta_{k,i}^{(0)}}{m_{k,i} R_{k,i}} f_{k,i}(G_{k,i} E_{k,i})$*.*

Thanks to Theorem 1, one can propose Algorithm 1 for the computation of the optimal resource allocation parameters $\{\gamma_{k,i}, E_{k,i}\}_{k=1\ldots K, i=1\ldots I_k}$. In Algorithm 1, the fact that $\mathit{\Lambda}_k^{(0)}$ defined in Lemma 5 is equal to the unique solution on $\mathbb{R}_+$ to $Q_{k,\mathit{\Lambda}}\left(F_{k,i}^{-1}\left(G_{k,i}\mathit{\Lambda}\right)\right) = Q_k^{(0)}$ is proved in Appendix C. Moreover, the size of the increment step should be a trade-off between the speed of convergence on one side and the precision of the resulting solution (λ and the associated resource allocation parameters) on the other.

Algorithm 1 Optimal resource allocation algorithm for Problem 3

```
for all k ∈ {1,...,K} do
  Λ_k^(0) ← the unique solution on R_+ to Q_{k,Λ}(F_{k,i}^{-1}(G_{k,i}Λ)) = Q_k^(0)
end for
Λ ← 0
repeat
  C(Λ) ← { k | Λ > Λ_k^(0) }
  for all k ∈ C(Λ) do
    G_{k,i_k} E_{k,i_k}(Λ) ← the unique solution on [s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}Λ)) to Eq. (37)
    for all i ∈ {1,...,I_k} \ {i_k} do
      E_{k,i} ← (1/G_{k,i}) X_{k,i,Λ}(G_{k,i_k} E_{k,i_k}(Λ)),
      γ_{k,i} ← η_{k,i}^(0) / (m_{k,i} R_{k,i}) f_{k,i}(G_{k,i} E_{k,i})
    end for
  end for
  for all k ∈ complement(C(Λ)) do
    for all i ∈ {1,...,I_k} do
      E_{k,i} ← (1/G_{k,i}) F_{k,i}^{-1}(G_{k,i}Λ),
      γ_{k,i} ← η_{k,i}^(0) / (m_{k,i} R_{k,i}) f_{k,i}(G_{k,i} E_{k,i})
    end for
  end for
  Increment Λ
until Σ_{k=1}^{K} Σ_{i=1}^{I_k} γ_{k,i} ≤ 1
return {γ_{k,i}, E_{k,i}}_{k=1...K, i=1...I_k}
```

V. Solving the per-link power-constrained problem

As stated in Section III, Problem 2 can be seen as a special case of Problem 3 applied to an equivalent network with $K' \stackrel{\text{def}}{=} \sum_{k=1}^{K} I_k$ one-link nodes obtained by a one-to-one mapping from (k,i) (with $I_k \geq 1$) to $k' \in \{1,\ldots,K'\}$ (with $I'_{k'} = 1$). Interestingly, the approach developed in Section IV can be simplified before being applied to solve Problem 2. The first simplification comes from the fact that Lemma 4 is obviously no longer needed. Moreover, the computation of the resource allocation parameters for links with an active power constraint is easier than in the previous section. Indeed, when $I'_{k'} = 1$ for $k' \in \{1,\ldots,K'\}$ in the equivalent network, Eq. (37) associated with the corresponding link (k,i) in the original network simplifies to

$$\frac{\eta_{k,i}^{(0)}}{m_{k,i} R_{k,i} G_{k,i}} x f_{k,i}(x) = Q_{k,i}^{(0)} , \tag{40}$$

which has a unique solution on $[s_{k,i}, +\infty)$ equal to $G_{k,i} Q_{k,i}^{(0)} / \gamma_{k,i,Q_{k,i}^{(0)}}^{(0)}$ (*cf.* Eq. (13)) provided that the feasibility condition in Lemma 1 is satisfied (Assumption 1 is no longer needed). The resource allocation parameters of links with active transmit power constraints are thus given by $\gamma_{k,i,Q_{k,i}^{(0)}}^{(0)}$ (as defined above) and $E_{k,i}^{(0)} \stackrel{\text{def}}{=} G_{k,i} Q_{k,i}^{(0)} / \gamma_{k,i,Q_{k,i}^{(0)}}^{(0)}$. Note that a *necessary* condition for a link (k,i) to have an active power constraint is that $E_{k,i}^{(0)}$ does *not* violate inequality (25). This condition is equivalent to $\lambda > \mathit{\Lambda}_{k,i}^{(0)}$, where

$$\mathit{\Lambda}_{k,i}^{(0)} \stackrel{\text{def}}{=} \frac{1}{G_{k,i}} F_{k,i}\left(G_{k,i} E_{k,i}^{(0)}\right) . \tag{41}$$

This condition is also *sufficient* as it leads to a strictly-positive $\nu_{k,i}$ in the per-link version of Eq. (24). It is thus useful to define for any $\mathit{\Lambda} \geq 0$ the following subset:

$$\tilde{\mathcal{C}}(\mathit{\Lambda}) \stackrel{\text{def}}{=} \left\{(k,i) \,\middle|\, k \in \{1,\ldots,K\}, i \in \{1,\ldots,I_k\}, \mathit{\Lambda} > \mathit{\Lambda}_{k,i}^{(0)}\right\}, \tag{42}$$

and the following function on $\mathbb{R}_+$:

$$\begin{aligned}\tilde{\mathit{\Gamma}}(\mathit{\Lambda}) \stackrel{\text{def}}{=} &\sum_{(k,i)\in\tilde{\mathcal{C}}(\mathit{\Lambda})} \gamma_{k,i,Q_{k,i}^{(0)}}^{(0)} \\ &+ \sum_{(k,i)\in\overline{\tilde{\mathcal{C}}(\mathit{\Lambda})}} \frac{\eta_{k,i}^{(0)}}{m_{k,i} R_{k,i}} f_{k,i}\left(F_{k,i}^{-1}(G_{k,i}\mathit{\Lambda})\right) .\end{aligned} \tag{43}$$

When $\mathit{\Lambda} = \lambda$ (the Lagrange multiplier associated with constraint (11c)), $\tilde{\mathcal{C}}(\lambda)$ is the subset of links whose transmit power constraints should be active while $\tilde{\mathit{\Gamma}}(\lambda)$ is equal to the total sum of the optimal bandwidth sharing factors. In Appendix E, we prove that this function is continuous and decreasing on its domain. Incorporating the above simplifications into Theorem 1 leads to the following theorem.

Theorem 2. *Let the feasibility condition in Lemma 1 hold. The optimal solution to Problem 2 is as follows.*

1) *If* $\tilde{\mathit{\Gamma}}(0) \leq 1$*, then* $\forall (k,i) \in \tilde{\mathcal{C}}(0)$*, we have* $\gamma_{k,i} =$

$\gamma^{(0)}_{k,i,Q^{(0)}_{k,i}}$ *and* $E_{k,i} = E^{(0)}_{k,i}$, *and* $\forall (k,i) \in \overline{\tilde{\mathcal{C}}(0)}$, $E_{k,i} = \frac{1}{G_{k,i}} F^{-1}_{k,i}(0)$ *and* $\gamma_{k,i} = \frac{\eta^{(0)}_{k,i}}{m_{k,i} R_{k,i}} f_{k,i}(G_{k,i} E_{k,i})$.

2) *Else,* $\forall (k,i) \in \tilde{\mathcal{C}}(\lambda)$ *we have* $\gamma_{k,i} = \gamma^{(0)}_{k,i,Q^{(0)}_{k,i}}$ *and* $E_{k,i} = E^{(0)}_{k,i}$, *and* $\forall (k,i) \in \overline{\tilde{\mathcal{C}}(\lambda)}$, $E_{k,i} = \frac{1}{G_{k,i}} F^{-1}_{k,i}(G_{k,i}\lambda)$ *and* $\gamma_{k,i} = \frac{\eta^{(0)}_{k,i}}{m_{k,i} R_{k,i}} f_{k,i}(G_{k,i} E_{k,i})$, *with* $\lambda > 0$ *the unique solution in* $\mathbb{R}^*_+$ *to* $\tilde{\Gamma}(\Lambda) = 1$.

Note that λ as defined in Theorem 2 is unique due to the fact that function $\tilde{\Gamma}$ is decreasing on $\mathbb{R}_+$ as opposed to function Γ in Theorem 1 which is not necessarily monotone. With Theorem 2 in hand, we propose Algorithm 2 for the computation of the optimal resource allocation parameters $\{\gamma_{k,i}, E_{k,i}\}_{k=1\ldots K, i=1\ldots I_k}$.

Algorithm 2 Optimal resource allocation algorithm for Problem 2

$\Lambda \leftarrow 0$
for all $k \in \{1,\ldots,K\}, i \in \{1,\ldots,I_k\}$ **do**
  $\gamma^{(0)}_{k,i,Q^{(0)}_{k,i}} \leftarrow$ the RHS of Eq. (13), $E^{(0)}_{k,i} \leftarrow \frac{Q^{(0)}_{k,i}}{\gamma^{(0)}_{k,i,Q^{(0)}_{k,i}}}$,
  $\Lambda^{(0)}_{k,i} \leftarrow \frac{1}{G_{k,i}} F_{k,i}\left(G_{k,i} E^{(0)}_{k,i}\right)$
end for
repeat
  $\tilde{\mathcal{C}}(\Lambda) \leftarrow \left\{(k,i) \middle| \Lambda > \Lambda^{(0)}_{k,i}\right\}$
  for all $(k,i) \in \tilde{\mathcal{C}}(\Lambda)$ **do**
    $E_{k,i} \leftarrow E^{(0)}_{k,i}$,
    $\gamma_{k,i} \leftarrow \gamma^{(0)}_{k,i,Q^{(0)}_{k,i}}$
  end for
  for all $(k,i) \in \overline{\tilde{\mathcal{C}}(\Lambda)}$ **do**
    $E_{k,i} \leftarrow \frac{1}{G_{k,i}} F^{-1}_{k,i}\left(G_{k,i}\Lambda\right)$,
    $\gamma_{k,i} \leftarrow \frac{\eta^{(0)}_{k,i}}{m_{k,i} R_{k,i}} f_{k,i}\left(G_{k,i} E_{k,i}\right)$
  end for
  Increment Λ
until $\sum_{k=1}^{K} \sum_{i=1}^{I_k} \gamma_{k,i} \leq 1$
return $\{\gamma_{k,i}, E_{k,i}\}_{k=1\ldots K, i=1\ldots I_k}$

VI. MCS SELECTION

Let $\mathcal{M}$ designate the set of indices of the available modulation schemes and $\mathcal{R}$ the set of coding rates of the available codes. When we fix in advance the MCS ($\mathbf{m} \overset{\text{def}}{=} [m_{1,1},\ldots,m_{K,I_K}]^{\mathrm{T}}$ and $\mathbf{R} \overset{\text{def}}{=} [R_{1,1},\ldots,R_{K,I_K}]^{\mathrm{T}}$) of the different links, Algorithms 1 and 2 return the optimal solution $\gamma_{1,1},\ldots,\gamma_{K,I_K}, E_{1,1},\ldots,E_{K,I_K}$ to their corresponding optimization problems. Define $\mathcal{Q}^*_T(\mathbf{m},\mathbf{R}) \overset{\text{def}}{=} \sum_{k=1}^{K}\sum_{i=1}^{I_k} \gamma_{k,i} E_{k,i}$ as the minimal total transmit power when the corresponding optimization problem is feasible. Otherwise, set $\mathcal{Q}^*_T(\mathbf{m},\mathbf{R}) = +\infty$. The optimal selection of the MCSs in the network is the solution to the following combinatorial optimization problem.

Problem 4. $(\mathbf{m}^*,\mathbf{R}^*) = \arg\min_{(\mathbf{m},\mathbf{R}) \in \mathcal{M}^{K'} \times \mathcal{R}^{K'}} \mathcal{Q}^*_T(\mathbf{m},\mathbf{R})$, *where* $K' \overset{\text{def}}{=} \sum_{k=1}^{K} I_k$.

The global solution to Problem 4 can be found by an exhaustive search that becomes prohibitively costly in computations even for moderate numbers of links. Instead, we resort to the suboptimal but computationally-efficient greedy MCS selection algorithm used in [11] and inspired by [22]. Let $\mathcal{M}$ and $\mathcal{R}$ be sorted such that $\mathcal{M} = \{m_1,\ldots,m_{|\mathcal{M}|}\}$ and $\mathcal{R} = \{R_1,\ldots,R_{|\mathcal{R}|}\}$ with $m_1 \leq \cdots \leq m_{|\mathcal{M}|}$ and $R_1 \leq \cdots \leq R_{|\mathcal{R}|}$. The idea behind the algorithm is to perform MCS selection iteratively by changing the MCS of only one link per iteration. This is done by assigning to each link the next MCS in the ordered set $\mathcal{M} \times \mathcal{R}$ and by selecting the link whose MCS modification results in the lowest sum transmit power. This approach is greedy in the sense that it continues while the so-obtained transmit power decreases, and stops otherwise. In the following, $k' \leq K'$ is used to index the K' links of the network. Let $\mathbf{m}^{(0)} = [m(1),\ldots,m(K')]^{\mathrm{T}}$ and $\mathbf{R}^{(0)} = [R(1),\ldots,R(K')]^{\mathrm{T}}$ be such that $m(k') = m_{j_{k'}}$ and $R(k') = R_{p_{k'}}$ with $j_{k'} = p_{k'} = 1$ for any $k' \in \{1,\ldots,K'\}$. The proposed greedy approach is described by Algorithm 3 which runs for at most $K'|\mathcal{M}||\mathcal{R}|$ iterations.

Algorithm 3 Greedy approach to solving Problem 4

$\mathbf{m} \leftarrow \mathbf{m}^{(0)}$, $\mathbf{R} \leftarrow \mathbf{R}^{(0)}$
repeat
  $\mathcal{Q}^*_T \leftarrow \mathcal{Q}^*_T(\mathbf{m},\mathbf{R})$
  $(\mathbf{m}^*,\mathbf{R}^*) \leftarrow (\mathbf{m},\mathbf{R})$
  for all $k' \in \{1,\ldots,K'\}$ **do**
    $\mathbf{m}^{(k')} \leftarrow m(k') = m_{\min\{j_{k'}+1,|\mathcal{M}|\}}$ and
    $j_{k'} \leftarrow \min\{j_{k'}+1,|\mathcal{M}|\}$
    $\mathbf{R}^{(k')} \leftarrow R(k') = R_{\min\{p_{k'}+1,|\mathcal{R}|\}}$ and
    $p_{k'} \leftarrow \min\{p_{k'}+1,|\mathcal{R}|\}$
  end for
  $k'^* \leftarrow \arg\min_{1\leq k' \leq K'} \min\{\mathcal{Q}^*_T(\mathbf{m}^{(k')},\mathbf{R}),$
    $\mathcal{Q}^*_T(\mathbf{m},\mathbf{R}^{(k')}), \mathcal{Q}^*_T(\mathbf{m}^{(k')},\mathbf{R}^{(k')})\}$
  $(\mathbf{m},\mathbf{R}) \leftarrow \arg\min\{\mathcal{Q}^*_T(\mathbf{m}^{(k'^*)},\mathbf{R}),$
    $\mathcal{Q}^*_T(\mathbf{m},\mathbf{R}^{(k'^*)}), \mathcal{Q}^*_T(\mathbf{m}^{(k'^*)},\mathbf{R}^{(k'^*)})\}$
until $\mathcal{Q}^*_T < \mathcal{Q}^*_T(\mathbf{m},\mathbf{R})$
return $\mathbf{m}^*,\mathbf{R}^*$

VII. SIMULATIONS

We considered a network with $K = 5$ active nodes with $I_k = I = 2$ links each. The bandwidth is equal to $W = 5$ MHz centered around the carrier frequency $f_0 = 2400$ MHz. Each information packet is of length $n_b = 128$ bits. The distance $D_{k,i}$ of link (k,i) is randomly drawn from a uniform distribution on [100m, 1km]. The term $\varsigma^2_{k,i}$ follows a free-space model so that $\varsigma^2_{k,i} = 1/\left(4\pi f_0 D_{k,i}/c\right)^2$, where c is the speed of light in vacuum. For the sake of simplicity, each link has the same target efficiency $\eta^{(0)}$ so that the required sum rate is equal to $IK\eta^{(0)}$. Finally, we fix $N_0 = -170$ dBm/Hz.

Hereafter, we assume all the links use the same MCS *i.e.,* $m_{k,i} = m, R_{k,i} = R, \forall (k,i)$. The values of m and R are chosen from Table I depending on the target efficiency. The modulation is based on a m-QAM constellation with $m \in \mathcal{M} \overset{\text{def}}{=} \{1,2,4,6\}$ while error control consists in a

CC-HARQ based on the convolutional code given in [17] with an initial rate equal respectively to 1/2 and 1. In other words, $R \in \mathcal{R} \overset{\text{def}}{=} \{1/2, 1\}$. In all the cases, we set $L = 3$. In Figure 1, we plot the sum transmit power

MCS name	1	2	3	4	5
m (bits)	1	2	4	6	4
R	1/2	1/2	1/2	1/2	1
max. sumrate (Mbps)	2.5	5	10	15	20
$WQ^{(0)}$	15	18	24	32	48
$\log_{10} g_{k,i,l}$ $l = 1, 2, 3$ (CC)	0.55 -0.22 -0.49	0.95 1.05 0.64	2.21 2.87 3.02	2.88 3.28 3.19	1.47 1.76 1.79
$\log_{10} g_{k,i,l}$ $l = 1, 2, 3$ (IR)	-0.40 -3.63 -5.89	1.60 -1.11 -2.76	4.92 2.23 1.55	6.51 3.80 2.90	2.75 4.90 4.03

Table I
THE MCSS, THE TRANSMIT POWER CONSTRAINTS AND THE CONSTANTS $g_{k,i,l}$ USED IN SIMULATIONS

$W \sum_{k=1}^{K} \sum_{i=1}^{I_k} \gamma_{k,i} E_{k,i}$ as obtained by Algorithm 2 with each link subject to a transmit power constraint equal to $WQ^{(0)}$ where $Q^{(0)}$ is set as given in Table I. The same table provides the constants $\{g_{k,i,l}\}_{l=1\ldots L}$ for goodput computations using Eq. (3). Each point was obtained using 100 Monte-Carlo runs, and each subinterval in Figure 1 is associated with a given MCS. For the sake of comparison, we also plot the

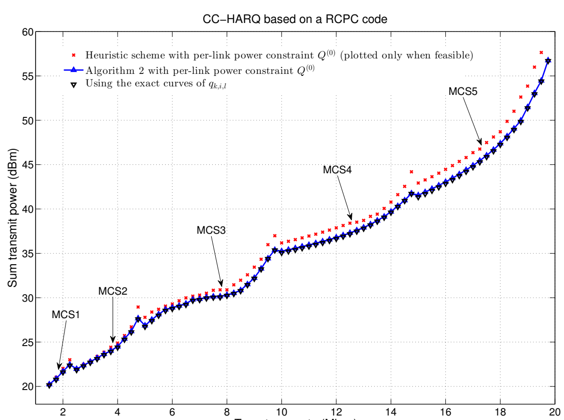

Figure 1. Sum transmit power vs. sum target rate for the proposed algorithm and the heuristic one when CC-HARQ is implemented and MCS is fixed in advance.

total transmit power resulting from the following heuristic resource allocation scheme. The **first step** of this scheme consists in fixing $\gamma_{k,i}^{\text{trivial}} = \frac{\eta_{k,i}^{(0)}/(m_{k,i} R_{k,i})}{\sum_{l=1}^{K} \sum_{j=1}^{I_l} \eta_{l,j}^{(0)}/(m_{l,j} R_{l,j})}$ (which trivially satisfies constraint (11c)) and in choosing $E_{k,i}^{\text{trivial}}$ to be equal to the minimal value such that constraint (11b) is respected. If the resulting transmit power $\gamma_{k_0,i_0}^{\text{trivial}} E_{k_0,i_0}^{\text{trivial}}$ on some link (k_0, i_0) violates constraint (11d), the **second step** of the heuristic scheme consists increasing the bandwidth sharing factor assigned to such links by setting $\gamma_{k_0,i_0}^{\text{trivial}} = \gamma_{k_0,i_0}^{(0)}$ (where $\gamma_{k_0,i_0}^{(0)}$ is defined in Eq. (13)) and in accordingly reducing the sharing factors of the other links (so that constraint (11c) is still respected). Note that after performing the second step, there is no guarantee that constraint (11b) is still respected on the links whose sharing factors have been reduced. Roughly speaking, the heuristic scheme fails half of the time in the far-right part of each subinterval. In addition to this feasibility issue, the advantage of using our algorithm over the heuristic scheme in terms of total transmit power is clear from Figure 1.

Finally, we investigate in the same figure the potential increase in the total transmit power due to the fact that the proposed resource allocation algorithms are developed using the upper bound $q_{k,i,l} \leq \tilde{\pi}_{k,i,l}$ (*cf* Eq. (3)). Therefore, a smaller transmit power parameter $E_{k,i}^{\text{quasi-optimal}}$ associated with the exact curves $q_{k,i,l}$ can be computed by solving the following equation in variable E for each link (k, i):

$$\eta_{k,i}^{(0)} = m_{k,i} R_{k,i} \gamma_{k,i} \frac{1 - q_{k,i,L}(G_{k,i} E)}{1 + \sum_{l=1}^{L-1} q_{k,i,l}(G_{k,i} E)} ,$$

where $\{\gamma_{k,i}\}_{(k,i)}$ are the bandwidth parameters returned by our resource allocation algorithms. We can see from Fig. 1 that the loss $\sum_{k=1}^{K} \sum_{i=1}^{I_k} \gamma_{k,i} \left(E_{k,i} - E_{k,i}^{\text{quasi-optimal}} \right)$ turns out to be negligible for all practical values of the target sum data rate.

In Figure 2, we plot the sum transmit power resulting from Algorithm 3 under the per-link power constraint of Table I applied to the IR-HARQ scheme based on the two nested convolutional codes from [21] with an initial rate equal respectively to 1/2 and 1. Here, the results are given for both $L = 3$ and $L = 4$. From Fig. 2, we can see the significant gain we get from the proposed MCS selection method with respect to the case where the MCS is fixed trivially.

We also compute the best lower bound (under the assumptions made in Section II) on the sum transmit power obtained by Algorithm 3. Since the wireless channels in our system model are fast fading, the lower bound is reached by endowing the links of the network with the possibility of achieving their *ergodic capacity*. More precisely, we now assume that a data rate of $\eta_{k,i}^{(0)}$ bits/s/Hz is possible provided that:

$$C_{k,i}(\gamma_{k,i}, E_{k,i}) \overset{\text{def}}{=} \gamma_{k,i} \mathbb{E} \left[\log_2 \left(1 + G_{k,i} E_{k,i} X_e \right) \right] \geq \eta_{k,i}^{(0)} \tag{44}$$

where $C_{k,i}$ is the ergodic capacity associated with link (k, i) and where the expectation is taken w.r.t. the unit-mean exponentially-distributed random variable X_e. Under this ideal assumption, we should modify Problem 2 by replacing constraint (11b) with Eq. (44). The optimal resource allocation parameters can then be obtained by replacing $f_{k,i}(x)$ and $F_{k,i}(x)$ for any $x \in \mathbb{R}_+^*$ in Algorithm 2 with $f_e(x) = \frac{1}{\mathbb{E}[\log(1+xX_e)]}$ and $F_e(x) = \frac{\log(1+xX_e)}{\mathbb{E}[X_e/(1+xX_e)]} - x$. In Figure 2, we note that Algorithm 3, while suboptimal, significantly reduces the optimality gap to the ergodic lower bound as compared with the case where the MCSs are fixed in advance. Moreover, setting $L = 4$ yields, as expected, a total transmit power that is closer to the lower bound as compared to $L = 3$.

VIII. CONCLUSIONS

In this article, we proposed near-optimal resource allocation algorithms to minimize sum power consumption in OFDMA-based wireless networks which use Type-II HARQ schemes

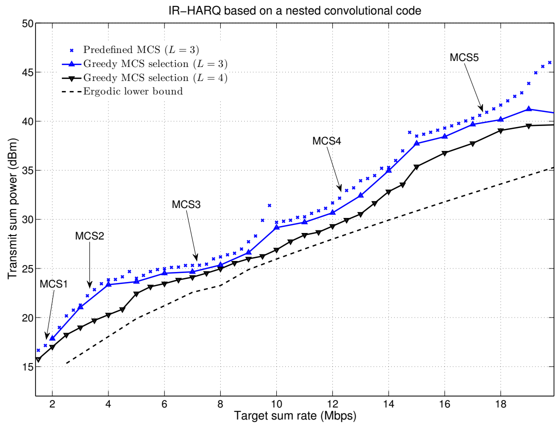

Figure 2. Sum transmit power vs. sum target rate for the proposed algorithm and the ergodic capacity based one when IR-HARQ is implemented and MCS is optimized.

subject to per-link goodput constraints and two kinds of individual transmit power limitations. Under the assumption of statistical CSI and practical MCSs, our algorithms return the optimal power and bandwidth parameters by resorting to a tight approximation of the links' goodput. Finally, a computationally-efficient algorithm for MCS selection was provided that, when coupled with the previous resource allocation algorithms, yields significant reductions in the total power emitted by the network.

Appendix A
Proof of Lemma 3

For any $k \in \{1,\ldots,K\}, i \in \{1,\ldots,I_k\}$, function $F_{k,i}$ can be written $\forall x > g_{k,i,L}^{1/d_{k,i,L}}$ as $F_{k,i}(x) = x\left(1/\left(\mathcal{F}_{k,i}(x) + \frac{d_{k,i,L}g_{k,i,L}/x^{d_{k,i,L}}}{1-g_{k,i,L}/x^{d_{k,i,L}}}\right) - 1\right)$ with $\mathcal{F}_{k,i}(x) \overset{\text{def}}{=} \frac{\sum_{l=1}^{L-1} d_{k,i,l}g_{k,i,l}/x^{d_{k,i,l}}}{1+\sum_{l=1}^{L-1} g_{k,i,l}/x^{d_{k,i,l}}}$. It is straightforward to show that $x \mapsto \frac{d_{k,i,L}g_{k,i,L}/x^{d_{k,i,L}}}{1-g_{k,i,L}/x^{d_{k,i,L}}}$ is decreasing on $\left(g_{k,i,L}^{1/d_{k,i,L}}, +\infty\right)$. As for the proof that $\mathcal{F}_{k,i}$ is decreasing on $\mathbb{R}_+^*$, we note that $\forall x \in \mathbb{R}_+^*$,

$$\mathcal{F}'_{k,i}(x) = \frac{-\sum_{l=1}^{L-1} \frac{d_{k,i,l}^2 g_{k,i,l}}{x^{d_{k,i,l}+1}} + \sum_{l,m=1}^{L-1} \frac{g_{k,i,l}g_{k,i,m}d_{k,i,l}(d_{k,i,m}-d_{k,i,l})}{x^{d_{k,i,l}+d_{k,i,m}+1}}}{\left(1+\sum_{l=1}^{L-1}\frac{g_{k,i,l}}{x^{d_{k,i,l}}}\right)^2}. \tag{45}$$

The first term in the numerator of the RHS of Eq. (45) is clearly negative. We now prove that the second term is negative too. To that end, we define for any $x > 0$:

$$a_{l,m} = g_{k,i,l}g_{k,i,m}\frac{d_{k,i,l}(d_{k,i,m}-d_{k,i,l})}{x^{d_{k,i,l}+d_{k,i,m}+1}}, 1 \le l,m \le L-1\,. \tag{46}$$

Note that the second term in the numerator of the RHS of Eq. (45) is equal to $\sum_{l,m=1}^{L-1} a_{l,m}$ and that $a_{l,l} = 0$. The computation of this sum is easier if we introduce $n \overset{\text{def}}{=} l+m$:

$$\begin{aligned}
\sum_{l,m=1}^{L-1} a_{l,m} &= \sum_{n=2}^{L-1}\Bigg(\sum_{\substack{l+m=n\\ l<m}} a_{l,m} + \sum_{\substack{l+m=n\\ l>m}} a_{l,m}\Bigg)\\
&\quad+\sum_{n=L}^{2L-2}\Bigg(\sum_{\substack{l+m=n\\ l<m}} a_{l,m} + \sum_{\substack{l+m=n\\ l>m}} a_{l,m}\Bigg)\\
&= \sum_{n=2}^{L-1}\Bigg(\sum_{l=1}^{\lfloor\frac{n}{2}\rfloor} a_{l,n-l} + \sum_{m=1}^{\lfloor\frac{n}{2}\rfloor} a_{n-m,m}\Bigg)\\
&\quad+\sum_{n=L}^{2L-2}\Bigg(\sum_{m=\lfloor\frac{n}{2}\rfloor+1}^{L} a_{n-m,m} + \sum_{l=\lfloor\frac{n}{2}\rfloor+1}^{L} a_{l,n-l}\Bigg)\\
&= -\sum_{n=2}^{L-1}\sum_{j=1}^{\lfloor\frac{n}{2}\rfloor} g_{k,i,j}g_{k,i,n-j}\frac{(d_{k,i,n-j}-d_{k,i,j})^2}{x^{d_{k,i,j}+d_{k,i,n-j}+1}}\\
&\quad-\sum_{n=L}^{2L-2}\sum_{j=\lfloor\frac{n}{2}\rfloor+1}^{L} g_{k,i,j}g_{k,i,n-j}\frac{(d_{k,i,j}-d_{k,i,n-j})^2}{x^{d_{k,i,j}+d_{k,i,n-j}+1}}
\end{aligned}$$

We thus conclude that $\sum_{l,m=1}^{L-1} a_{l,m} < 0$ and that as a result $\mathcal{F}'_{k,i}(x) < 0$ for all $x \in \mathbb{R}_+^*$.

Putting all pieces together, function $x \mapsto 1/\left(\mathcal{F}_{k,i}(x) + \frac{d_{k,i,L}g_{k,i,L}/x^{d_{k,i,L}}}{1-g_{k,i,L}/x^{d_{k,i,L}}}\right) - 1$ is thus increasing on $\left(g_{k,i,L}^{1/d_{k,i,L}}, +\infty\right)$ from -1 to $+\infty$. Therefore, there exists $s_{k,i} > g_{k,i,L}^{1/d_{k,i,L}}$ such that $F_{k,i}(s_{k,i}) = 0$ and $F_{k,i}(x) \ge 0$ for all $x \ge s_{k,i}$.

Appendix B
Proof of Lemma 4

For any $x \in \left[s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)\right]$, the LHS of Eq. (34) as function of $\mathcal{X}$ increases on $\left[s_{k,i}, F_{k,i}^{-1}(G_{k,i}\Lambda)\right]$ from $\frac{G_{k,i_k}\Lambda - F_{k,i_k}(x)}{F_{k,i_k}(x)+x}s_{k,i}$ to $\frac{G_{k,i_k}\Lambda + x}{F_{k,i_k}(x)+x}G_{k,i}\Lambda + \frac{G_{k,i_k}\Lambda - F_{k,i_k}(x)}{F_{k,i_k}(x)+x}F_{k,i}^{-1}(G_{k,i}\Lambda) \ge G_{k,i}\Lambda$. The last inequality holds because $x \le F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)$ so that $\frac{G_{k,i_k}\Lambda + x}{F_{k,i_k}(x)+x} \ge 1$. A sufficient condition for Eq. (34) to have a solution on $\left[s_{k,i}, F_{k,i}^{-1}(G_{k,i}\Lambda)\right]$ is then:

$$\frac{G_{k,i_k}\Lambda - F_{k,i_k}(x)}{F_{k,i_k}(x)+x}s_{k,i} \le G_{k,i}\Lambda, \forall s_{k,i_k} \le x \le F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)\,. \tag{47}$$

Since $x \mapsto \frac{G_{k,i_k}\Lambda - F_{k,i_k}(x)}{F_{k,i_k}(x)+x}$ is decreasing on $\left[s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)\right]$, we only need to verify the inequality in Eq. (47) at $x = s_{k,i_k}$: $\frac{G_{k,i_k}\Lambda - F_{k,i_k}(s_{k,i_k})}{F_{k,i_k}(s_{k,i_k})+s_{k,i_k}}s_{k,i} = \frac{G_{k,i_k}\Lambda}{s_{k,i_k}}s_{k,i} \le G_{k,i}\Lambda$, where the last inequality holds since $i_k = \arg\max_{i\in\{1,\ldots,I_k\}} \frac{s_{k,i}}{G_{k,i}}$.

Appendix C
Proof of Lemma 5

We need to show that for any $k \in \{1,\ldots,K\}$, the function $x \mapsto Q_{k,\Lambda}(x)$ defined by Eq. (35) is increasing on $\left[s_{k,i_k}, F_{k,i_k}^{-1}(G_{k,i_k}\Lambda)\right]$ for any $\Lambda \ge 0$. For that sake, we first prove that $\mathcal{X} \mapsto \mathcal{X} f_{k,i}(\mathcal{X})$ is increasing on $\left[s_{k,i}, F_{k,i}^{-1}\left(G_{k,i}\Lambda\right)\right]$ for any $i \in \{1,\ldots,I_k\}$. Indeed, one can show after some manipulations that $(\mathcal{X} f_{k,i}(\mathcal{X}))' = \frac{f_{k,i}(\mathcal{X})}{F_{k,i}(\mathcal{X})+\mathcal{X}} F_{k,i}(\mathcal{X})$, so that the derivative of $\mathcal{X} \mapsto \mathcal{X} f_{k,i}(\mathcal{X})$ is positive on $\left(s_{k,i}, F_{k,i}^{-1}\left(G_{k,i}\Lambda\right)\right]$ due to Lemma 3. The next step is to note by referring to Eq. (34) that $x \mapsto \mathcal{X}_{k,i,\Lambda}(x)$ is increasing on $\left[s_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right]$ for any $\Lambda \geq 0$ and that the image of $\left[s_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right]$ w.r.t. this function is $\left[s_{k,i}, F_{k,i}^{-1}\left(G_{k,i}\Lambda\right)\right]$. It follows that $\mathcal{Q}_{k,\Lambda}(x)$ as defined in Eq. (35) is increasing on $\left[s_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right]$. There is hence at most one solution on the latter interval to Eq. (37) *i.e.,* to equation $\mathcal{Q}_{k,\Lambda}(x) = Q_k^{(0)}$. This solution exists if and only if

$$\mathcal{Q}_{k,\Lambda}(s_{k,i_k}) \leq Q_k^{(0)} \quad \& \quad \mathcal{Q}_{k,\Lambda}\left(F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right) \geq Q_k^{(0)} \,. \tag{48}$$

Moreover, this solution x satisfies $\frac{\eta_{k,i_k}^{(0)}}{m_{k,i_k}R_{k,i_k}G_{k,i_k}} x f_{k,i_k}(x) \leq Q_k^{(0)}$ so that it is bounded $\forall \Lambda \geq 0$. Now note that $\Lambda \mapsto \mathcal{Q}_{k,\Lambda}\left(F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right)$ is increasing on $\mathbb{R}_+$ and that it increases from $\mathcal{Q}_{k,\Lambda}(s_{k,i_k})$ to $+\infty$. As $\mathcal{Q}_{k,\Lambda}(s_{k,i_k}) \leq Q_k^{(0)}$, there exists a unique $\Lambda_k^{(0)}$ in $\mathbb{R}_+$ such that $\mathcal{Q}_{k,\Lambda}\left(F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right) = Q_k^{(0)}$. Note that the inequality $\Lambda \geq \Lambda_k^{(0)}$ is equivalent to the second inequality in Eq. (48) due to the monotonicity of function $\Lambda \mapsto \mathcal{Q}_{k,\Lambda}\left(F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right)$. We now want to establish a condition that does not involve Λ for the first inequality in Eq. (48) to hold. To that end, we note that $\Lambda \to \mathcal{X}_{k,i,\Lambda}(s_{k,i_k})$ is increasing on $\mathbb{R}_+$ as $\mathcal{X}_{k,i,\Lambda}(s_{k,i_k})$ is equal to the unique solution in variable $\mathcal{X}$ on $\left[s_{k,i}, F_{k,i}^{-1}\left(G_{k,i}\Lambda\right)\right]$ to the equation $\left(G_{k,i_k} + \frac{s_{k,i_k}}{\Lambda}\right) F_{k,i}(\mathcal{X}) + G_{k,i_k}\mathcal{X} = s_{k,i_k}G_{k,i}$. We can show by standard convergence arguments that the increasing function $\Lambda \mapsto \mathcal{X}_{k,i,\Lambda}(s_{k,i_k})$ is continuous on $\mathbb{R}_+$ so that $\lim_{\Lambda\to+\infty} \mathcal{X}_{k,i,\Lambda}(s_{k,i_k}) = \mathcal{X}_{k,i,\infty}(s_{k,i_k})$, $\mathcal{X}_{k,i,\infty}(s_{k,i_k})$ being the unique solution on $[s_{k,i}, +\infty]$ to the equation $G_{k,i_k}F_{k,i}(\mathcal{X}) + G_{k,i_k}\mathcal{X} = s_{k,i_k}G_{k,i}$. It follows that $\forall \Lambda \geq 0$, $\mathcal{X}_{k,i,\Lambda}(s_{k,i_k}) \leq \mathcal{X}_{k,i,\infty}(s_{k,i_k})$ leading to:

$$\begin{aligned} &\mathcal{Q}_{k,\Lambda}(s_{k,i_k}) = \\ &\quad \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\Lambda}(s_{k,i_k}) f_{k,i}\left(\mathcal{X}_{k,i,\Lambda}(s_{k,i_k})\right) \leq \\ &\quad \sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\infty}(s_{k,i_k}) f_{k,i}\left(\mathcal{X}_{k,i,\infty}(s_{k,i_k})\right) \stackrel{\text{def}}{=} \\ &\qquad\qquad \mathcal{Q}_{k,\infty}(s_{k,i_k}) \,. \end{aligned} \tag{49}$$

We thus conclude that $\left\{\mathcal{Q}_{k,\infty}(s_{k,i_k}) \leq Q_k^{(0)}\right\}$ implies $\left\{\mathcal{Q}_{k,\Lambda}(s_{k,i_k}) \leq Q_k^{(0)}, \forall \Lambda \geq 0\right\}$. In other words, Eq (36) implies Eq. (48). This concludes the proof of Lemma 5.

Appendix D
Proof of Theorem 1

We first prove that $\Gamma(\Lambda)$ is continuous on $\mathbb{R}_+$. To that end, note that for any link (k,i) and for any $x \in \left[s_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda\right)\right)$, function $\Lambda \mapsto \mathcal{X}_{k,i,\Lambda}(x)$ is continuous on $\mathbb{R}_+$ by standard convergence arguments applied to Eq. (34). Similar arguments can be used to prove that $E_{k,i_k}(\Lambda)$ defined by Eq. (38) is continuous on $[0, \Lambda_k^{(0)})$ and on $(\Lambda_k^{(0)}, +\infty)$, where $\Lambda_k^{(0)}$ is defined in Lemma 5. We now show that it is continuous at $\Lambda = \Lambda_k^{(0)}$. On one hand, it is clear from Eq. (38) that $\lim_{\Lambda \nearrow \Lambda_k^{(0)}} G_{k,i_k}E_{k,i_k}(\Lambda) = F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda_k^{(0)}\right)$. On the other hand, continuity of $\Lambda \mapsto \mathcal{X}_{k,i,\Lambda}(x)$ can be used in Eq. (36) to show that $\lim_{\Lambda \searrow \Lambda_k^{(0)}} G_{k,i_k}E_{k,i_k}(\Lambda)$ is the unique solution on $\left[s_{k,i_k}, F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda_k^{(0)}\right)\right]$ to

$$\sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\Lambda_k^{(0)}}(x) f_{k,i}\left(\mathcal{X}_{k,i,\Lambda_k^{(0)}}(x)\right) = Q_k^{(0)} \,. \tag{50}$$

Now by definition of $\Lambda_k^{(0)}$ (see Appendix C) we have

$$\begin{aligned} &\sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}G_{k,i}} \mathcal{X}_{k,i,\Lambda_k^{(0)}}\left(F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda_k^{(0)}\right)\right) \\ &\quad \times f_{k,i}\left(\mathcal{X}_{k,i,\Lambda_k^{(0)}}\left(F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda_k^{(0)}\right)\right)\right) = Q_k^{(0)} \,. \end{aligned} \tag{51}$$

Comparing Eqs. (50) and (51) and referring to the properties of functions $x \mapsto \mathcal{X}_{k,i,\Lambda}(x)$ and $\mathcal{X} \mapsto \mathcal{X} f_{k,i}(\mathcal{X})$ in Appendix C, we get $\lim_{\Lambda \searrow \Lambda_k^{(0)}} G_{k,i_k}E_{k,i_k}(\Lambda) = F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda_k^{(0)}\right) = \lim_{\Lambda \nearrow \Lambda_k^{(0)}} G_{k,i_k}E_{k,i_k}(\Lambda)$. Now define k_j ($j \in \{1,\ldots,K\}$) as a one-to-one mapping from $\{1,\ldots,K\}$ on itself that results in a nondecreasing ordering of the values $\{\Lambda_k^{(0)}\}_{1\leq k\leq K}$ *i.e.,* $\Lambda_{k_1}^{(0)} \leq \ldots \leq \Lambda_{k_K}^{(0)}$. Note that the subset $\mathcal{C}(\Lambda)$ does not change as Λ changes inside any interval of the form $\left[\Lambda_{k_j}^{(0)}, \Lambda_{k_{j+1}}^{(0)}\right)$ $\forall j \in \{1,\ldots,K-1\}$. Combining this with the fact that functions $f_{k,i}(x)$, $F_{k,i}(x)$ and $E_{k,i_k}(\Lambda)$ are continuous, we get that $\Gamma(\Lambda)$ is continuous on any interval $\left[\Lambda_{k_j}^{(0)}, \Lambda_{k_{j+1}}^{(0)}\right)$. Finally, the continuity of $\Gamma(\Lambda)$ at $\{\Lambda_k^{(0)}\}_{k=1\ldots K}$ is a direct result of the fact that $\lim_{\Lambda\to\Lambda_k^{(0)}} G_{k,i_k}E_{k,i_k}^{(0)}(\Lambda) = F_{k,i_k}^{-1}\left(G_{k,i_k}\Lambda_k^{(0)}\right)$.

The second step is to prove that $\lim_{\Lambda\to+\infty}\Gamma(\Lambda) = \Gamma_\infty$, where Γ_∞ is defined in Assumption 1. This can be done in a straightforward manner using standard convergence arguments applied to $\Lambda \mapsto \sum_{k=1}^{K}\sum_{i=1}^{I_k} \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} f_{k,i}\left(\mathcal{X}_{k,i,\Lambda}\left(G_{k,i_k}E_{k,i_k}(\Lambda)\right)\right)$. Now recall that $\Gamma_\infty \leq 1$ provided that Assumption 1 is satisfied. As a result, we conclude that one of the following two cases holds.

Either $\Gamma(0) \leq 1$, then the value of the Lagrange multiplier associated with constraint (14c) is $\lambda = 0$. Consequently, the subset of nodes with active transmit power constraints is $\mathcal{C}(0)$. This means that for any $k \in \mathcal{C}(0)$, the optimal parameters $(E_{k,i_k}, \gamma_{k,i_k})$ of the reference link i_k can be obtained by first finding the unique solution to Eq. (37) as dictated by Lemma 5. The resource allocation parameters $(E_{k,i}, \gamma_{k,i})$ of the other links of node $k \in \mathcal{C}(0)$ are obtained respectively by Eq. (28) and Eq. (29) due to Lemma 4. As for nodes $k \in \overline{\mathcal{C}(0)}$, $E_{k,i} = \frac{1}{G_{k,i}}F_{k,i}^{-1}(0)$ and $\gamma_{k,i} = \frac{\eta_{k,i}^{(0)}}{m_{k,i}R_{k,i}} f_{k,i}(F_{k,i}^{-1}(0))$ due to Eq. (31). **Else**, $\Gamma(0) > 1$. Since Γ is continuous and $\Gamma_\infty \leq 1$, $\exists \lambda > 0$ s.t. $\Gamma(\lambda) = 1$. In this case, the value of the multiplier associated with constraint (14c) is λ. Therefore, for each $k \in \mathcal{C}(\lambda)$, parameters $(E_{k,i_k}, \gamma_{k,i_k})$ are obtained by solving

Eq. (37) (due to Lemma 5) and $(E_{k,i},\gamma_{k,i})_{i\in\{1,\ldots,I_k\}\setminus\{i_k\}}$ are given by Eqs. (28) and (29) (due to Lemma 4). As for $k \in \overline{\mathcal{C}(\lambda)}$, parameters $(E_{k,i},\gamma_{k,i})_{i=1\ldots I_k}$ are given by Eqs. (31).

APPENDIX E
BEHAVIOR OF FUNCTION $\tilde{\Gamma}(\Lambda)$ ON $\mathbb{R}_+$:

Let $k' \in \{1,\ldots,K'\}$ be used to index the K' links of the network. Define k'_j as a one-to-one mapping from $\{1,\ldots,K'\}$ on itself that results in a nondecreasing ordering of the values $\{\Lambda^{(0)}_{k'}\}_{1\leq k'\leq K'}$ defined by Eq. (41) *i.e.,* $\Lambda^{(0)}_{k'_1} \leq \ldots \leq \Lambda^{(0)}_{k'_{K'}}$. Due to Eq. (42), subsets $\tilde{\mathcal{C}}(\Lambda)$ do not change as Λ changes within any interval of the form $\left[\Lambda^{(0)}_{k'_j},\Lambda^{(0)}_{k'_{j+1}}\right)$ $\forall 1\leq j\leq K'-1$. The sum $\sum_{k'\in\tilde{\mathcal{C}}(\Lambda)}\gamma^{(0)}_{k'}$ is thus constant on any such interval. Next, $\sum_{k'\in\overline{\tilde{\mathcal{C}}(\Lambda)}}\frac{\eta^{(0)}_{k'}}{m_{k'}R_{k'}}f_{k'}\left(F^{-1}_{k'}(G_{k'}\Lambda)\right)$ is continuous and decreasing on any interval $[\Lambda^{(0)}_{k'_j},\Lambda^{(0)}_{k'_{j+1}})$ since $f_{k'}$ is decreasing and $F^{-1}_{k'}$ is increasing on $\mathbb{R}^*_+$ and both are continuous. Therefore, function $\tilde{\Gamma}(\Lambda)$ is piecewise continuous decreasing on $\left[0,\Lambda^{(0)}_{k'_{K'}}\right)$. Moreover, it is continuous at the points $\Lambda^{(0)}_{k'_1},\ldots,\Lambda^{(0)}_{k'_{K'}}$ as a direct result of the continuity of function $\Gamma(\Lambda)$. Finally, $\tilde{\Gamma}(\Lambda)$ is constant over $\left[\Lambda^{(0)}_{k'_{K'}},+\infty\right)$ since $\overline{\tilde{\mathcal{C}}(\Lambda)}=\emptyset$ for any $\Lambda\geq\Lambda^{(0)}_{k'_{K'}}$. Putting all pieces together, we get that $\tilde{\Gamma}$ is continuous decreasing on $\mathbb{R}_+$.

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