Energy-Efficient Resource Allocation for HARQ With Statistical CSI

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***Abstract*—This paper addresses the resource allocation (RA) problem for orthogonal frequency division multiple access and single carrier frequency division multiple access based networks using hybrid automatic repeat request (HARQ) when considering energy efficiency (EE) as the metric to optimize. The RA is performed considering the Rayleigh channel, and assuming that only statistical channel state information is available, which fits well device-to-device (D2D) communications setup. Moreover, the use of practical modulation and coding schemes is taken into account. We formulate several RA problems with different EE criteria, subject to constraints on the minimum goodput and the maximum transmit power per-link. Based on a packet error probability approximation, we provide a framework which allows us to derive algorithms maximizing most of the common EE criteria, namely the sum of the EE of the links, the product of the EE, the EE of the link with the worst EE, and the EE of the whole network. The complexity of the proposed algorithms is also provided. The performance related to each criterion are evaluated through simulations and ranked according to the network configurations. We also illustrate on a practical example the real effectiveness of the EE criterion to achieve a better user experience than other common criteria such as the maximum sum rate and the minimum transmit power.**

***Index Terms*—HARQ, energy efficiency, resource allocation, fractional programming, geometric programming, successive convex approximation, alternating optimization.**

I. Introduction

THE hybrid automatic repeat request (HARQ) is a mechanism combining automatic repeat request (ARQ) and forward error correction (FEC) in order to increase the robustness of wireless communications under time-varying channel. This mechanism is nowadays used in several standards including 4G long term evolution [2] in a multiuser context.

To apply these schemes in real systems, a resource manager (RM) selects the resource parameters that go along with the HARQ such as per-user transmit power, modulation and coding schemes (MCS) and bandwidth proportion (assuming orthogonal frequency division multiple access (OFDMA) or single frequency division multiple access (SC-FDMA) techniques). For instance, the RM may be the basestation in the case of cellular-assisted device-to-device (D2D) communications system [3].

The resource allocation (RA) usually solves an optimization problem that consists in a criterion to be optimized subject to constraints. Two conventional criteria that have been widely considered are the maximum sum rate [4], or the minimization of the links' transmit power [5]. A third criterion, energy efficiency (EE), has gained a lot of interest in the last decade [6], [7]. Indeed, when optimizing energy consumption is at stake (which can be directly related to our everyday life cellphone battery discharge), this criterion is the most relevant among the aforementioned criteria. It is obvious for the maximum sum rate since this is achieved using the maximum transmit power. Compared to the minimum transmit power, it is less straightforward since one can imagine that minimizing the transmit power will lead to minimizing the energy consumption. This is actually wrong in general, as it will be illustrated in this paper.

Thus, the aim of this paper is to provide computable solutions to perform RA maximizing various EE related criteria assuming Type-II HARQ schemes with practical MCS. This work also encompasses the Type-I HARQ case, as well as schemes with no retransmission [8]. The allocated parameters are per-link transmit power and bandwidth proportion. We assume a random Rayleigh channel model without instantaneous channel state information (CSI) feedback, meaning that only statistical CSI is used to perform the optimization. This situation may happen for example in D2D communications, when RA is performed in an assisted fashion, i.e., there is a RM performing RA [3]. Indeed, as the RM is usually neither the transmit node nor the receiver node of the link, there is a delay between the time the nodes send their CSI to the RM and the time the RM sends their RA to them. This delay is typically larger than the channel's coherence time and unfortunately renders impossible to perform RA with full CSI. On the other hand, the channel's statistics are expected to remain constant for time duration much longer than the channel's coherence time and thus an accurate version of statistical CSI can be communicated to the RM. Thus, we assume that the RM uses statistical CSI to perform RA [9], [10].

A. Related Works

There is a large number of works related to RA problem with EE criteria without using HARQ. In [11]–[16], the au-

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thors consider capacity achieving codes as well as perfect CSI at the transmitter. More precisely, in [11], the EE of an orthogonal frequency division multiplexing (OFDM) based system is maximized in the single-user context. In the multiuser context using OFDMA, the sum of the users' EE (SEE) is maximized in [12]. The minimum EE (MEE) maximization, called max-min problem, is solved in [13]. Suboptimal heuristic algorithms are derived for SEE and max-min problems in [14]. In multi-cell context, RA problems are addressed in [15] where several EE criteria are treated, namely the SEE, the product of the users' EE (PEE), and the global EE (GEE) defined as the EE at the network level. In [16], centralized and decentralized algorithms are proposed for GEE. It is worth emphasizing that considering capacity achieving codes for RA while using practical MCS yields EE performance degradations, as illustrated in [17].

In the remainder of the paper, the maximization of the SEE, PEE, MEE and GEE criteria will be referred to as the maximum SEE (MSEE), maximum PEE (MPEE), max-min EE (MMEE), and maximum GEE (MGEE) respectively.

When HARQ is taken into account, there is a lot of work investigating the single user context [8], [18]–[26], whereas only few works address the multiuser context [1], [27]–[29]. Let us first focus on the single-user context. In [8], [18]–[26], the authors consider statistical CSI at the transmitter. In [18]–[22], EE of HARQ is analyzed by considering capacity-achieving codes. In contrast, in [8], [23]–[26], EE of HARQ is studied with practical MCS. In [23], the EE of a relay-assisted scheme is numerically evaluated. In [8], the energy per transmit bit is minimized for a Type-I HARQ in a frequency flat channel context. This work is extended to chase combining (CC)-HARQ in [24], where the optimal power per HARQ round is derived for frequency flat and slow varying channel. In [25], the EE for CC-HARQ is maximized with respect to the transmit power and the block length for frequency flat and block varying channel. In [26], the EE is maximized with respect to the code rate for Type-I and CC-HARQ. Let us now focus on the multiuser context, which is the scope of this paper. In [1], [27]–[29], the authors consider OFDMA. In [27], the GEE is optimized assuming perfect CSI at the transmitter and capacity achieving codes. In [28], EE is analyzed with respect to some predefined power allocation assuming capacity achieving codes. In [29], a suboptimal algorithm is proposed to optimize the harmonic mean of EE assuming relay assisted systems using OFDMA and Type-I HARQ with statistical CSI at the transmitter and practical MCS. Finally, in our conference paper [1], we solved the MSEE problem assuming statistical CSI at the transmitter and practical MCS for Type-II HARQ (see Section IV-A). To the best of our knowledge, no previous work address the EE of HARQ considering SC-FDMA.

Thus, except in [29] and [1], there is no other work dealing with RA problem for HARQ in the multiuser context with practical MCS, statistical CSI, and EE optimization.

B. Contributions and Paper Organization

The main contributions of this paper are the following ones.

- We derive optimal and computationally tractable algorithms solving the MSEE, the MPEE and the MMEE problems under goodput (defined as the information bit rate without error) and maximum transmit power per OFDMA or SC-FDMA symbol constraints when considering HARQ and practical MCS, which have never been addressed in the literature. Indeed, when considering HARQ with practical MCS, the resulting RA problems are not in the form of combinations of ratios between concave and convex functions and thus conventional fractional programming tools cannot be directly applied to solve them. Our main technical contribution is to transform all these problems that have no special properties (like convexity) because of the HARQ retransmission mechanism and practical MCS into equivalent convex problems, for which efficient solvers exist. We also propose two suboptimal procedures to solve the GEE optimization problem. In addition, we analyze the complexity of the proposed algorithms.
- We analyze the results of the proposed criteria through simulations of relevant practical scenarios. We also compare these results with other existing conventional criteria that are not related to EE: the one from [30] minimizing the sum of the users' transmit power, called minimum power (MPO), and the one maximizing the sum of the users' goodput, called maximum goodput (MGO). These two schemes actually achieve rather bad performance in EE. We find out that the MPEE criterion achieves a good performance-fairness tradeoff and thus is suitable for D2D communications context.
- We also go further in the analysis, illustrating the effect of these differences on the battery drain of a smartphone in terms of the amount of transmitted bits and battery autonomy. The results confirm the fact that the scheme with the best PEE outperforms the others by allowing to transmit the largest amount of information bits with the least battery drain.

The rest of this paper is organized as follows: the system model is described in Section II. The optimization problems are mathematically expressed in Section III. Optimal and/or suboptimal procedures to solve these optimization problems are proposed in Section IV. The complexity analysis is done in Section V. Numerical results and performance analysis of the proposed RA policies are conducted in Section VI. Finally, concluding remarks are drawn in Section VII.

II. System Model

A. Channel Model

In this paper, we assume a network of K active links sharing a bandwidth B that is divided in N_c subcarriers using either the OFDMA or the SC-FDMA, without multiuser interference. We suppose that the links' statistical CSI are centralized in a RM entity, whose task is to perform RA.

For link k, the stream of transmit symbols $\{X_k(j)\}_{j=1}^{+\infty}$ is split into blocks of length n_k: $\mathbf{X}_k(j) := [X_k(jn_k+1), \cdots, X_k((j+1)n_k)]^T$ where n_k is the number of subcarriers allocated to the kth link and $(.)^T$ stands for the transposition operator. The signal sent by the kth link during the jth OFDMA or SC-FDMA symbol can be written as

$$\mathbf{S}_k(j) := \mathbf{C}_p \text{IFFT}_{N_c}(\Theta_k(\text{FFT}_{\mathcal{M}}\mathbf{X}_k(j))) \quad (1)$$

where $\text{FFT}_{\mathcal{M}}$ is the $\mathcal{M} \times \mathcal{M}$ Fourier transform matrix (with $\mathcal{M} = 1$ for OFDMA and $\mathcal{M} = n_k$ for SC-FDMA), Θ_k is a $N_c \times n_k$ matrix mapping the output of the Fourier transform onto the subcarriers allocated to link k, IFFT_{N_c} is the $N_c \times N_c$ inverse Fourier transform matrix, and $\mathbf{C}_p$ is a matrix adding the cyclic prefix (CP) at the beginning of the sent block.

We assume that each link is modelled as a time-varying multipath Rayleigh channel which is constant within the duration of an OFDMA or SC-FDMA symbol, and changes independently from symbol to symbol. This model will be referred to as block fading (BF) assumption. Let $\mathbf{h}_k(j) = [h_k(j, 0), ..., h_k(j, L-1)]^T$ be the sampled channel impulse response of link k during the jth OFDMA (or SC-FDMA) symbol, where L is the channel impulse response length. We make the common assumption of uncorrelated taps, i.e., $\mathbf{h}_k(j) \sim \mathcal{CN}(0, \Sigma_k)$, where $\mathcal{CN}(0, \Sigma_k)$ stands for the multi-variate circularly-symmetric complex-valued normal distribution with zero mean and covariance matrix $\Sigma_k := \text{diag}_{L \times L}(\sigma^2_{k,0}, ..., \sigma^2_{k,L-1})$.

At the receiver side, after removing the CP and applying the matrix FFT_{N_c}, the received signal on link k on the nth subcarrier at symbol j is

$$Y_k(j, n) = H_k(j, n)\mathcal{X}_k(j, n) + Z_k(j, n), \quad (2)$$

where $H_k(j, n)$ is the nth coefficient of the discrete Fourier transform of $\mathbf{h}_k(j)$ computed on N_c points, $\mathcal{X}_k(j, n)$ is the nth coefficient of $\Theta_k(\text{FFT}_{\mathcal{M}}\mathbf{X}_k(j))$, and $Z_k(j, n) \sim \mathcal{CN}(0, N_0 B/N_c)$, with N_0 the noise level in the power spectral density. The coefficients $H_k(j, n)$ are identically distributed random variables: $H_k(j, n) \sim \mathcal{CN}(0, \zeta_k^2)$ where $\zeta_k^2 := \text{Tr}(\Sigma_k)$. We now define the average gain-to-noise ratio (GNR) as

$$G_k := \frac{\mathbb{E}[|H_k(j, n)|^2]}{N_0} = \frac{\zeta_k^2}{N_0}. \quad (3)$$

Since statistical CSI is assumed, the RM only knows the average GNR of each link to perform RA.

B. The HARQ Mechanism

Each link uses a HARQ mechanism, which works as follows. The infinite stream of information bits of link k is arranged into packets of length $\mathcal{L}_k$ bits, and each packet is sent on the channel at most M times. The transmitted packets' content depends on the considered HARQ scheme: for both Type-I and CC-HARQ, the transmitted packets' content is identical, and is obtained by encoding the information bits by a FEC with rate R_k. For incremental redundancy (IR)-HARQ, we consider that the information bits are encoded by a FEC called mother code with rate R_k/M, and the resulting encoded stream is split into identical-length redundancy blocks following the rate compatible coding principle.

The receiver decodes the information bits after the reception of the pth packet. The decoding process depends once again on the considered HARQ scheme. For Type-I HARQ, the receiver uses only the pth received packet to decode the information bits. For CC-HARQ, the maximum ratio combining with the p received packets is performed by the receiver before decoding. For IR-HARQ, the receiver concatenates all the received redundancy blocks, and decodes the information bits. Finally, the receiver checks the decoded information bits validity using a cyclic redundancy check (CRC), and sends an acknowledgement (ACK) if no error is detected, and a negative ACK (NACK) if not. The CRC and the ACK/NACK are assumed to be error free.

C. Energy Consumption Model

We suppose that a quadrature amplitude modulation (QAM) modulation with m_k bits is used on link k. Let $\gamma_k := n_k/N_c$ be the proportion of bandwidth allocated to link k. Since only statistical CSI is available to perform RA and all subcarriers are identically distributed, the same power is used on all the subcarriers. We then define $P_k := \mathbb{E}[|\mathcal{X}_k(j, n)|^2]$ as the allocated power per subcarrier to the kth link.

The total energy consumed to send and receive one packet is equal to the sum of the transmit packet energy and the circuitry consumption of both the emitter and the receiver. The power used by the kth link to send and receive one OFDMA or SC-FDMA symbol is

$$P_{T,k} := N_c \gamma_k \frac{P_k}{\kappa_k} + P_{ctx,k} + P_{crx,k}, \quad (4)$$

where $\kappa_k \leq 1$ is the power amplifier (PA) efficiency, and $P_{ctx,k}$ (resp. $P_{crx,k}$) is the per-symbol circuitry power consumption at the transmitter (resp. receiver).

D. Energy Efficiency

The EE is defined as the ratio between the goodput and the power consumption, which can be written as

$$\mathcal{E}_k := \frac{\eta_k \text{ [bits/s]}}{P_{T,k} \text{ [W]}}, \quad (5)$$

where η_k is the goodput of the kth link. From [31] and [30, Eq. (9)], we know that for the considered HARQ schemes the goodput is given by

$$\eta_k := B\alpha_k\gamma_k \frac{D_k(E_k)}{S_k(E_k)}, \quad (6)$$

where $\alpha_k := m_k R_k$, $E_k := P_k N_c/B$ is the energy consumed by the link k to transmit its part of an OFDMA or SC-FDMA symbol, $D_k(E_k) := (1 - q_{k,M}(E_k))$, $S_k(E_k) := (1 + \sum_{m=1}^{M-1} q_{k,m}(E_k))$ with $q_{k,m}(E_k)$ the probability that the first m transmissions are all received in error. By plugging (4) and (6) into (5), we obtain

$$\mathcal{E}_k(E_k, \gamma_k) = \frac{\alpha_k\gamma_k D_k(E_k)}{S_k(E_k)(\gamma_k E_k \kappa_k^{-1} + E_{c,k})}, \quad (7)$$

where $E_{c,k} := (P_{ctx,k} + P_{crx,k})/B$. Similarly, the GEE, defined as the ratio between the sum of the links' goodput and the sum of their power consumption, is given by

$$\mathcal{G}(\mathbf{E},\boldsymbol{\gamma}) := \frac{\sum_{k=1}^{K}\eta_k}{\sum_{k=1}^{K}P_{T,k}} = \frac{\sum_{k=1}^{K}\alpha_k\gamma_k\frac{D_k(E_k)}{S_k(E_k)}}{\sum_{k=1}^{K}(\gamma_k E_k\kappa_k^{-1}+E_{c,k})}, \quad (8)$$

with $\mathbf{E} := [E_1,\cdots,E_K]$ and $\boldsymbol{\gamma} := [\gamma_1,\cdots,\gamma_K]$.

Notice that the case $M = 1$ corresponds to no retransmission, but also to the Type-I HARQ. Indeed, due to the BF assumption, for Type-I HARQ, $q_{k,m} = q_{k,1}^m$ and thus $\eta_k = B\alpha_k\gamma_k(1-q_{k,1}(E_k))$, which is equivalent to (6) when $M = 1$. In that case, the GEE problem simplifies and drops down to the maximization of a ratio between a concave and a convex function problem over convex set which can be optimally solved using the Dinkelbach's algorithm [6] (not detailed in this paper). However, it does not allow to simplify the derivations of the MSEE, MPEE and MMEE problems since the complexity of the expressions with regard to $\mathbf{E}$ and $\boldsymbol{\gamma}$ are the same for any M.

E. Error Probability Approximation

The first difficulty in the theoretical study of HARQ with practical MCS is the absence of closed form expression for the error probability $q_{k,m}$. Here, we overcome this issue by considering the following upper bound [32]

$$q_{k,m}(E_k) \le \pi_{k,m}(E_k), \quad (9)$$

where $\pi_{k,m}(E_k)$ is the probability of decoding failure when m packets are available. In this paper, we assume that we use bit interleaved coded modulation (BICM) briefly described below [33]: at the transmitter side, a sequence of bits is generated and encoded using a FEC. The resulting bits are then passed through an interleaver. The interleaved bits are modulated and send through the propagation channel. At the receiver side, after possible processing such as channel equalization, the receiver performs a soft-demodulation of the received symbols, resulting in log-likelihood ratios (LLR). These LLR are passed through a desinterleaver, and the desinterleaved LLR are used to perform decoding.

In this case, when OFDMA is considered along with zero-forcing (ZF) one-tap equalizer followed by a soft decoding, as in [30], we use the following tight upper bound of $\pi_{k,m}(E_k)$ for medium-to-high signal-to-noise ratio (SNR).

$$\pi_{k,m}(E_k) \le \tilde{\pi}_{k,m}(E_k) := \frac{g_{k,m}}{(G_k E_k)^{d_{k,m}}} \quad (10)$$

where $g_{k,m}$ and $d_{k,m}$ are fitting coefficients obtained through least square estimation which depend both on the packet length and the considered MCS. Notice that these coefficients capture the effect of the frequency correlation due to multipath as well as the effect of the BICM. When SC-FDMA is considered, (10) is still valid for ZF equalizer followed by a soft decoding with different fitting coefficients [34]. The accuracy of the upper bound (10) is checked in Section VI-B.

Thanks to (9) and (10), we can now derive the approximated expressions of the metrics of interest, replacing $q_{k,m}$ with its upper bound $\tilde{\pi}_{k,m}$, in (6) for the goodput, in (7) for the EE, and in (8) for the GEE, leading to, $\forall k$:

$$\tilde{\eta}_k := B\gamma_k\alpha_k\frac{1-\tilde{\pi}_{k,M}(E_k)}{1+\sum_{m=1}^{M-1}\tilde{\pi}_{k,m}(E_k)}, \quad (11)$$

$$\tilde{\mathcal{E}}_k(E_k,\gamma_k) := \frac{\alpha_k\gamma_k(1-\tilde{\pi}_{k,M}(E_k))}{(1+\sum_{m=1}^{M-1}\tilde{\pi}_{k,m}(E_k))(\gamma_k E_k\kappa_k^{-1}+E_{c,k})}, \quad (12)$$

$$\tilde{\mathcal{G}}(\mathbf{E},\boldsymbol{\gamma}) := \frac{\sum_{k=1}^{K}\alpha_k\gamma_k\frac{(1-\tilde{\pi}_{k,M}(E_k))}{(1+\sum_{m=1}^{M-1}\tilde{\pi}_{k,m}(E_k))}}{\sum_{k=1}^{K}(\gamma_k E_k\kappa_k^{-1}+E_{c,k})}. \quad (13)$$

For the rest of this paper, all the derivations will be performed based on these approximations.

III. Problem Formulation

In this Section, we mathematically formulate the four EE based optimization problems we solve in Section IV. Since our objective is to allocate to each link a transmit energy and a proportion of the bandwidth, the optimization variables will be represented by vectors $\mathbf{E}$ and $\boldsymbol{\gamma}$.

A. Constraints

In our RA problems, we impose a quality of service (QoS) requirement consisting in a per-link minimum goodput constraint, denoted by $\eta_k^{(1)}$ for link k. In addition, we also consider a per-link maximum transmit power constraint, denoted by $P_{\max,k}$ for link k, in order to account for the PA characteristics and to limit the power consumption. Last, we also need to introduce a structural constraint on the bandwidth to ensure that the allocated bandwidth is less or equal to the available one.

1) Per-Link Minimum Goodput Constraint:

$$\tilde{\eta}_k \ge \eta_k^{(1)}, \forall k. \quad (14)$$

which can be rewritten as:

$$\alpha_k\gamma_k\frac{1-\tilde{\pi}_{k,M}(E_k)}{1+\sum_{k=1}^{M-1}\tilde{\pi}_{k,m}(E_k)} \ge \eta_k^{(0)}, \forall k. \quad (15)$$

with $\eta_k^{(0)} := \eta_k^{(1)}/B$.

2) Per-Link Maximum Transmit Power Constraint:

$$E_k\gamma_k \le P_{\max,k}, \forall k. \quad (16)$$

3) Bandwidth Constraint:

$$\sum_{k=1}^{K}\gamma_k \le 1. \quad (17)$$

Notice that due to the upper bound of the approximation (9)–(10), if (15) holds then necessarily $\eta_k \ge \eta_k^{(1)}$. This means that the actual goodputs obtained after RA satisfy the QoS constraints.

In the rest of this paper, we assume that we can always find $\mathbf{E}$ and $\boldsymbol{\gamma}$ such that constraints (15)–(17) are simultaneously satisfied, i.e., the feasible set is never empty, by choosing carefully $\eta_k^{(0)}$ and $P_{\max,k}$ for any k. We can now formulate RA optimization problems.

B. EE-Related Metrics

We present now the four considered EE-related metrics along with the associated optimization problems. The first three metrics are aggregation of the links individual EE while the fourth is the GEE. In details, Problem 1 (P1) is the MSEE problem, Problem 2 (P2) is the MPEE problem,[1] Problem 3 (P3) is the MMEE problem while Problem 4 (P4) is the MGEE one:

$$\textbf{P1:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{k=1}^{K} \frac{\alpha_k \tilde{D}_k(E_k)}{\tilde{S}_k(E_k)(E_k\kappa_k^{-1} + E_{c,k}\gamma_k^{-1})}, \tag{18}$$

$$\text{s.t.} \quad (15), (16), (17), \tag{19}$$

$$\textbf{P2:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{k=1}^{K} \log\left(\frac{\alpha_k \tilde{D}_k(E_k)}{\tilde{S}_k(E_k)(E_k\kappa_k^{-1} + E_{c,k}\gamma_k^{-1})}\right), \tag{20}$$

$$\text{s.t.} \quad (15), (16), (17), \tag{21}$$

$$\textbf{P3:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \min_{k\in\{1,\cdots,K\}} \frac{\alpha_k \tilde{D}_k(E_k)}{\tilde{S}_k(E_k)(E_k\kappa_k^{-1} + E_{c,k}\gamma_k^{-1})}, \tag{22}$$

$$\text{s.t.} \quad (15), (16), (17), \tag{23}$$

$$\textbf{P4:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \frac{\sum_k^K \alpha_k \gamma_k \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)}}{\sum_{k=1}^K (\gamma_k E_k \kappa_k^{-1} + E_{c,k})}, \tag{24}$$

$$\text{s.t.} \quad (15), (16), (17), \tag{25}$$

with $\tilde{D}_k(E_k) := 1 - \tilde{\pi}_{k,M}(E_k)$ and $\tilde{S}_k(E_k) := 1 + \sum_{m=1}^{M-1} \tilde{\pi}_{k,m}(E_k)$.

These criteria have different pros and cons according to the system's objectives [6]. The MSEE is flexible in terms of fairness and individual EE value since links' priority can be adapted by considering a weighted sum instead of the sum. Note that the solution provided herafter for the MSEE can be straightforwardly extended to its weighted version. The discussion about the weights selection is out of the scope of this paper. The MPEE is expected to provide a better tradeoff between individual EE and fairness than the MSEE. The MMEE enables us to achieve the highest degree of fairness. Finally, the GEE is a measure of the network EE and therefore the MGEE of interest as long as network performance is at stake.

Next, we explain the methodology for solving these four optimization problems.

C. Solution Methodology

As they are stated, P1 to P4 are not computationally tractable, meaning that they cannot be solved analytically or numerically with affordable complexity, i.e., in polynomial time. One of the main contribution of this paper is to transform these problems into equivalent ones, for which standard convex optimization tools are applicable, e.g. the interior point method (IPM). This Section is dedicated to the methodology used to achieve this purpose.

Problems P1-P4 can be written in the general form

$$\max_{\mathbf{E},\boldsymbol{\gamma}} \quad J(\mathbf{E},\boldsymbol{\gamma}), \tag{26}$$

$$\text{s.t.} \quad (15), (16), (17), \tag{27}$$

where J is a generic function representing the objective function of one of the considered problem.

We notice that the feasible set for P1-P4 defined by the constraints (15)–(17) is not convex due to the non-convexity of constraint (16), thus preventing us to use convex optimization tools. To overcome this issue, we first remark that constraints (16) and (17) are posynomials.[2] We also see that constraint (15) can be transformed into posynomial form as follows: dividing both sides by γ_k and multiplying both sides of the resulting inequality by $(1 + \sum_{m=1}^{M-1} \tilde{\pi}_{k,m}(E_k))$ yields the following posynomial form of (15), $\forall k$,

$$\eta_k^{(0)}\gamma_k^{-1}\left(1 + \sum_{m=1}^{M-1} a_{k,m} E_k^{-d_{k,m}}\right) + \alpha_k a_{k,M} E_k^{-d_{k,M}} \le \alpha_k, \tag{28}$$

with $a_{k,m} := g_{k,m}/G_k^{d_{k,m}} > 0$. Consequently, these posynomial constraints can be transformed into convex ones through a change of variables [35] detailed in the next Section. After this change of variables, Problem (26)–(27) now writes:

$$\max_{\mathbf{x},\mathbf{y}} \quad J(\mathbf{x},\mathbf{y}), \tag{29}$$

$$\text{s.t.} \quad (15)', (16)', (17)', \tag{30}$$

where $(\mathbf{x},\mathbf{y}) := \mathcal{U}(\mathbf{E},\boldsymbol{\gamma})$ with $\mathcal{U}$ a one-to-one mapping, and (15)'–(17)' are constraints (15)–(17) after the change of variables. For the rest of this paper, the new corresponding problems will be referred to as Pi' (i = 1,...,4).

In a second step, for P1'-P3', we identify properties of the new objective functions allowing us to use optimal convex optimization procedures. Concerning P4', we do not find such properties, leading us to consider preferably P4 whose structure enables us to apply two suboptimal procedures.

D. Change of Variable Leading to a Convex Feasible Set

The change of variable we apply to our problems is the one of the geometric programming (GP) [35], which writes $\mathcal{U}(\mathbf{E},\boldsymbol{\gamma}) = [\nu(E_1),\cdots,\nu(E_K),\nu(\gamma_1),\cdots,\nu(\gamma_K)]$ with $\nu(u) := \log(u)$. Hence, we have

$$\forall k, E_k = e^{x_k}, \tag{31}$$

$$\forall k, \gamma_k = e^{y_k}. \tag{32}$$

[1] This problem can also be seen as the sum of the log of the individual EE and so is related to the proportional fairness.

[2] A posynomial function is a function of the form $f(x_1,\ldots,x_n) = \sum_{k=1}^K c_k x_1^{b_{1,k}} \ldots x_K^{b_{n,k}}$ where $c_k \in \mathbb{R}^+$ and $b_{i,j} \in \mathbb{R}$.

Applying (31)–(32) and (28), (16) and (17), the constraints (15)'–(17)' write, $\forall k$:

$$\eta_k^{(0)} e^{-y_k} \left(1 + \sum_{m=1}^{M-1} a_{k,m} e^{-x_k d_{k,m}}\right) + \alpha_k a_{k,M} e^{-x_k d_{k,M}} \leq \alpha_k, \tag{33}$$

$$e^{x_k + y_k} \leq P_{\max,k}, \tag{34}$$

$$\sum_{k=1}^{K} e^{y_k} \leq 1. \tag{35}$$

The next result establishes the convexity of the set defined by the three previous equations.

Result 1: Let us define $\mathbf{x} := [x_1, \ldots, x_K]$ and $\mathbf{y} := [y_1, \ldots, y_K]$. The following set is convex

$$\mathcal{F} = \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^K \times \mathbb{R}^K \mid \text{(33)–(35) are satisfied}\}.$$

Proof: We use the following two properties: *i*) the composition of a convex function with an affine function is convex, and *ii*) the sum of convex functions is convex [35]. We see that constraints (33)–(35) are sums of functions, which are convex since they can be expressed as the composition of the exponential function, which is convex, and affine functions. Thus, contraints (33)–(35) are sums of convex functions and as a result $\mathcal{F}$ is convex. ■

IV. Solutions of the Problems

In this Section, we propose several algorithms to solve P1 to P4. Notice that we will resort several times to the epigraph form of convex optimization problems [35], which is based on the following equivalence:

$$\max_{\mathbf{x}} \mathcal{Q}(\mathbf{x}) \Leftrightarrow \max_{\mathbf{x},\lambda} \lambda, \text{ s.t. } \lambda \leq \mathcal{Q}(\mathbf{x}). \tag{36}$$

A. SEE Maximization (Problem 1)

We solve P1 by solving P1', which can be written as

$$\max_{\mathbf{x},\mathbf{y}} \quad \sum_{k=1}^{K} \frac{f_k(x_k)}{g_k(x_k, y_k)}, \tag{37a}$$

$$\text{s.t.} \quad (33), (34), (35). \tag{37b}$$

with $f_k(x_k) := \alpha_k(1 - a_{k,M} e^{-x_k d_{k,M}})$ and $g_k(x_k, y_k) := (1 + \sum_{m=1}^{M-1} a_{k,m} e^{-x_k d_{k,m}})(\kappa_k^{-1} e^{x_k} + E_{c,k} e^{-y_k})$. This equivalent problem is characterized through the following result.

Result 2: P1' is the maximization of a sum of ratios whose numerators are concave and denominators are convex, over a convex set.

Proof: The convexity of the feasible set is given by Result 1. The concavity of the f_k can be established by computing its second order derivative. Finally, the convexity of the g_k can be proved using same steps as for the proof of Result 1. ■

From Result 2, we know that the algorithm developed in [36] enables us to optimally solve P1'. This algorithm was successfully applied in RA context in several works such as [12], [37], [38]. According to [36], P1' can be optimally solved by alternating the following two steps, which are iterated until convergence to the global optimum.

1) Find the optimal solution of the problem defined by:

$$\max_{\mathbf{x},\mathbf{y}} \quad \sum_{k=1}^{K} u_k^* \left(f_k(x_k) - \beta_k^* g_k(x_k, y_k)\right), \tag{38a}$$

$$\text{s.t.} \quad (33), (34) \text{ and } (35). \tag{38b}$$

with $\forall k$, $u_k^* \geq 0$ and $\beta_k^* \geq 0$ depends on iteration the result of the previous iteration. The problem defined by (38a)–(38b) is concave (i.e., Result 2) and can thus be optimally solved using the IPM.

2) Compute $\forall k$, u_k^* and β_k^* using the modified Newton (MN) method detailed in [12, Eqs. (33)-(34)].

Numerical results for the number of iterations to reach convergence as well as the overall algorithm complexity are provided in Section V.

B. PEE Maximization (Problem 2)

We solve P2 by solving P2', which can be written as

$$\max_{\mathbf{x},\mathbf{y}} \quad \sum_{k=1}^{K} \left(\log\left(f_k(x_k)\right) - \log\left(g_k(x_k, y_k)\right)\right), \tag{39a}$$

$$\text{s.t.} \quad (33), (34), (35). \tag{39b}$$

In the following result, we exhibit a property of P2' allowing us to find its optimal solution.

Result 3: P2' is the maximization of a concave function over a convex set.

Proof: The convexity of the feasible set is ensured by Result 1. The objective function (39a) can be written as $\sum_{k=1}^{K} \mathcal{W}_k(x_k, y_k)$, with $\mathcal{W}_k(x_k, y_k) := \log(f_k(x_k)) - \log(g_k(x_k, y_k))$. Let us prove that $\mathcal{W}_k(x_k, y_k)$ is concave. To do so, first, we remind that the logarithm of a concave function is concave [35]. As a consequence, since $f_k(x_k)$ is concave (see, i.e., Result 2), $\log(f_k(x_k))$ is concave. Second, we prove that $\log(g_k(x_k, y_k))$ is convex and hence $-\log(g_k(x_k, y_k))$ is concave. The proof is based on the following key property: the log-sum-exp function, defined by

$$L(x_1, \cdots, x_n) := \log\left(\sum_{k=1}^{n} e^{x_k}\right),$$

is convex [35]. Using this property, we can then infer that $\log(g_k(x_k, y_k))$ is convex. Hence, $\mathcal{W}_k$ is concave and finally, $\sum_{k=1}^{K} \mathcal{W}_k(x_k, y_k)$ is concave. ■

This problem is simpler than the MSEE problem since it can be optimally solved directly using numerical algorithms such as the IPM [35], requiring no additional computation.

C. Max-Min Fairness (Problem 3)

We solve P3 by solving P3', which can be written as

$$\max_{\mathbf{x},\mathbf{y}} \quad \min_{k \in \{1,\cdots,K\}} \frac{f_k(x_k)}{g_k(x_k, y_k)}, \tag{40a}$$

$$\text{s.t.} \quad (33), (34), (35). \tag{40b}$$

Due to Results 1 and 2, one can check that this problem is the maximization of the minimum of a set of ratios with concave numerators and convex denominators, over a convex set. Hence, this problem falls within the generalized fractional programming framework, and could be solved with the Generalized Dinkelbach's (GD) algorithm, as done for instance in [13]. The GD algorithm is iterative, and it requires to alternately solve a concave maximization problem, and update a parameter. However, by taking a closer look at our objective function (40a), we are able to find a simpler procedure (not iterative) to solve this problem. To do so, we observe that each f_k and g_k in (40a) are a combination of exponentials. Hence, our idea is to introduce a monomial[3] auxiliary optimization variable and to perform the change of variable of GP in this new variable to obtain a convex optimization problem.

Following (36), we introduce the optimization variable ϕ, and the following constraint $\phi \leq \min_k \frac{f_k(x_k)}{g_k(x_k,y_k)}$. Noticing that $\phi \leq \min_k \frac{f_k(x_k)}{g_k(x_k,y_k)} \Leftrightarrow \phi \leq \frac{f_k(x_k)}{g_k(x_k,y_k)}, \forall k$, we can rewrite the problem equivalently as

$$\max_{\mathbf{x},\mathbf{y},\phi} \quad \phi, \tag{41a}$$

$$\text{s.t.} \quad \phi g_k(x_k,y_k) - f_k(x_k) \leq 0, \quad \forall k, \tag{41b}$$

$$(33), (34), (35). \tag{41c}$$

In this new problem, the objective function is linear and hence concave, but the K new constraints given by (41b) are not convex due to the product between ϕ and g_k. To render them convex, we can remark that g_k is a sum of exponential in x_k and y_k. Performing the change of variable of the GP on ϕ, i.e., $\phi := e^z$, enables us to obtain convex constraints since we exhibit a linear combination of exp–sum with non-negative weights. After this change of variable, the following equivalent problem can be written

$$\max_{\mathbf{x},\mathbf{y},z} \quad e^z, \tag{42a}$$

$$\text{s.t.} \quad e^z g_k(x_k,y_k) - f_k(x_k) \leq 0, \quad \forall k, \tag{42b}$$

$$(33), (34), (35). \tag{42c}$$

All the constraints defined by (42b)–(42c) are now convex. However, the objective function (42a) is not concave anymore. To overcome this issue, one can remark that maximizing e^z is equivalent to minimizing $1/e^z = e^{-z}$, which is convex. The resulting equivalent optimization problem now writes in the following convex form

$$\min_{\mathbf{x},\mathbf{y},z} \quad e^{-z}, \tag{43a}$$

$$(42b), (33), (34), (35). \tag{43b}$$

The problem defined by (43a)–(43b) is the minimization of a convex function over a convex set, and thus it can be optimally solved using the IPM.

[3]A monomial function is a function of the form $f(x_1, \cdots, x_n) = cx_1^{b_1} \ldots x_K^{b_n}$ where $c \in \mathbb{R}^+$ and $b_i \in \mathbb{R}$.

D. GEE Maximization (Problem 4)

Unlike P1-P3 we do not succeed to find optimal procedures to solve P4, whereas in the literature, the MGEE problem without multiuser interference is easily solved since the capacity is considered as the measure of the goodput with no HARQ. As a consequence the GEE reduces to a ratio between a concave and a convex function, which can be efficiently maximized using the Dinkelbach's algorithm [39]. However, in our case, the MGEE problem is more complicated due to the consideration of the HARQ mechanism and practical MCS because one can prove that the numerator of the GEE is not necessarily a concave function even after the change of variables (31)–(32). For this reason, we propose two suboptimal solutions (meaning that we do not have global optimality guarantee) to solve this problem, one based on successive convex approximation (SCA), and the other one based on alternating optimization (AO). Even if we cannot ensure these algorithms to find the global optimum of P4, we can prove that the SCA based algorithm produces a sequence of point converging to a point satisfying the Karush-Kuhn-Tucker (KKT) conditions [35] of P4, and that the AO algorithm converges. It is worth noticing that, contrarily to P1-P3, we address the solution of P4 starting from the problem before the change of variables (31)–(32) since we are able to observe specific structure of this problem. Let us first explain the SCA based solution.

1) Successive Convex Approximation: When dealing with a minimization problem with a non-convex objective function, a conventional procedure is to resort to the SCA approach. For instance, in [40], the authors use the SCA procedure to provide suboptimal MSEE solution in a multicell context, when capacity achieving codes are assumed without HARQ.

The SCA approach is an iterative one: for a given iteration, it consists in approximating a non-convex problem around a feasible point by a convex problem we optimally solve, and to use this optimal solution as the initialization for the next iteration. Under some conditions on the approximation (detailed below), it can be proved that the SCA procedure produces a sequence of solutions converging to a point satisfying the KKT conditions [6, Proposition 4.2].

Let $\bar{\mathbf{x}}$ denote the feasible point around which the problem is approximated. The convergence conditions of the SCA procedure are the following ones:

1) The objective function of the approximate problem is an upper bound of the original one.
2) The approximate objective function is equal to the original one at $\bar{\mathbf{x}}$.
3) The gradient of the approximate objective function is equal to the gradient of the original one at $\bar{\mathbf{x}}$.

Looking at our optimization problem, we see that all the constraints and the denominator of the objective function are posynomials, but the numerator is not posynomial. This problem is close to the framework proposed in [41], where a SCA procedure, called single condensation method for GP, is proposed to solve the problem of the minimization of a ratio of posynomials with posynomial constraints. Hence, our idea is to transform our optimization problem in order to use [41]. The first step is to transform the numerator of the objective function into a posynomial. To do so, similarly to Section IV-C and following (36), we introduce K new optimization variables $[z_1, \cdots, z_K]$ along with K new constraints $z_k \leq \alpha_k \gamma_k \tilde{D}_k(E_k)/\tilde{S}_k(E_k)$, which will be shown to be posynomials. The second step is to transform the maximization problem into a minimization one, by taking the inverse of the resulting objective function. After these two steps, P4 can be rewritten equivalently as

$$\min_{\mathbf{E},\boldsymbol{\gamma},\mathbf{z}} \quad \frac{\sum_{k=1}^{K}(\gamma_k E_k \kappa_k^{-1} + E_{c,k})}{\sum_{k=1}^{K} z_k}, \tag{44a}$$

$$\text{s.t.} \quad z_k \leq \alpha_k \gamma_k \tilde{D}_k(E_k)/\tilde{S}_k(E_k), \tag{44b}$$

$$(15), (16), (17), \tag{44c}$$

with $\mathbf{z} := [z_1, \cdots, z_K]$. We can see that the problem defined by (44a)–(44c) is the minimization of a ratio of posynomials with posynomial constraints since constraint (44b) can be rewritten equivalently as

$$z_k \gamma_k^{-1}\left(1 + \sum_{m=1}^{M-1} a_{k,m} E_k^{-d_{k,m}}\right) + \alpha_k a_{k,M} E_k^{-d_{k,M}} \leq \alpha_k \tag{45}$$

which is posynomial. The solution proposed in [41] is to replace the denominator in (44a), at each iteration, with its best monomial lower bound in the sense of its Taylor approximation about the solution found at the previous iteration. To do so, let us first define $\mathbf{E}^{*(i)} := [E_1^{*(i)}, \cdots, E_K^{*(i)}]$ and $\boldsymbol{\gamma}^{*(i)} := [\gamma_1^{*(i)}, \cdots, \gamma_K^{*(i)}]$ the optimal solution at the end of the ith iteration of the SCA procedure. To derive the lower bound of the denominator of the ratio at the $(i+1)$th iteration, the authors of [41] take advantage of the arithmetic-geometric mean inequality to write

$$\sum_{k=1}^{K} z_k \geq \prod_{k=1}^{K}\left(\frac{z_k}{\nu_k^{(i)}}\right)^{\nu_k^{(i)}} \tag{46}$$

where

$$\nu_k^{(i)} := \frac{\mathcal{H}_k(E_k^{*(i)}, \gamma_k^{*(i)})}{\sum_{k=1}^{K}\mathcal{H}_k(E_k^{*(i)}, \gamma_k^{*(i)})} \tag{47}$$

with $\mathcal{H}_k(E_k, \gamma_k) := \alpha_k \gamma_k \tilde{D}_k(E_k)/\tilde{S}_k(E_k)$. In [41], it is proved that this lower bound meets the assumptions from [42] ensuring the convergence of the SCA to a KKT solution.

The problem defined by (44a)–(44c) is then approximated by replacing $\sum_{k=1}^{K} z_k$ in (44a) with its lower bound given in (46). The resulting approximated problem writes

$$\min_{\mathbf{E},\boldsymbol{\gamma},\mathbf{z}} \quad \sum_{k=1}^{K}(\gamma_k E_k \kappa_k^{-1} + E_{c,k})\left(\prod_{k=1}^{K}\frac{z_k}{\nu_k^{(i)}}\right)^{-\nu_k^{(i)}}, \tag{48a}$$

$$\text{s.t.} \quad (45), (15), (16), (17). \tag{48b}$$

The problem defined by (48a)–(48b) is the minimization of a posynomial function (48a) with posynomial constraints (48b). Then, it can be optimally solved by applying the change of variables of the GP, i.e., (31)–(32) for $\mathbf{E}$ and $\boldsymbol{\gamma}$, and $\Phi_k := e^{z_k}$, and by using the IPM on the resulting problem. We initialize the algorithm using the algorithm provided in [30] solving the

Algorithm 1: SCA Based MGEE Solution.

Initialize $\mathbf{E}^{*(0)}, \boldsymbol{\gamma}^{*(0)}$
For all k, compute $\nu_k^{(0)}$ using (47)
Find $(\mathbf{E}^{*(1)}, \boldsymbol{\gamma}^{*(1)})$ solving the problem defined by (48a)–(48b)
For all k, compute $\nu_k^{(1)}$ using (47)
Set $\epsilon_{SCA} > 0, i = 1$
while $||[\mathbf{E}^{*(i)}, \boldsymbol{\gamma}^{*(i)}] - [\mathbf{E}^{(i-1)}, \boldsymbol{\gamma}^{(i-1)}]|| > \epsilon_{SCA}$ **do**
    Find $(\mathbf{E}^{*(i+1)}, \boldsymbol{\gamma}^{*(i+1)})$ solving the problem defined by (48a)–(48b)
    For all k, compute $\nu_k^{(i+1)}$ using (47)
    $i = i + 1$;
end while

MPO problem. Finally, the SCA procedure is summarized in Algorithm 1, which creates a sequence of points converging to a KKT solution of P4. Numerical results for the number of iterations for Algorithm 1 to reach convergence as well as the overall algorithm complexity analysis are provided in Section V.

2) Alternating Optimization: in the AO framework, the optimization is performed alternately with respect to the variables $\mathbf{E}$ and $\boldsymbol{\gamma}$ until convergence is reached [43]. We first explain how to perform the optimization w.r.t. $\boldsymbol{\gamma}$ with fixed $\mathbf{E}$, and next the procedure to optimize w.r.t. $\mathbf{E}$ for given $\boldsymbol{\gamma}$.

a) Optimization w.r.t. $\boldsymbol{\gamma}$: when $\mathbf{E}$ is fixed, the optimization problem writes as

$$\max_{\boldsymbol{\gamma}} \quad \frac{\sum_{k=1}^{K} A_k \gamma_k}{\sum_{k=1}^{K}(B_k \gamma_k + E_{c,k})}, \tag{49a}$$

$$\text{s.t.} \quad \gamma_k \geq \gamma_{\min,k}, \tag{49b}$$

$$\gamma_k \leq \gamma_{\max,k}, \tag{49c}$$

$$\sum_{k=1}^{K}\gamma_k \leq 1, \tag{49d}$$

with $A_k := \alpha_k \tilde{D}_k(E_k)/\tilde{S}_k(E_k)$, $B_k := E_k/\kappa_k$, $\gamma_{\min,k} := \eta_k^{(0)} \tilde{S}_k(E_k)/(\alpha_k \tilde{D}_k(E_k))$ and $\gamma_{\max,k} := P_{\max,k}/E_k$.

The problem defined by (49a)–(49d) is a linear fractional programming problem, i.e., an optimization problem whose objective function (49a) is the ratio of two linear functions and whose constraints are all linear. Hence, it can be efficiently solved using the Charnes-Cooper transformation [44], for which we add the following $(K+1)$ new optimization variables

$$r_k := \frac{\gamma_k}{\sum_{k=1}^{K}(B_k \gamma_k + E_{c,k})}, \quad \forall k, \tag{50}$$

$$t := \frac{1}{\sum_{k=1}^{K}(B_k \gamma_k + E_{c,k})}. \tag{51}$$

The problem defined by (49a)–(49d) can then be equivalently rewritten in a linear form [44], which can be optimally solved using numerical algorithms such as the simplex method [35] or the IPM. The optimal solution of the original problem

(49a)–(49d) can then be deduced from (50)–(51) as follows

$$\gamma_k^* = \frac{r_k^*}{t^*}, \quad \forall k, \tag{52}$$

where r_k^* and t^* are the optimal solution of the equivalent linear problem.

b) Optimization w.r.t. $\mathbf{E}$: the optimization problem to solve P4 when $\boldsymbol{\gamma}$ is fixed is the following one

$$\max_{\mathbf{E}} \quad \frac{\sum_{k=1}^K C_k \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)}}{\sum_{k=1}^K (F_k E_k + E_{c,k})}, \tag{53a}$$

$$\text{s.t.} \quad \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)} \geq M_k, \quad \forall k, \tag{53b}$$

$$E_k \leq E_{\max,k}, \quad \forall k, \tag{53c}$$

where $C_k := \alpha_k \gamma_k$, $F_k := \gamma_k \kappa_k$, $M_k := \eta_k^{(0)}/(\alpha_k \gamma_k)$ and $E_{\max,k} := P_{\max,k}/\gamma_k$.

The problem defined by (53a)–(53c) is a fractional programming problem and hence, it can be solved using the Dinkelbach's algorithm since both the numerator and denominator of the objective function (53a) are positive differentiable functions, and the feasible set is compact [6, pp. 243]. The Dinkelbach's algorithm is iterative and, for a given iteration $(i+1)$, it requires to optimally solve the following problem:

$$\max_{\mathbf{E}} \quad \sum_{k=1}^K \left(C_k \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)} - \lambda_D^{(i)} (F_k E_k + E_{c,k}) \right), \tag{54a}$$

$$\text{s.t.} \quad (53b), (53c), \tag{54b}$$

where $\lambda_D^{(i)} \geq 0$ depends on the optimal solution of the ith iteration (the update rule for this parameter is given in Algorithm 2, line 5). This problem defined by (54a)–(54b) is not concave due to the non concavity of the objective function (54a) and then we cannot apply the IPM to solve it. However, we are able to optimally solve it for certain configurations, detailed later. To do so, we first remark that this problem is separable into K subproblems since there is no coupling constraints between the elements of $\mathbf{E}$. The K resulting subproblems can be written as

$$\max_{E_k} \quad C_k \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)} - \lambda_D^{(i)} F_k E_k, \quad \forall k, \tag{55a}$$

$$\text{s.t.} \quad (53b), (53c). \tag{55b}$$

The objective functions (55a) of the K subproblems are not posynomials, but following (36), it is possible to alleviate this issue by introducing K auxiliary optimization variables (one per subproblem) w_k along with K new constraints similarly to Section IV-C, leading to the following K subproblems

$$\max_{E_k, w_k} \quad w_k \tag{56a}$$

$$\text{s.t.} \quad (\lambda_D^{(i)} F_k E_k + w_k)\tilde{S}_k(E_k) - C_k \tilde{D}_k(E_k) \leq 0, \quad \forall k, \tag{56b}$$

$$M_k \tilde{S}_k(E_k) - \tilde{D}_k(E_k) \leq 0, \quad \forall k, \tag{56c}$$

$$E_k \leq E_{\max,k}, \quad \forall k. \tag{56d}$$

Algorithm 2: AO Based MGEE Solution w.r.t. $\mathbf{E}$.

1: Set $\epsilon_D > 0$, $\lambda_D^{(0)} = 0$, $i = 0$
2: For all k, find $E_k^{*(1)}$ solving the problem defined by (57a)–(57b)
3: **while** $\mathcal{G}_D^{(i)}(\lambda_D^{(i)}) > \epsilon_D$ **do**
4: $\quad i = i + 1$;
5: $\quad \lambda_D^{(i)} = \frac{\sum_{k=1}^K C_k \frac{\tilde{D}_k(E_k^{*(i)})}{\tilde{S}_k(E_k^{*(i)})}}{\sum_{k=1}^K (F_k E_k^{*(i)} + E_{c,k})}$
6: $\quad$ For all k, find $E_k^{*(i+1)}$ solving the problem defined by (57a)–(57b)
7: **end while**

Similarly to (28), (56b) and (56c) can be rewritten in a posynomial form. As a consequence, the problem defined by (56a)–(56d) is the maximization of a monomial function with posynomials constraints, and it can be turned into a standard GP as follows

$$\min_{E_k, w_k} \quad w_k^{-1}, \tag{57a}$$

$$\text{s.t.} \quad (56b), (56c), (56d). \tag{57b}$$

The problem defined by (57a)–(57b) is a geometric program and thus it can be optimally solved performing the change of variable (31) on E_k, by setting $w_k = e^{\Psi_k}$, and using the IPM on the resulting equivalent problem.

Notice that this approach does not work if the maximum of the subproblem defined by (55a)–(55b) is negative since it implies $w_k \leq 0$ and as a result, we cannot apply the change of variable $w_k = e^{\Psi_k}$. If this case occurs, it is always possible to switch the SCA based procedure using the end of the last feasible iterations of the AO based procedure for initialization.

Finally, the procedure to optimally solve the problem defined by (54a)–(54b) is given in Algorithm 2 for which we define the function $\lambda \mapsto \mathcal{G}_D^{(i)}(\lambda) := \sum_{k=1}^K (C_k \frac{\tilde{D}_k(E_k^{*(i+1)})}{\tilde{S}_k(E_k^{*(i+1)})} - \lambda(F_k E_k^{*(i+1)} + E_{c,k}))$, with $E_k^{*(i+1)}$ the optimal solution of the problem defined by (57a)–(57b). Numerical results for the number of iterations for Algorithm 2 to reach convergence are provided in Section V.

c) AO based algorithm: Finally, the AO based suboptimal solution of P4 is depicted in Algorithm 3, whose convergence is guaranteed since it creates a monotonically increasing sequence of GEE. Notice that, unlike for the SCA based solution, we do not have guarantee that the resulting solution satisfies the KKT conditions. Numerical results for the number of iterations for Algorithm 3 to reach convergence as well as the overall algorithm complexity analysis are provided in Section V.

V. Complexity Analysis

Here, we analyze the proposed solutions' complexity. We first remind that they are all iterative, and at each iteration, they use the IPM. The IPM numerically solves the KKT conditions using the Newton method [35]. There exist a number of different ver-

Algorithm 3: AO Based MGEE Solution.

Set $\epsilon > 0$, $i = 0$, $C_D = \epsilon + 1$.
Find initial feasible $\mathbf{E}^{(0)} := [E_1^{(0)}, \cdots, E_L^{(0)}]$ and $\boldsymbol{\gamma}^{(0)} := [\gamma_1^{(0)}, \cdots, \gamma_L^{(0)}]$.
while $C_D > \epsilon$ **do**
  Find $\mathbf{E}^{(i+1)} := [E_1^{(i+1)}, \cdots, E_L^{(i+1)}]$ the optimal solution of the problem defined by (53a)–(53c) with $\boldsymbol{\gamma}^{(i)}$ using Algorithm 2.
  Find $\boldsymbol{\gamma}^{(i+1)} := [\gamma_1^{(i+1)}, \cdots, \gamma_L^{(i+1)}]$ the optimal solution of the problem defined by (49a)–(49d) with $\mathbf{E}^{(i+1)}$ solving the linear program resulting from the Charnes-Cooper transformation (i.e., Section IV-D2a).
  Set $C_D = ||[\mathbf{E}^{(i+1)}, \boldsymbol{\gamma}^{(i+1)}] - [\mathbf{E}^{(i)}, \boldsymbol{\gamma}^{(i)}]||$.
  Set $i = i + 1$.
end while

sions of the IPMs (barrier or primal dual methods for instance), with their own convergence rate.

In [45, pp. 4], an upper bound on the IPM complexity is given by $\rho := V(V^3 + C)$, where V (resp. C) is the number of optimization variables (resp. constraints) of the optimization problem. Moreover, if we define N as the number of times the IPM is used for a given algorithm, the overall complexity to reach the convergence for the algorithms solving the MSEE, the MPEE, the MMEE and the SCA based MGEE problems is

$$N\mathcal{O}(\rho).$$

Concerning the AO based algorithm solving the MGEE, the complexity is given by

$$N_{out}\mathcal{O}(\rho_\gamma + NK\mathcal{O}(\rho_E)),$$

where N_{out} is the number of times the algorithm alternates between the optimization w.r.t. $\mathbf{E}$ and $\boldsymbol{\gamma}$, $\rho_\gamma := V_\gamma(V_\gamma^3 + C_\gamma)$ where C_γ (resp. V_γ) is the number of constraints (resp. variables) of the optimization problem w.r.t. $\boldsymbol{\gamma}$, and $\rho_E := V_E(V_E^3 + C_E)$ where C_E (resp. V_E) is the number of constraints (resp. variables) of the optimization problem w.r.t. $\mathbf{E}$. In Table I, we report the values of C and V, the average values of N and N_{out} for the considered algorithms, and the total number of flops for the setup used in Section VI.

According to Table I, the complexity of the algorithms can be split into two classes. The first class includes the algorithms solving the MSEE and the MGEE-SCA problems which are the most complex algorithms because of their high number of iterations to converge. The second class gathers the algorithms solving the MPEE, the MMEE and the MGEE-AO problems which are less complex. Among all the proposed algorithms, the algorithm solving the MPEE is the less complex one.

VI. Numerical Results

A. Simulation Setup

In this paper, we consider only the use of the OFDMA as the multiple access technology, but results with SC-FDMA are quite similar and lead to the same conclusions. We use the IR-HARQ scheme based on a convolutional code with initial rate $R_k = 1/2$ described in [46], and we use a quadrature phase shift keying (QPSK) modulation. The number of links is $K = 8$ and the link distances δ_k are uniformly drawn in [50 m, 1 km]. We set $B = 5$ MHz, $N_0 = -170$ dBm/Hz and $\mathcal{L}_k = 128$. The carrier frequency is $f_c = 2400$ MHz and we put $\zeta_k^2 = (4\pi f_c/c)^{-2}\delta_k^{-3}$ where c is the speed of light in vacuum. We assume that the required goodput per-link is equal for all links, and unless otherwise stated, is equal to $\eta_k^{(1)} = 62.5$ kbits/s. Also, unless in Section VI-E, we put $M = 3$. We also consider that for all k, $P_{ctx,k} = P_{crx,k} = 0.4$ W [13] and $\kappa_k = 0.5$. All points have been obtained by averaging through 50 random networks configurations.

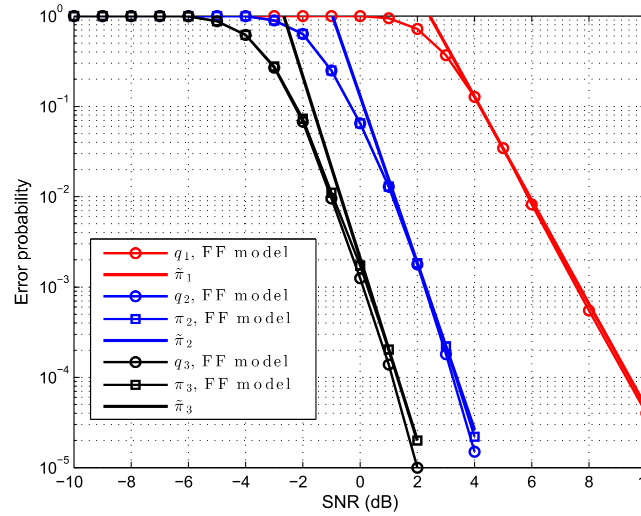

Fig. 1. Error probability approximation, FF model.

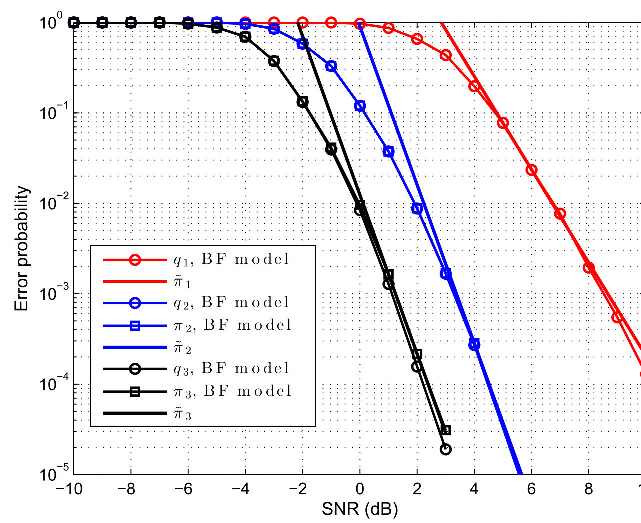

Fig. 2. Error probability approximation, BF model.

B. Tightness of the Error Probability Approximation

In Figs. 1 and 2, we plot the error probability along with the approximation, whose coefficients are reported in Table II for the FF and BF models, respectively. The fast fading (FF) model

TABLE I
PROBLEMS DIMENSIONALITY AND NUMBER OF ITERATIONS

	MSEE	MPEE	MMEE	MGEE-SCA	MGEE-AO (γ: left, E: right)	
V	$2K$	$2K$	$2K+1$	$3K$	$K+1$	1
C	$2K+1$	$2K+1$	$3K+1$	$3K+1$	$2K+3$	3
N	979.1	1	1	839.18	1	3.65
N_{out}	-	-	-	-	3	
Total flops ($K=8$)	64 432 613	65 808	83 946	27 892 329	20 546	

TABLE II
FITTING COEFFICIENTS

	$g_{k,p}$, FF model	$d_{k,p}$, FF model	$g_{k,p}$, BF model	$d_{k,p}$, BF model
$p=1$	25.04	5.73	29.33	5.16
$p=2$	0.13	9.23	0.91	8.76
$p=3$	0.0021	10.07	0.012	8.79

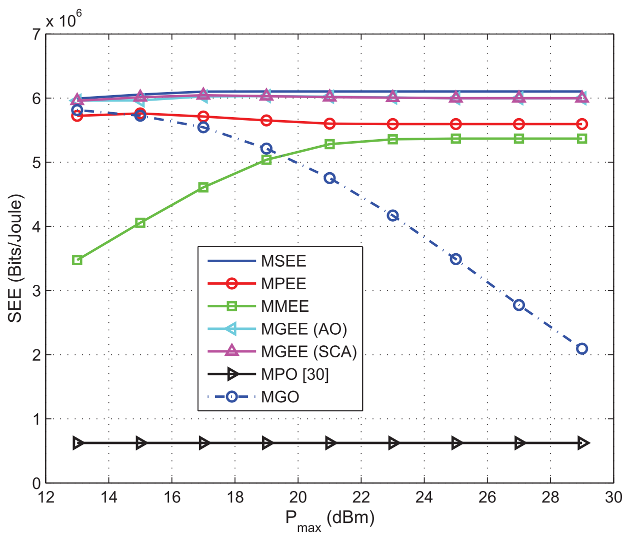

Fig. 3. SEE obtained with the proposed algorithms versus $P_{\max}$.

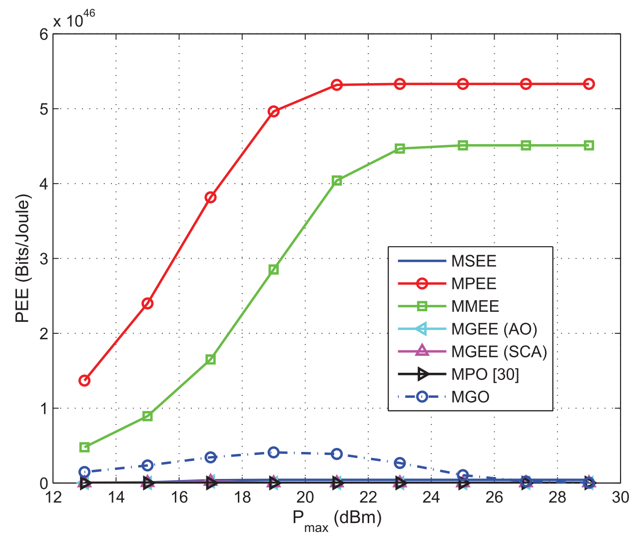

Fig. 4. PEE obtained with the proposed algorithms versus $P_{\max}$.

corresponds to the ideal case in which the interleaving allows each modulated symbols to act over independent frequency bins realizations. The BF Rayleigh channel is simulated with $L=10$ and $\sigma^2_{k,\ell} = \frac{\zeta_k^2}{L}, \forall k, \ell$. We use 256 subcarriers with 20 randomly chosen subcarriers allocated to the link of interest. The codeword of 128 modulated symbols is thus spanned over 7 OFDMA symbols.

We can see in Figs. 1 and 2 that the approximation, whose coefficients are reported in Table II, is tight for both models for medium-to-high SNR which means that our paper applies in both cases. Second, the frequency correlation induces 1 dB performance loss of BF model compared to FF model and a small loss in diversity as it can be seen in Table II (i.e., the values of $d_{k,m}$ are very close under both models).

All the results exhibited in the rest of this Section are obtained considering the case of ideal FF channel.

C. Performance Analysis

In Figs. 3 to 6, we plot as the function of the maximum transmit power the SEE, PEE, MEE, and GEE obtained with the proposed algorithms, the MPO, and the MGO.

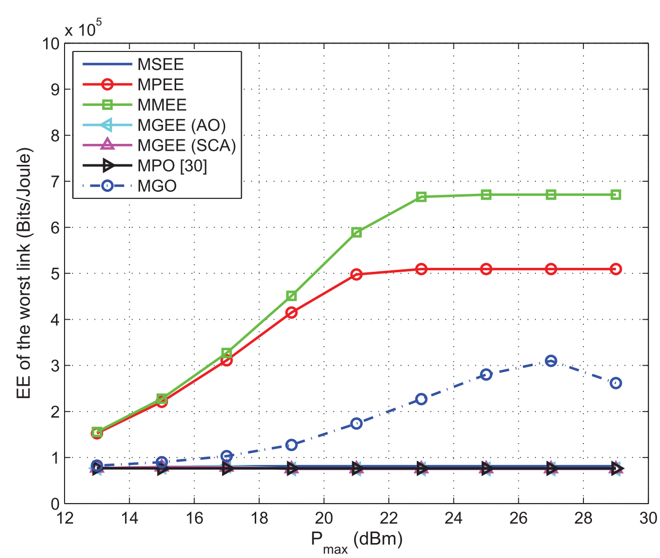

Fig. 5. MEE obtained with the proposed algorithms versus $P_{\max}$.

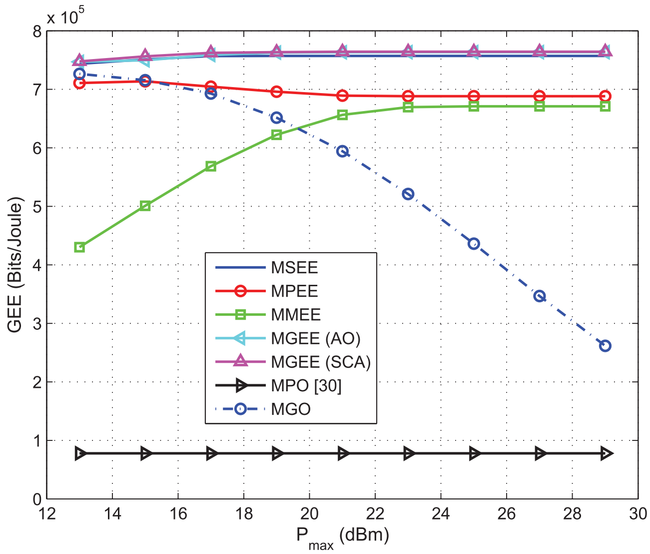

Fig. 6. GEE obtained with the proposed algorithms versus $P_{\max}$.

The comparison between EE related criteria with MPO and MGO shows that: *i*) the MPO gives systematically the worst performance, *ii*) the MGO gives bad MEE and PEE whereas it is comparable to SEE and GEE for low $P_{\max}$ but degrades when $P_{\max}$ increases. Both behaviors can be explained because: the EE given by (12) is a unimodal function[4] of E_k for fixed γ, with a unique maximizer $E_k^* := \arg\max \mathcal{E}_k$, and the E_k obtained by MPO (resp. MGO) is much lower (resp. larger) than E_k^*. As a consequence, these two criteria achieve low EE values.

The comparison between the EE related criteria leads to the following observations: *i*) the results are in agreement with what is expected, i.e., maximizing a given criterion leads to the highest values with regard to this criterion. *ii*) Regarding the MGEE criterion, both SCA and AO achieve almost the same performance. Since we established that the AO has much less complexity, we recommend to use it for practical implementation. *iii*) Among all the criteria, the MPEE achieves almost the best performance for all the metrics. Moreover, since it has the lowest complexity, it makes it attractive for practical implementations.

From the above observations, we provide the following recommendations for applying our algorithms to communication systems when EE is concerned. For D2D communications, maximizing individual EE is of interest, and thus the MMEE is a good candidate. However, MMEE performs badly for the other criteria which means that the individual EE is low. Thus we recommend the use of MPEE, because of observation (*iii*) in the previous paragraph, and the fact that its performance is close to MMEE in terms of MEE. For this reason, we consider only this criterion in the rest of this Section.

D. Application to the Smartphone Case

In this Section, we illustrate on a practical example the real effectiveness of the EE criterion to achieve a better user experience than other common criteria such as MGO and MPO. We consider the case of a smartphone which has to send messages one after the other and evaluate for different criteria the performance achieved in terms of number of transmitted messages and battery drain. Let us consider a battery with capacity $Q_0 = 3000$ mAh, with voltage $U = 3.85$ V, which are typical values for recent smartphones. The battery drain equation as a function of time for user k is given by $Q_k(t) = Q_0 - \frac{P_u t}{U}$, with P_u the power consumption. We investigate two scenarios: in the first one, each link has to transmit 10^7 messages corresponding to packets of length $\mathcal{L}_k = 128$ bits. In the second one, each link sends messages until its battery is empty. For both scenarios, we compute the following metrics, averaged over all users: Q_r the remaining battery (in %), T_t the time to transmit the messages (in s), N_p the number of transmitted messages, n_t the average number of HARQ rounds, η_k^A the goodput (in kbits/s), and $\bar{\gamma}$ the average used bandwidth (in %). We compare the MPEE and the two conventional criteria: the MGO and the MPO when $P_{\max} = 25$ dBm. The results are reported in Table III.

In the first scenario, the MPEE is the best one since it transmits all the messages within the shortest duration, with the least energy consumption. It is followed by the MGO which also succeeds to transmit all the messages but in a longer duration and with more energy consumption. The longer duration is explained because the sum of the links' goodput is proportional to the harmonic mean of the transmit duration, but maximizing this harmonic mean does not guarantee that the mean of the transmit duration is minimized. The MPO criterion gives the worst result since the battery goes flat before succeeding to transmit all the messages. We can first see from $\bar{\gamma}$ that the MPO allocates little proportion of the bandwidth to the users which implies that the transmit duration of each message is long as observed through T_t. This actually explains the small goodput. Second, the MPO succeeds to use low transmit power taking advantage of the retransmission capability of HARQ to achieve the target goodput at the expense of the time duration. Finally the tradeoff between the (very low) transmit power and the (very large) time duration is disastrous for the energy consumed by the MPO for sending the pre-fixed number of messages.

In the second scenario, the MPEE transmits more packets than the other criteria when the whole battery is used. The battery lifetime for the MPEE is also longer than for the MGO. Indeed, the average goodput is almost the same for the MPEE and the MGO, but the energy consumption is much lower for the MPEE, which gives a better tradeoff between the energy consumption and the goodput. The results for the MPO are identical to the ones for the first scenario since the baterry was already empty.

To summarize, when RA is performed using the MPEE criterion, either the links can transmit more packets in average than when using the MPO and the MGO at the end of the battery lifetime, or the nodes have higher battery levels in average for the same number of transmit messages. This clearly demonstrates the practical relevance of considering the EE when designing a RA procedure.

E. Impact of the Retransmission Number in the HARQ Process

Here, we discuss the impact of the maximum number of transmissions M of the HARQ process. To do so, we compute the gain in terms of PEE when $M = 2$ and $M = 3$ compared

[4]as already observed in [6] for a different framework.

TABLE III
COMPARISON OF MPEE WITH CONVENTIONAL CRITERIA IN TERMS OF BATTERY LIFETIME AND TIME TO TRANSMIT INFORMATION

Scenario	Criterion	Q_r	T_t (s)	N_p	η_k^A (kbits/s)	n_t	$\bar{\gamma}$ (%)
10^7 sent messages	MPEE	83%	2150	1×10^7	6150	1.02	100%
	MGO	72%	2222	1×10^7	6250	1	100%
	MPO	0%	14 271	7×10^6	63	1.2	12.2%
Full battery drain	MPEE	0%	12 937	6×10^7	6150	1.02	100%
	MGO	0%	8063	4×10^7	6250	1	100%
	MPO	0%	14 271	7×10^6	63	1.2	12.2%

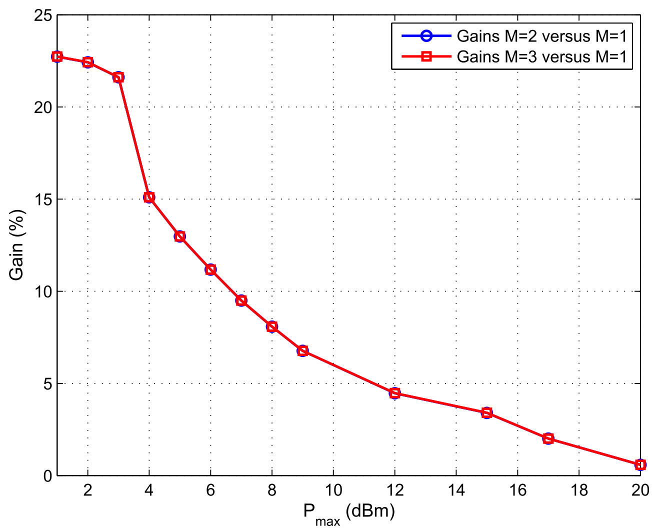

Fig. 7. PEE gains when $M = 2$ and $M = 3$ compared to $M = 1$ versus $P_{\max}$.

with $M = 1$, defined as:

$$100 \times \left(\frac{\text{PEE}_j}{\text{PEE}_1} - 1 \right), \quad j = 2, 3,$$

where PEE_i, $i = 1, 2, 3$ stands for the optimal PEE value obtained for $M = i$. Fig. 7 represents this gain as a function of $P_{\max}$ for $\eta_k^{(1)} = 1.25$ kbits/s.

First of all, one can prove that the EE is increasing with respect to M since the sufficient condition on the $q_{k,m}$s given in [32], which writes can be written as:

$$\frac{q_{k,m+1}}{q_{k,m}} \leq \frac{q_{k,m}}{q_{k,m-1}}, \quad \forall m \geq 1, \tag{58}$$

holds in our setup, explaining the strictly positive gains observed in Fig. 7.

Second, we remark that the PEE gains obtained when $M = 2$ is numerically almost identical to the one when $M = 3$ for the considered range of $P_{\max}$. This is because the throughput resulting from RA for $M = 2$ and $M = 3$ are very close. In contrast, we observe a large gain between $M = 1$ and $M = 2$ at low and medium $P_{\max}$. For instance, when $P_{\max} = 0$ dBm, the gain is about 22%. We deduce that HARQ is a relevant way to increase the PEE. For our setup, we advocate to choose $M = 2$ instead of $M = 3$.

VII. CONCLUSION

We provided a framework for energy efficient RA for HARQ with practical MCS and statistical CSI in a multiuser context. We formulated four EE optimization problems with a constraint on the minimum goodput and on the maximum transmit power per link. We succeeded to transform the problems such that they can be resolved using standard convex optimization tools. The MSEE, MPEE, and MMEE problems were optimally solved, whereas two suboptimal algorithms were proposed for the MGEE problem. We analyzed the performance of the proposed criteria along with the complexity of the corresponding algorithms. Our objective for future work is to take into account other statistical channel models, such as the Rician channel which is especially suited to millimeter waves environment.

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