Estimation of the Ricean K Factor in the presence of shadowing

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***Abstract*—We address the estimation of the Ricean K factor when the available complex channel samples are noisy and subject to Nakagami-m shadowing, i.e., the line-of-sight component is modeled as a Nakagami-m random variable. We propose two estimators: one based on the expectation-maximization (EM) procedure, and a second one based on the method of moment (MoM). The MoM estimator can be used to initialize the EM procedure. We show by simulations that the two proposed estimators outperform the existing ones.**

***Index Terms*—Channel, Estimation, Ricean, Shadowing.**

I. Introduction

The propagation channel in mobile communications is modeled by a random variable characterized by deterministic parameters such as its mean and its variance. Estimating these parameters is of great interest since knowing them enables the system designer to perform efficient resource allocation [1].

A wireless channel can be analyzed at different scale: at the large scale, the deterministic *pathloss* is dominant. At the medium scale, *shadowing* has to be taken into account. At the small scale, the most common stochastic model for the wireless channel is to consider the Ricean model where the impulse response follows a non-zero mean Gaussian distribution [2]. The Ricean model is characterized by the so-called Ricean K factor which is related to the proportion of the mean with respect to the random part. This parameter is an important indicator of the link quality. For instance, when $K = 0$, the Ricean channel boils down to the Rayleigh one.

The objective of this letter is to estimate the Ricean K factor when *i)* shadowing is considered at the medium scale, and *ii)* noisy complex channels' samples are available at the receiver.

Many estimators have been developed in the literature under the assumptions of noiseless complex-valued channels' samples without shadowing [3]–[8]. When noisy samples without shadowing are considered, the existing estimators are based either on real-valued channels' samples, i.e., the magnitude of the channels' components [9], [10] or on the complex-valued channels' samples [11], [12]. Only a few works investigate the case with shadowing [13], [14]. More precisely, in [13], an estimator based on noiseless magnitude of the channels' components is proposed and follows the method of moments (MoM) approach [15, Chapter 9]. This estimator requires a prior estimation step for the shadowing's statistical parameters. Very recently, in [14], another MoM approach is proposed by considering high-order statistics (up to 6th order). As a consequence, the algorithm requires a large observation window, typically, up to $100,000$ samples for standard operating signal-to-noise ratio (SNR) where the system operates. In both papers, the shadowing is modeled with a Nakagami-m distribution, and changes between two consecutive channel samples independently. This shadowing model was first proposed in [16] and then often used, see [17], [18] for instance.

The contribution of this paper is as follows: we propose two estimators for the Ricean K factor based on the complex-valued noisy channels samples and assuming a Nakagami-m stochastic model for the shadowing with arbitrary coherence time. Our first estimator is based on the expectation-maximization (EM) procedure whereas the second one is based on the MoM. The MoM may be used to initialize the EM.

The rest of the paper is organized as follows. In Section II, we provide the system and channel models. In Section III, we introduce our proposed estimators. These estimators are numerically evaluated and compared to the state-of-the-art in Section IV. Concluding remarks are drawn in Section V.

II. System and channel models

We estimate the Ricean K factor for an arbitrary point-to-point link using an orthogonal frequency division multiple access (OFDMA)-type communication where only a subset of subcarriers is assigned to this link.

We assume that the received signal on the n_sth subcarrier of the i_oth OFDMA symbol can be written as:

$$Y(i_o, n_s) = H(i_o, n_s)X(i_o, n_s) + W(i_o, n_s), \qquad (1)$$

where $X(i_o, n_s)$ is the ith transmit symbol on subcarrier n, $W(i_o, n_s) \sim \mathcal{CN}(0, 2\sigma_n^2)$ is a circularly-symmetric complex-valued white Gaussian noise with zero-mean and variance $2\sigma_n^2$ which is assumed to be known by the receiver[1], and $H(i_o, n_s)$ is the channel frequency response on subcarrier n_s for the i_oth OFDMA symbol. In this letter, we model $H(i_o, n_s)$ using the channel model introduced in [16], which is a Ricean channel whose line of sight (LoS) is subject to partial blockage and thus it is random. More precisely, this LoS is assumed to follow a Nakagami-m distribution. The resulting channel model can be written as follows:

$$H(i_o, n_s) = ac(i_0)e^{j\theta_0} + H_R(i_0, n_s), \qquad (2)$$

where $H_R(i_0, n)$ is the diffuse component which is a circularly symmetric Gaussian random variable with zero mean and variance $2\sigma_h^2$, a is the magnitude of the LoS component, $c(i_0)$ is the shadowing attenuation for OFDMA symbol i_o [16] whose power is assumed to be normalized without any loss of generality (the power of the shadowing component is taken into account in a), and θ_0 represents the common phase. As in [16], the θ_0 is assumed to be constant which is realistic if the number of samples to estimate the Ricean factor

[1]The estimation of the noise variance in OFDMA systems is addressed for instance in [19]. It can be performed prior to our estimation problem, and it is out of the scope of this paper.

is small enough. For instance, if the receiver is moving at $v = 50$ km/h with carrier frequency $f_c = 2.4$ GHz, the phase difference is upper-bounded by 4^o when the time interval between two pilots is $T_s = 1.01$ μs and the number of pilots is $N = 100$. We have observed that the proposed estimators are robust to such a variation. Notice that θ_0 is assumed to change independently from sample to sample in [14], yielding the authors to use samples magnitude to estimate K and to use large values of N. We assume that $c(i_o)$ is constant over N_s OFDMA symbols, and varies independently every N_s symbols following a Nakagami-m distribution, whose probability density function (PDF) is:

$$f_{c_n}(x) = \frac{2m^m}{\Gamma(m)} x^{2m-1} e^{-mx^2}, \ \forall x \geq 0, \tag{3}$$

If $N_s = \infty$, there is no shadowing, and we fix $c(i_o) = 1$, $\forall i_o$.

From (2), we define the Ricean K factor of the channel as the ratio between the average power of the LoS and the average power of the non-LoS component as in [13], [14], i.e., $K := a^2/(2\sigma_h^2)$. This definition is consistent with the case without shadowing.

We assume that the channel is estimated from (1) using pilot symbols, meaning that $X(i_o, n_s)$ is known from the receiver. The channel samples $H(i_o, n_s)$ can be estimated as $\tilde{H}(i_o, n_s) = Y(i_o, n_s)/X(i_o, n_s)$. Assuming a normalized quadrature phase shift keying (QPSK) for the pilots, we get

$$\tilde{H}(i_o, n_s) = H(i_o, n_s) + \hat{W}(i_o, n_s), \tag{4}$$

where $\hat{W}(i_o, n_s) \sim \mathcal{CN}(0, 2\sigma_n^2)$ is the error in the channel estimation. The number of pilot symbols per OFDMA symbol is denoted by i_p whereas the number of available OFDMA symbols is denoted by i_s. The total number of available estimated channel samples is thus $N := i_s \times i_p$. We assume that the frequency spacing between two pilot symbols within one OFDMA symbol is larger than the channel's coherence bandwidth (which can be evaluated with [20]), and thus we neglect the channel's frequency correlation. We also neglect the channel's fast fading time correlation. We assume $N_s \leq i_s$ in order to see the shadowing effect. Let us rewrite our problem in a more compact shape: let $N_c := i_s/N_s$ (assumed to be an integer for the sake of simplicity) be the number of different realizations of the shadowing, let $N_\ell := N_s i_p$ be the number of pilots per shadowing realization. We define the entries of the $N_\ell \times N_c$ matrix $\hat{\mathbf{H}}$ as follows

$$\hat{H}(i,n) = \tilde{H}(1+(i-1 \bmod i_p), 1+(n-1)N_s + \left\lfloor \frac{i-1}{i_p} \right\rfloor) \tag{5}$$

with $\lfloor x \rfloor$ the floor function, and $x \bmod y$ the modulo operator.

One can prove that

$$\hat{\mathbf{H}}_n \sim \mathcal{CN}\left(c_n a e^{j\theta_0} \mathbf{1}_{N_c}, \text{diag}_{N_\ell \times N_\ell}(2\sigma_h^2 + 2\sigma_n^2)\right)$$

where $\hat{\mathbf{H}}_n$ is the nth column of $\hat{\mathbf{H}}$, $\mathbf{1}_{N_\ell}$ is the column vector composed by N_ℓ ones, $\{c_n := c(1 + (n \bmod N_s))\}_{n=1,\ldots,N_s}$ are independent and identically distributed (i.i.d.) random variables whose PDF is given by (3). Moreover, $\forall n_1 \neq n_2$, $\mathbb{E}[(\hat{\mathbf{H}}_{n_1} - c_{n_1}\mathbf{1}_{N_\ell})^*(\hat{\mathbf{H}}_{n_2} - c_{n_2}\mathbf{1}_{N_\ell})] = 0$, where $(.)^*$ stands for the transpose-conjugate operator.

Our matrix model encompasses the case without shadowing by setting $N_\ell = N$, $N_c = 1$ and $c_1 = 1$, or the case considered in [13] and [14] by setting $2\sigma_n^2 = 0$, $N_\ell = 1$ and $N_c = N$.

Our goal is to estimate $K = a^2/(2\sigma_h^2)$ from the channel estimates $\hat{\mathbf{H}}$ when θ_0, a, σ_h^2, and m are unknown whereas σ_n^2 is known. The vector of unknown parameters is denoted by $\boldsymbol{\theta} = [\theta_0, a, 2\sigma_h^2, m]$.

III. Proposed estimators of the Ricean K factor

We propose to estimate the Ricean K factor using the EM procedure, which has been originally proposed in [21] and then widely used for channel statistical parameters estimation [22]–[24]. This procedure aims to find local maximum of the likelihood function iteratively. It is especially interesting when analytical maximization of the log-likelihood function is intractable, but is rendered possible by fixing some parameters.

Let $\mathbb{L}_{\hat{\mathbf{H}}}(\hat{\mathbf{H}};\boldsymbol{\theta})$ be the likelihood of $\hat{\mathbf{H}}$ for the parameters $\boldsymbol{\theta}$. The maximum-likelihood estimator is given by $\arg\max_{\boldsymbol{\theta}} \mathbb{L}_{\hat{\mathbf{H}}}(\hat{\mathbf{H}};\boldsymbol{\theta})$. As the columns of $\hat{\mathbf{H}}$ are independent, we get

$$\log\left(\mathbb{L}_{\hat{\mathbf{H}}}\left(\hat{\mathbf{H}};\boldsymbol{\theta}\right)\right) = \sum_{n=1}^{N_c} \log\left(\mathbb{L}_{\hat{\mathbf{H}}_n}\left(\hat{\mathbf{H}}_n;\boldsymbol{\theta}\right)\right), \tag{6}$$

where $\mathbb{L}_{\hat{\mathbf{H}}_n}(\hat{\mathbf{H}}_n;\boldsymbol{\theta})$ is the likelihood of $\hat{\mathbf{H}}_n$. Like in [14], we use the law of total probability which yields:

$$\mathbb{L}_{\hat{\mathbf{H}}_n}(\hat{\mathbf{H}}_n;\boldsymbol{\theta}) = \int_0^{+\infty} \mathbb{L}_{\hat{\mathbf{H}}_n|c_n}(\hat{\mathbf{H}}_n|x;\boldsymbol{\theta}) f_{c_n}(x) dx, \tag{7}$$

where $\mathbb{L}_{\hat{\mathbf{H}}_n|c_n}(\hat{\mathbf{H}}_n|x;\boldsymbol{\theta})$ is the likelihood of $\hat{\mathbf{H}}_n$ knowing that $c_n = x$. Then we have

$$\mathbb{L}_{\hat{\mathbf{H}}_n|c_n}(\hat{\mathbf{H}}_n|x;\boldsymbol{\theta}) = \frac{e^{-\frac{\sum_{i=1}^{N_\ell} |\hat{H}(i,n) - xae^{j\theta_0}|^2}{2\sigma_h^2 + 2\sigma_n^2}}}{(\pi(2\sigma_h^2 + 2\sigma_n^2))^{N_\ell}}. \tag{8}$$

Plugging (8) and (3) into (7) and using [25, 3.462] yields

$$\mathbb{L}_{\hat{\mathbf{H}}_n}(\hat{\mathbf{H}}_n;\boldsymbol{\theta}) = \frac{\mathcal{B}_{1,n}\Gamma(2m) e^{\frac{\mathcal{B}_{3,n}^2}{8\mathcal{B}_{2,n}}}}{(2\mathcal{B}_{2,n})^m} . D_{-2m}\left(\frac{\mathcal{B}_{3,n}}{\sqrt{2\mathcal{B}_{2,n}}}\right) \tag{9}$$

with

$$\begin{aligned}
\mathcal{B}_{1,n} &= \left(\frac{2m^m}{(\pi(2\sigma_h^2 + 2\sigma_n^2))^{N_\ell}\Gamma(m)}\right) e^{-\sum_{i=1}^{N_\ell} \frac{|\hat{H}(i,n)|^2}{2\sigma_h^2 + 2\sigma_n^2}}, \\
\mathcal{B}_{2,n} &= N_\ell \frac{a^2}{2\sigma_h^2 + 2\sigma_n^2} + m, \\
\mathcal{B}_{3,n} &= -\frac{a}{\sigma_h^2 + \sigma_n^2} \sum_{i=1}^{N_\ell} \Re\left(\hat{H}(i,n) e^{-j\theta_0}\right),
\end{aligned}$$

and where $D_{-2m}(\bullet)$ is the parabolic cylinder function [25, 9.24-9.25]. This function is due to the presence of the Nakagami-m shadowing and prevents us from maximizing the likelihood function (9) analytically. To overcome this issue, we propose to resort to the EM procedure, by seeing the shadowing $\mathbf{c} = [c_1, \ldots, c_{N_c}]$ as *nuisance parameters*. The next two steps are performed at iteration t of the EM procedure:

expectation (E) step: let $\hat{\boldsymbol{\theta}}^{(t)} = \left[\hat{a}^{(t)}, 2\hat{\sigma}_h^{2,(t)}, \hat{\theta}_0^{(t)}, \hat{m}^{(t)}\right]$ be the available parameters' estimation at iteration t. This step consists in computing the following expectation:

$$Q_{\rm EM}\left(\boldsymbol{\theta},\hat{\boldsymbol{\theta}}^{(t)}\right)=\mathbb{E}_{\mathbf{c}|\hat{\mathbf{H}},\hat{\boldsymbol{\theta}}^{(t)}}\left[\log\left(\mathbb{L}_{\hat{\mathbf{H}},\mathbf{c}}(\hat{\mathbf{H}},\mathbf{c};\boldsymbol{\theta})\right)\right], \quad (10)$$

maximization (M) step: this step consists in finding $\hat{\boldsymbol{\theta}}^{(t+1)}$ as follows

$$\hat{\boldsymbol{\theta}}^{(t+1)}=\arg\max_{\boldsymbol{\theta}}\left\{Q_{\rm EM}\left(\boldsymbol{\theta},\hat{\boldsymbol{\theta}}^{(t)}\right)\right\}. \quad (11)$$

According to [21], the EM procedure converges to a local maximum of the likelihood function given in (6). Our goal is now to apply this procedure to our estimation problem, i.e., to characterize $Q_{\rm EM}$ and to maximize it in closed-form.

A. The expectation step

First, we express in closed-form $\log\left(\mathbb{L}_{\hat{\mathbf{H}},\mathbf{c}}(\hat{\mathbf{H}},\mathbf{c};\boldsymbol{\theta})\right)$. To do so, we decompose it as follows:

$$\log\left(\mathbb{L}_{\hat{\mathbf{H}},\mathbf{c}}(\hat{\mathbf{H}},\mathbf{c};\boldsymbol{\theta})\right)=\log\left(\mathbb{L}_{\hat{\mathbf{H}}|\mathbf{c}}(\hat{\mathbf{H}}|\mathbf{c};\boldsymbol{\theta})\right)+\log\left(\mathbb{L}_{\mathbf{c}}(\mathbf{c};\boldsymbol{\theta})\right), \quad (12)$$

where $\mathbb{L}_{\mathbf{c}}(\mathbf{c};\boldsymbol{\theta})$ is the likelihood function of $\mathbf{c}$. For the first term in right hand side (RHS), we get

$$\log\left(\mathbb{L}_{\hat{\mathbf{H}}|\mathbf{c}}\left(\hat{\mathbf{H}}|\mathbf{c};\boldsymbol{\theta}\right)\right)=-N_cN_\ell\log\left(\pi\left(2\sigma_h^2+2\sigma_n^2\right)\right)-\frac{\sum_{i=1}^{N_\ell}\sum_{n=1}^{N_c}|\hat{H}(i,n)-c_nae^{j\theta_0}|^2}{2\sigma_h^2+2\sigma_n^2}. \quad (13)$$

For the second term in RHS, as the components of $\mathbf{c}$ are i.i.d. whose PDF are given by (3), we obtain

$$\begin{aligned}\log\left(\mathbb{L}_{\mathbf{c}}(\mathbf{c};\boldsymbol{\theta})\right) &= N_c\left(m\log(m)+\log(2)-\log\left(\Gamma(m)\right)\right)\\ &+ (2m-1)\sum_{n=1}^{N_c}\log\left(c_n\right)-m\sum_{n=1}^{N_c}c_n^2. \quad (14)\end{aligned}$$

Plugging (13) and (14) into (10) yields

$$\begin{aligned}Q_{\rm EM}\left(\boldsymbol{\theta},\hat{\boldsymbol{\theta}}^{(t)}\right)=&-N_cN_\ell\log\left(\pi\left(2\sigma_h^2+2\sigma_n^2\right)\right)-\\ &\frac{\sum_{i=1}^{N_\ell}\sum_{n=1}^{N_c}|\hat{H}(i,n)|^2}{2\sigma_h^2+2\sigma_n^2}+\\ &\frac{\sum_{i=1}^{N_\ell}\sum_{n=1}^{N_c}T_1^{(t)}(n)a\Re(\hat{H}(i,n)e^{-j\theta_0})}{\sigma_h^2+\sigma_n^2}-\\ &N_\ell\sum_{n=1}^{N_c}T_2^{(t)}(n)\frac{a^2}{2\sigma_h^2+2\sigma_n^2}+\\ &N_c\left(m\log(m)+\log(2)-\log\left(\Gamma(m)\right)\right)+\\ &(2m-1)\sum_{n=1}^{N_c}T_3^{(t)}(n)-m\sum_{n=1}^{N_c}T_2^{(t)}(n),\end{aligned} \quad (15)$$

with $T_1^{(t)}(n):=\mathbb{E}_{c_n|\hat{\mathbf{H}},\hat{\boldsymbol{\theta}}^{(t)}}[c_n]$, $T_2^{(t)}(n):=\mathbb{E}_{c_n|\hat{\mathbf{H}},\hat{\boldsymbol{\theta}}^{(t)}}[c_n^2]$, and $T_3^{(t)}(n):=\mathbb{E}_{c_n|\hat{\mathbf{H}},\hat{\boldsymbol{\theta}}^{(t)}}[\log(c_n)]$.

According to Bayes' rule, we have

$$T_k^{(t)}(n)=\frac{U_k^{(t)}(n)}{U_0^{(t)}(n)}, \quad k=1,2,3 \quad (16)$$

where $U_k^{(t)}(n)=\int_0^{+\infty}\mathbb{L}_{\hat{\mathbf{H}}_n|c_n}\left(\hat{\mathbf{H}}_n|x;\hat{\boldsymbol{\theta}}^{(t)}\right)f_{c_n}(x)g_k(x)dx$, with $g_k(x)=x^k$ for $k=\{0,1,2\}$, and $g_3(x)=\log(x)$.

After some derivations which are omitted due to space limitation, we get:

$$\begin{aligned}U_k^{(t)}(n) &= \mathcal{B}_{1,n}^{(t)}\left(2\mathcal{B}_{2,n}^{(t)}\right)^{-\frac{2m+k}{2}}\Gamma(2\hat{m}^{(t)}+k)e^{\frac{\left(\mathcal{B}_{3,n}^{(t)}\right)^2}{8\mathcal{B}_{2,n}^{(t)}}}\\ &\times D_{-2\hat{m}^{(t)}-k}\left(\frac{\mathcal{B}_{3,n}^{(t)}}{\sqrt{2\mathcal{B}_{2,n}^{(t)}}}\right)\end{aligned} \quad (17)$$

for $k=\{0,1,2\}$, and

$$\begin{aligned}U_3^{(t)}(n) &= \mathcal{B}_{1,n}^{(t)}e^{\frac{\left(\mathcal{B}_{3,n}^{(t)}\right)^2}{8\mathcal{B}_{2,n}^{(t)}}}\Gamma(2\hat{m}^{(t)})\left(2\mathcal{B}_{2,n}^{(t)}\right)^{-\hat{m}^{(t)}}\\ &\times \left(-\frac{1}{2}\log\left(2\mathcal{B}_{2,n}^{(t)}\right)D_{-2\hat{m}^{(t)}}\left(\frac{\mathcal{B}_{3,n}^{(t)}}{\sqrt{2\mathcal{B}_{2,n}^{(t)}}}\right)\right.\\ &+ \psi_0\left(2\hat{m}^{(t)}\right)D_{-2\hat{m}^{(t)}}\left(\frac{\mathcal{B}_{3,n}^{(t)}}{\sqrt{2\mathcal{B}_{2,n}^{(t)}}}\right)\\ &+ \left.\frac{\partial}{\partial w}D_{-2\hat{m}^{(t)}-w}\left(\frac{\mathcal{B}_{3,n}^{(t)}}{\sqrt{2\mathcal{B}_{2,n}^{(t)}}}\right)_{|w=0}\right),\end{aligned} \quad (18)$$

where $\psi_0(x)$ is the digamma function, and $\mathcal{B}_{i,n}^{(t)}$ is defined as $\mathcal{B}_{i,n}$ where values of $\boldsymbol{\theta}$ have been replaced with these of $\boldsymbol{\theta}^{(t)}$. We evaluate the derivative as follows $\frac{\partial}{\partial w}f(w)_{|w=0}\approx(f(\varepsilon)-f(-\varepsilon))/(2\varepsilon)$ for any function f. In our simulations in Section IV, we use $\epsilon=10^{-3}$.

B. The maximization step

The optimization over $\boldsymbol{\theta}$ of $Q_{\rm EM}(\boldsymbol{\theta},\hat{\boldsymbol{\theta}}^{(t)})$ given (15) is obtained by computing its derivative. After some algebraic manipulations, we obtain the following estimates:

$$\hat{\theta}_0^{(t+1)}=\angle\left(\sum_{i=1}^{N_\ell}\sum_{n=1}^{N_c}T_1^{(t)}(n)\hat{H}(i,n)\right), \quad (19)$$

$$\hat{a}^{(t+1)}=\frac{\sum_{i=1}^{N_\ell}\sum_{n=1}^{N_c}T_1^{(t)}(n)\Re\left(\hat{H}(i,n)e^{-j\hat{\theta}_0^{(t+1)}}\right)}{N_\ell\sum_{n=1}^{N_c}T_2^{(t)}(n)}, \quad (20)$$

$$\begin{aligned}2\hat{\sigma}_h^{2,(t+1)}=&\frac{1}{N_\ell N_c}\sum_{i=1}^{N_\ell}\sum_{n=1}^{N_c}\left(|\hat{H}(i,n)|^2+T_2^{(t)}(n)(\hat{a}^{(t+1)})^2\right.\\ &\left.-2T_1^{(t)}(n)\hat{a}^{(t+1)}\Re\left(\hat{H}(i,n)e^{-j\hat{\theta}_0^{(t+1)}}\right)\right)-2\sigma_n^2,\end{aligned} \quad (21)$$

and

$$\begin{aligned}\hat{m}^{(t+1)} &= \arg\max_m\left\{\log\left(\frac{m^m}{\Gamma(m)}\right)-\frac{m}{N_c}\sum_{n=1}^{N_c}T_2^{(t)}(n)\right.\\ &+ \left.\frac{2m-1}{N_c}\sum_{n=1}^{N_c}T_3^{(t)}(n)\right\},\end{aligned} \quad (22)$$

where $\angle(z)$ is the phase of the complex number z. The estimators of θ_0, a and $2\sigma_h^2$ at iteration t have been obtained in closed-form while the estimator of m still requires a one dimension maximization, which can be efficiently performed using the Newton method or the bisection method.

C. Starting and stopping points for the EM procedure

We propose not to initialize the EM procedure randomly, but to consider the MoM adapted to shadowing case. Notice that this MoM based estimator is also new.

Let us first remind that $\hat{H}(i,n)$ can be expressed as follows:

$$\hat{H}(i,n) = c_n a e^{j\theta_0} + H_c(i,n), \tag{23}$$

with $H_c(i,n) \sim \mathcal{CN}(0, 2\sigma_h^2 + 2\sigma_n^2)$. Like [13], (23) leads to obtain closed-form expressions for $\mu_1 = \mathbb{E}[\hat{H}(i,n)]$, $\mu_2 = \mathbb{E}[|\hat{H}(i,n)|^2]$, and $\mu_4 = \mathbb{E}[|\hat{H}(i,n)|^4]$ as follows

$$\begin{aligned} \mu_1 &= \frac{a}{\sqrt{m}}\left(\frac{\Gamma(m+0.5)}{\Gamma(m)}\right)e^{j\theta_0}, && (24)\\ \mu_2 &= 2\sigma_h^2 + 2\sigma_n^2 + a^2, && (25)\\ \mu_4 &= 2\left(2\sigma_h^2 + 2\sigma_n^2\right)^2 + 4\left(2\sigma_h^2 + 2\sigma_n^2\right)a^2 + \\ &\quad \frac{m+1}{m}a^4. && (26) \end{aligned}$$

Plugging (25) into (26) by removing $\sigma_h^2 + \sigma_n^2$, yields

$$m = \frac{a^4}{\mu_4 - 2\left(\mu_2 - a^2\right)^2 - 4\left(\mu_2 - a^2\right)a^2 - a^4}. \tag{27}$$

Plugging (27) into (24) provides the following estimator $\hat{a}^{(0)}$ for a by solving the following implicit function:

$$|\hat{\mu}_1|^2 = \frac{\left(\hat{a}^{(0)}\right)^2}{u\left(\hat{a}^{(0)}\right)}\left(\frac{\Gamma\left(u\left(\hat{a}^{(0)}\right)+0.5\right)}{\Gamma\left(u\left(\hat{a}^{(0)}\right)\right)}\right)^2, \tag{28}$$

where $u(x) = x^4/(\hat{\mu}_4 - 2\left(\hat{\mu}_2 - x^2\right)^2 - 4\left(\hat{\mu}_2 - x^2\right)x^2 - x^4)$, and $\hat{\mu}_1$, $\hat{\mu}_2$, and $\hat{\mu}_4$ are the empirical estimates of μ_1, μ_2, and μ_4 respectively. In our simulations, we solved the implicit function (28) as $\hat{a}^{(0)} = \arg\min_{\hat{a}\in\mathcal{A}} ||\hat{\mu}_1|^2 - \left(\hat{a}\right)^2/u\left(\hat{a}\right)\Gamma\left(u\left(\hat{a}\right)+0.5\right)/\Gamma\left(u\left(\hat{a}\right)\right)^2|$ with $\mathcal{A} = \{n \times 0.001\}_{n\in\{1,10^5\}}$.

Finally, from (27), (25), and (24), we respectively obtain

$$\begin{aligned} \hat{m}^{(0)} &= u\left(\hat{a}^{(0)}\right), && (29)\\ 2\hat{\sigma}_h^{2,(0)} &= \hat{\mu}_2 - 2\sigma_n^2 - \left(\hat{a}^{(0)}\right)^2, && (30)\\ \hat{\theta}_0^{(0)} &= \angle\left(\sqrt{\hat{m}^{(0)}}\frac{\hat{\mu}_1\Gamma\left(\hat{m}^{(0)}\right)}{\hat{a}^{(0)}\Gamma\left(\hat{m}^{(0)}+0.5\right)}\right). && (31) \end{aligned}$$

Therefore, the EM is initialized as follows:

$$\hat{K}_{\text{MoM}} = \frac{\left(\hat{a}^{(0)}\right)^2}{2\hat{\sigma}_h^{2,(0)}}, \tag{32}$$

and it is stopped at iteration t_{EM} when the condition $||\hat{\boldsymbol{\theta}}^{(t_{\text{EM}})} - \hat{\boldsymbol{\theta}}^{(t_{\text{EM}}-1)}|| < \epsilon_{\text{EM}}$ is fulfilled, providing the following Ricean K factor estimate:

$$\hat{K}_{\text{EM}} = \frac{\left(\hat{a}^{(t_{\text{EM}})}\right)^2}{2\hat{\sigma}_h^{2,(t_{\text{EM}})}}. \tag{33}$$

D. Complexity analysis

The EM is more complex than the MoM since it is iterative and initialized by the MoM. Each iteration of the EM requires to compute (19)-(22) whose complexity is driven by (22) which is solved by Newton or bisection methods. Moreover, we have observed through simulations that the EM converges to a nearly optimal point with about 15 iterations. Consequently, the overall complexity of the EM is roughly 15 times the complexity of the Newton or bisection methods.

IV. Numerical results

We perform simulations to compare the proposed EM based estimator $\hat{K}_{\text{EM}}$ as well as the proposed MoM estimator $\hat{K}_{\text{MoM}}$ derived in (32) with the following ones: $\hat{K}_{\text{WMoM}}$ from [13], and $\hat{K}_{\text{MML}}$ from [12]. We have also simulated $\hat{K}_{\text{LMoM}}$ from [14] but not displayed its performance which are very poor for the chosen values for N since this estimator uses the 6-th order moment of the samples and thus requires very large sample size. Notice that $\hat{K}_{\text{WMoM}}$ is implemented by knowing m as in [13] whereas $\hat{K}_{\text{EM}}$ use its own estimates for m, and $\hat{K}_{\text{MML}}$ does not require an estimate of m since it does not take into account the shadowing.

Our simulation setup is as follows: we set $N_c = N$, $N_\ell = 1$ as in [13], i.e., one subcarrier is used for channel estimation and the shadowing changes independently between consecutive OFDMA symbols. We set $N = 100$, and $\epsilon_{\text{EM}} = 10^{-5}$. The estimators' performance are averaged through $10,000$ Monte-Carlo simulations.

In Fig. 1, we set $m = 5$, $2\sigma_n^2 = 0$, i.e., the channel is perfectly known, and we plot the normalized mean square error (NMSE) for the different estimators versus K. Both proposed estimators perform better than these from [12], and [13]. This is rational since [12] does not take into account shadowing, and [13] does not use the phase information of the complex samples. For small K, $\hat{K}_{\text{MML}}$ yields better performance than other estimators. This can be explain as for low K, the deterministic component in the channel impulse response is not dominant and thus the shadowing has less impact as already observed in [14]. We see that the EM offers gains as compared to the MoM. For instance when $K = 5$, the NMSE of the EM is approximately 70 % lower than those of the MoM.

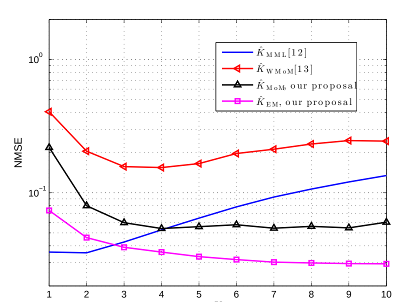

Fig. 1: Performance of the considered estimators versus K, $2\sigma_n^2 = 0$, $N = 100$, $m = 5$.

In Fig. 2, we plot the NMSE for the different estimators versus SNR, defined as $(a^2 + 2\sigma_h^2)/2\sigma_n^2$. Once again $\hat{K}_{\text{EM}}$

yields the best performance except for low SNR where the shadowing may be neglected and $\hat{K}_{\mathrm{MML}}$ is better.

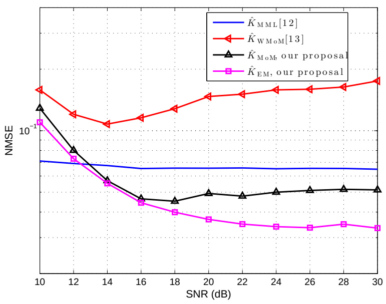

Fig. 2: Performance of the considered estimators versus the SNR, $N = 100$, $K = 5$, $m = 5$.

We have also observed through simulations that the two proposed estimators outperform the state of the art ones for N varying from 100 to 10^4 (not shown due to space limitation).

Now, we study the impact of m on the estimators' performance. In Fig. 3, we set $2\sigma_n^2 = 0$, $K = 5$, $N = 100$, and we plot the performance of the different estimators versus the value of m. We can see that *i)* the NMSE of the proposed estimators is once again better than those of [13], regardless of the value of m, and *ii)* $\hat{K}_{\mathrm{MML}}$ provides better performance than the proposed estimators when $m > 8$. Indeed, when m is large enough, the Nakagami-m distribution is close to a Dirac around its expectation and thus it is preferable to compute the best known deterministic estimator in the absence of shadowing ($\hat{K}_{\mathrm{MML}}$) than ours which need to estimate an extra parameter (m).

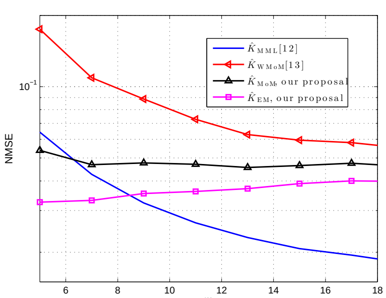

Fig. 3: Performance of the considered estimators versus m, $N = 100$, $2\sigma_n^2 = 0$.

Finally, we observed that: *i*) random initialization of the EM leads in some cases to divergence of the algorithm, *ii*) initializing EM using MoM always converges, and *iii*) when random initialization converges, both performance are close. Thus, we recommend initializating EM procedure using MoM.

V. Conclusion

We addressed the estimation of the Ricean K factor from noisy complex channel samples subject to Nakagami-m shadowing. We proposed two estimation procedures: one based on the EM and the other one based on the MoM. We provided numerical results and showed that both EM and MoM estimators outperform the existing ones from the literature.

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