

Optimal Resource Allocation for Type-II HARQ based OFDMA Ad Hoc Networks under Individual Rate and PER Constraints

Nassar Ksairi, Philippe Ciblat
Telecom ParisTech,
46 rue Barrault 75013 Paris

Email: {nassar.ksairi,philippe.ciblat}@telecom-paristech.fr

Christophe Le Martret
Thales Communications and Security,
4 rue des Louvresses 92622 Gennevilliers
Email: christophe.le_martret@thalesgroup.com

Abstract—In this paper, we address the problem of multiuser power and bandwidth allocation for Orthogonal Frequency Division Multiple Access (OFDMA) networks employing a Type-II Hybrid Automatic Repeat reQuest (HARQ) mechanism, practical Coding and Modulation Schemes (MCSs) and Bit Interleaved Coded Modulation (BICM). The problem is formulated as minimizing the sum power required to satisfy a goodput constraint for each link while its post-HARQ Packet Error Rate (PER) does not exceed a certain threshold. Assuming statistical Channel State Information (CSI), we propose a computationally-efficient algorithm to compute the corresponding optimal resource allocation. We finally provide a practical selection of the MCSs that significantly boosts the performance of the proposed resource allocation algorithm.

I. INTRODUCTION

HARQ is considered a powerful link-layer mechanism that allows reliable communications over time-varying channels. Among the different HARQ schemes, the so-called Type-II, which includes Chase Combining (CC-HARQ) and Incremental Redundancy (IR-HARQ) [1], is the most promising in terms of performance. On the physical layer, random Subcarrier Assignment Scheme (SAS) and BICM allow to harvest the inherent diversity in wireless links while OFDMA allows to handle multi-path and multi-user interference. It is thus of great interest to address sum transmit-power minimization for OFDMA networks that use BICM, MCSs and Type-II HARQ. Note that transmit power minimization is crucial to reduce energy consumption and to minimize the impact produced by the network on other systems through interference. Although the above problem arises in a wide class of wireless systems, we are mainly interested in this article in HARQ-based ad hoc networks. In such networks, generally a node called “resource manager” is elected to perform the resource allocation. The time delay between the initiation of a specific link and the reception by the resource manager of the CSI feedback associated with that link may last several frame periods. As a consequence, the resource manager has only outdated CSI whereas it can have accurate *statistical* CSI due to the much larger coherence time of the latter. We therefore assume that

the resource allocation should be done with only statistical CSI about the different wireless links.

The minimization of the sum transmit power emitted by the network should be done subject to some relevant statistical Quality-of-Service (QoS) constraints. In HARQ-based wireless systems, a widely-used QoS metric that well represents the data rate is the so-called *goodput* which is proportional to the useful data rate (after the discarded erroneous packets are accounted for). Although its mathematical expression depends on the PER, using the goodput as the only QoS measure does not always guarantee that the PER value is kept below a certain threshold [2]. This last requirement is nonetheless crucial in modern communications systems, especially for applications such as video streaming and voice communications where the packet drop rate has to be kept low enough. Therefore, we should make sure that both goodput and PER constraints are respected while performing the resource allocation process.

In the literature, few works that addressed multiuser resource allocation for communication systems utilizing HARQ *e.g.*, [2]-[9]. In [3], the objective is to determine user scheduling and resource allocation that minimize the sum of the information-theoretic data rates in a IR-HARQ based network assuming perfect CSI. The problem is simplified by separately performing power control and bandwidth assignment. The authors of [4] address the problem of sum power minimization for a similar network under average delay constraints with an information-theoretic approach that fails to take practical MCSs into account. In [5], [6], the system goodput is maximized assuming Type-II HARQ and outdated CSI. However, no more than one user can be scheduled at any given time. Moreover, practical MCSs are not considered. In [7], perfect CSIT and Type-I HARQ are considered along with practical MCSs. Nevertheless, the proposed resource allocation is suboptimal since subchannel assignment and power allocation are not jointly optimized. In the context of cognitive radio, some works have been devoted to resource allocation for secondary users when HARQ is employed [8]. In our previous work [9], transmit-power minimization is done in presence of statistical CSIT and practical MCS for Type-II HARQ but without PER constraints. Finally, in [2], the previous problem

is addressed with the additional per-link PER constraints but only for Type-I HARQ. In this context, the authors proposed a costly optimal resource allocation algorithm that involves exhaustive search for the links that should reach the PER constraint with equality. *In this paper, our main contribution is to extend [2] to the context of Type-II HARQ and to propose a computationally-efficient algorithm that does not resort to any exhaustive search.* The extension from Type-I HARQ to Type-II HARQ is not straightforward since the closed-form expressions for the performance metrics of the latter are much more complicated [9].

II. SYSTEM MODEL

We focus on network with K active links. One of its nodes is the resource manager which performs the proposed resource allocation algorithm. Each of the links is considered as a time-varying frequency-selective channel whose M time-domain taps are Rayleigh distributed. It is assumed that OFDM (with N subcarriers covering a total bandwidth of W Hz) is employed and that channels remain constant over one OFDM symbol but change independently between consecutive OFDM symbols. Let $\mathbf{h}_k(j) = [h_k(j, 0), \dots, h_k(j, M-1)]^T$ be the channel impulse response of link k during OFDM symbol j where the superscript $(\cdot)^T$ stands for the transposition operator. The multi-variate complex circular Gaussian distribution with mean \mathbf{a} and covariance matrix Σ is hereafter denoted by $\mathcal{CN}(\mathbf{a}, \Sigma)$. Let $\mathbf{H}_k(j) = [H_k(j, 0), \dots, H_k(j, N-1)]^T$ be the Fourier Transform of $\mathbf{h}_k(j)$. The received signal associated with OFDM symbol j for link k at subcarrier n is thus

$$Y_k(j, n) = H_k(j, n)X_k(j, n) + Z_k(j, n),$$

where $X_k(j, n)$ is the transmitted symbol and where $Z_k(j, n) \sim \mathcal{CN}(0, N_0W/N)$ is an additive noise with a power spectral density equal to N_0 . The time-domain channel taps $\{h_k(j, m)\}_{j,m}$ are independent random variables with variances $\zeta_{k,m}^2$ that are constant w.r.t the OFDM symbol index j i.e., $\mathbf{h}_k(j) \sim \mathcal{CN}(0, \Sigma_k)$ with $\Sigma_k \stackrel{\text{def}}{=} \text{diag}_{M \times M}(\zeta_{k,0}^2, \dots, \zeta_{k,M-1}^2)$. The subcarriers of a single link are thus identically distributed as $H_k(j, n) \sim \mathcal{CN}(0, \zeta_k^2)$ where $\zeta_k^2 = \text{Tr}(\Sigma_k)$. Finally, define the gain-to-noise ratio associated with link k :

$$G_k \stackrel{\text{def}}{=} \frac{\mathbb{E}[|H_k(j, n)|^2]}{N_0} = \frac{\zeta_k^2}{N_0}. \quad (1)$$

At the Medium Access Layer (MAC), each link k receives from the upper layer an infinite stream of information bits arranged in packets of n_b bits each. A Type-II HARQ scheme is then used to transmit each information packet in at most L transmissions. The content of each one of these L transmissions is called a MAC Packet (MP). It depends on the particular Type-II scheme in use. We examine two of these schemes, namely CC-HARQ and IR-HARQ [1], [10]. In either case, we denote by R_k the code rate associated with the first transmission: *i) CC-HARQ*: The MP is obtained by encoding the information packet with a Forward Error Correcting (FEC) code of rate R_k . At the end of each transmission $1 \leq l \leq L$,

the receiver combines the so-far received l MPs using *maximum ratio combining* [10]. *i) IR-HARQ*: The information packet is firstly encoded by a FEC code of rate R_k/L (known as the mother code). The resulting codeword is then split into L MPs using *rate compatible coding* [15]. After the reception of the l th MP, the receiver tries to decode the information packet by concatenating the l received MPs.

On the physical layer, the symbols are chosen from a 2^{m_k} -QAM constellation. The MCS associated with link k can thus be represented by the couple (m_k, R_k) . Let $\mathcal{E}_{k,l}$ be the event that decoding the information packet based on the first l MPs results in an error and define $\pi_{k,l} \stackrel{\text{def}}{=} \mathbb{P}\{\mathcal{E}_{k,l}\}$. If the BICM and random SAS techniques are tuned to the channel coherence time, the links can be considered as fast fading and the l th transmission can achieve the maximum diversity gain of $d_{k,l}$ defined as follows. In the case of IR-HARQ, $d_{k,1}, \dots, d_{k,L}$ are the minimal Hamming distances associated respectively with transmissions $l = 1, \dots, L$. As for CC-HARQ, $d_{k,l} = ld_{k,1}$ where $d_{k,1}$ can be obtained from [15] for several coding rates. The results of [11], [12] can thus be applied to show that:

$$\pi_{k,l}(\text{SNR}_k) \approx \frac{g_{k,l}(m_k, R_k)}{\text{SNR}_k^{d_{k,l}(R_k)}}, \quad (2)$$

where $g_{k,l}(m_k, R_k)$ is a constant designed to fit the simulated $\mathbb{P}\{\mathcal{E}_{k,l}\}$ curve. In the remainder, we use the simpler notation $g_{k,l}$ instead of $g_{k,l}(m_k, R_k)$. Finally, define $q_{k,l}(\text{SNR}_k) \stackrel{\text{def}}{=} \mathbb{P}\{\mathcal{E}_{k,1}, \dots, \mathcal{E}_{k,l}\}$ ($1 \leq l \leq L$) as the probability that the first l transmissions of a HARQ round are all received in error. In particular, $q_{k,L}$ is the PER associated with link k .

The resource manager is assumed to only know the gains G_k which are subcarrier-independent. It cannot thus decide which subset of subcarriers a link should use, but only how many. Let n_k designate the number of subcarriers assigned to link k and define parameter $\gamma_k \stackrel{\text{def}}{=} \frac{n_k}{N}$ that we allow to take any value in $(0, 1)$. It is also natural to use the same power $P_k \stackrel{\text{def}}{=} \mathbb{E}[|X_k(j, n)|^2]$ on all the n_k subcarriers. Let $E_k \stackrel{\text{def}}{=} \frac{P_k}{W/N}$ be the energy consumed to transmit one symbol on one subcarrier and define $\sigma_k^2 \stackrel{\text{def}}{=} N_0W/N$ as the noise variance. Note that each subcarrier of k undergoes an average signal-to-noise ratio (SNR) given by $\text{SNR}_k \stackrel{\text{def}}{=} \frac{\zeta_k^2 P_k}{\sigma_k^2} = G_k E_k$. Finally, we define the goodput $\eta_k(\gamma_k, E_k)$ of link k as the number of successfully-decoded information bits per channel use.

III. OPTIMAL POWER AND BANDWIDTH ALLOCATION ASSUMING FIXED MCSS

As the average energy consumed on any link k to send its part of the OFDM symbol is $N\gamma_k E_k$, our goal is to minimize the total average transmit power, proportional to $\sum_{k=1}^K \gamma_k E_k$, while a minimum goodput $\eta_k^{(0)}$ for each link k is guaranteed

$$\eta_k(\gamma_k, E_k) \geq \eta_k^{(0)}, \quad (3)$$

and while a maximum allowable PER value is respected

$$q_{k,L}(G_k E_k) \leq \pi_k^{(0)}, \quad (4)$$

It is difficult to get $q_{k,L}$ in closed-form, but it can be upper-bounded as follows: $q_{k,L} \leq \pi_{k,L}$, where $\pi_{k,L}$ is expressed analytically by Eq. (2). This bound is relatively tight for all practical values of L and the SNR [10]. From [10], we know that for any Type-II HARQ the goodput writes as:

$$\eta_k(\gamma_k, E_k) = \gamma_k m_k R_k \frac{1 - q_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} q_{k,l}(G_k E_k)}. \quad (5)$$

The factor γ_k in Eq. (5) reflects the fact that the goodput of link k is proportional to the number of its assigned subcarriers. Using once again the fact that $q_{k,L} \leq \pi_{k,L}$ leads to

$$\eta_k \geq \gamma_k m_k R_k \frac{1 - \pi_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(G_k E_k)}. \quad (6)$$

We can thus slightly modify the resource allocation problem by replacing the LHS of the PER constraint (4) with $\pi_{k,L}(\text{SNR}_k)$ and the RHS of its goodput constraint (3) with the LHS of Eq. (6) to get the following optimization problem.

Problem 1. $\min_{\gamma_1 \dots \gamma_K, E_1 \dots E_K} \sum_{k=1}^K \gamma_k E_k$ *subject to*

$$\gamma_k \frac{1 - g_{k,L}/(G_k E_k)^{d_{k,L}}}{1 + \sum_{l=1}^{L-1} g_{k,l}/(G_k E_k)^{d_{k,l}}} \geq \frac{\eta_k^{(0)}}{m_k R_k}, \quad (7a)$$

$$\frac{g_{k,L}}{(G_k E_k)^{d_{k,L}}} \leq \pi_k^{(0)}, \forall k \in \{1, \dots, K\}, \quad (7b)$$

$$\sum_{k=1}^K \gamma_k \leq 1, \quad (7c)$$

$$\gamma_k > 0, E_k > 0, \forall k \in \{1, \dots, K\}. \quad (7d)$$

Problem 1 is feasible if and only if:

$$\sum_{k=1}^K \frac{\eta_k^{(0)}}{m_k R_k} < 1. \quad (8)$$

Indeed, if we assume that the above condition holds then choosing E_k large enough and $\gamma_k = (\eta_k^{(0)} + \epsilon) / (m_k R_k)$ (for some sufficiently small positive ϵ) will result in a feasible problem. The converse is straightforward. Finally, one can show that the above inequality implies Slater's condition.

In general, Problem 1 is not convex due to the objective function and to constraint (7a). Nevertheless, by assuming the specific expression (2) for $\pi_{k,l}$, we obtain a *geometric program* [13]. Indeed, the LHS of Eq. (7b) and Eq. (7c) are straightforwardly posynomials in $\{\gamma_k, E_k\}_{k=1, \dots, K}$. Plugging Eq. (2) into Eq. (7a) leads to the following new expression which is clearly also a posynomial:

$$\frac{\eta_k^{(0)}}{m_k R_k} \gamma_k^{-1} + \sum_{l=1}^{L-1} \frac{\eta_k^{(0)} g_{k,l}}{m_k R_k G_k^{d_{k,l}}} \gamma_k^{-1} E_k^{-d_{k,l}} + \frac{g_{k,L}}{G_k^{d_{k,L}}} E_k^{-d_{k,L}} \leq 1.$$

Problem 1 is thus convex in $\{x_k, y_k\}_{1 \leq k \leq K}$ where $\gamma_k = e^{x_k}$ and $E_k = e^{y_k}$ [13] and the associated Karush-Kuhn-Tucker (KKT) conditions provide a global solution if condition (8) is satisfied. Let μ_k, ξ_k, λ be the Lagrangian multipliers associated with constraints (7a), (7b), (7c) respectively and define

function $x \mapsto f_k(x)$ for any SNR value $x \in \mathbb{R}_+^*$ as

$$f_k(x) \stackrel{\text{def}}{=} \frac{1 + \sum_{l=1}^{L-1} g_{k,l}/x^{d_{k,l}}}{1 - g_{k,L}/x^{d_{k,L}}}. \quad (9)$$

Note that the LHS of Eq. (7a) is equal to $\gamma_k / f_k(G_k E_k)$ and that f_k is decreasing on $(g_{k,L}^{1/d_{k,L}}, +\infty)$. Here, $g_{k,L}^{1/d_{k,L}}$ is the smallest value that the SNR $G_k E_k$ can take while the approximate PER (c.f. Eq. (2)) is less than one. The associated KKT conditions should be first derived in variables $\{x_k, y_k\}_{k=1 \dots K}$. Rewriting them in $\{\gamma_k, E_k\}_{k=1 \dots K}$ gives:

$$\gamma_k E_k - \mu_k \frac{\eta_k^{(0)}}{m_k R_k \gamma_k} \left(1 - \sum_{l=1}^{L-1} \frac{g_{k,l}}{G_k E_k^{d_{k,l}}} \right) + \lambda \gamma_k = 0, \quad (10)$$

$$\gamma_k E_k - \frac{\xi_k}{E_k} - \mu_k \left(\frac{\eta_k^{(0)}}{m_k R_k} \sum_{l=1}^{L-1} \frac{g_{k,l} d_{k,l}}{\gamma_k G_k E_k^{d_{k,l}}} - \frac{g_{k,L} d_{k,L}}{G_k E_k^{d_{k,L}}} \right) = 0, \quad (11)$$

$$\mu_k \left(\frac{\eta_k^{(0)}}{m_k R_k} - \frac{\gamma_k}{f_k(G_k E_k)} \right) = 0, \quad (12)$$

$$\lambda \left(\sum_{k=1}^K \gamma_k - 1 \right) = 0, \quad \xi_k \left(\frac{g_{k,L}}{(G_k E_k)^{d_{k,L}}} - \pi_k^{(0)} \right) = 0. \quad (13)$$

From Eq. (7d), we note that $\gamma_k E_k > 0$ and $\gamma_k > 0$. As $\lambda > 0$ because it is a Lagrange multiplier, then $\gamma_k E_k + \lambda \gamma_k > 0$. We thus get from Eq. (10) that $\mu_k \neq 0$, meaning that the goodput constraint (Eq. 7a) is always active. Eq. (12) thus yields:

$$\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k). \quad (14)$$

Thanks to Eq (14), the bandwidth parameter γ_k of any link k can be obtained as a function of the power parameter E_k . Optimal resource allocation thus boils down to the determination of parameters $\{E_k\}_{1 \leq k \leq K}$. **We propose to proceed by writing E_k as function of one Lagrange multiplier, namely λ** , by eliminating both μ_k and ξ_k . To that end we plug Eqs. (11) and (14) into Eq. (10) to get

$$\lambda = \frac{1}{G_k} F_k(G_k E_k) + \frac{\xi_k m_k R_k}{G_k E_k^2 f_k(G_k E_k) \eta_k^{(0)}} (F_k(G_k E_k) + G_k E_k), \quad (15)$$

where we defined for any SNR $x = G_k E_k \in (g_{k,L}^{1/d_{k,L}}, +\infty)$,

$$F_k(x) \stackrel{\text{def}}{=} \frac{x}{\sum_{l=1}^{L-1} d_{k,l} g_{k,l} / x^{d_{k,l}} + \frac{d_{k,L} g_{k,L} / x^{d_{k,L}}}{1 - g_{k,L} / x^{d_{k,L}}} - x}. \quad (16)$$

The following lemma states some properties of function F_k .

Lemma 1. $\forall k, \exists! s_k > g_{k,L}^{1/d_{k,L}} > 0$ s.t. i) $F_k(s_k) = 0$, ii) $F_k(x) < 0 \forall x < s_k$, iii) F_k is increasing from 0 to $+\infty$ on $[s_k, +\infty)$ so that its increasing inverse F_k^{-1} exists on $[0, +\infty)$.

For the moment, assume that a genie tells us the value of the Lagrange multiplier λ . In this case, due to Eq. (15), multiplier ξ_k can be eliminated by writing:

$$\xi_k = \frac{E_k^2 f_k(G_k E_k) \eta_k^{(0)}}{m_k R_k} \frac{F_k(G_k E_k) - G_k \lambda}{(G_k E_k + F_k(G_k E_k))}, \quad (17)$$

For links with active PER constraint, the transmit power parameter E_k is equal to $E_k^{(0)}$ defined as

$$E_k^{(0)} \stackrel{\text{def}}{=} \frac{1}{G_k} \left(g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}}, \quad (18)$$

while the bandwidth parameter is given by $\gamma_k = \gamma_k^{(0)}$

$$\gamma_k^{(0)} \stackrel{\text{def}}{=} \frac{\eta_k^{(0)}}{m_k R_k} f_k \left(\left(g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}} \right) \quad (19)$$

due to Eq. (14). As for links with inactive PER constraint, we plug $\xi_k = 0$ into Eq. (17) and we refer to Eq. (14) to obtain:

$$E_k = \frac{1}{G_k} F_k^{-1}(G_k \lambda), \quad \gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(G_k \lambda)). \quad (20)$$

We now can determine the subset of links, denoted as $\mathcal{A}(\lambda)$, for which $E_k = \frac{1}{G_k} \left(g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}}$. $\mathcal{A}(\lambda)$ is simply composed of the links k whose associated multiplier ξ_k , as given by Eq. (17), is strictly positive *i.e.*, $G_k \lambda < F_k(G_k E_k)$. We thus have

$$\mathcal{A}(\lambda) = \left\{ k \mid G_k \lambda < F_k \left(\left(g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}} \right) \right\}. \quad (21)$$

We now turn our attention to the determination of λ in order to obtain a practical resource allocation algorithm. To that end, we should extend the definition of $\mathcal{A}(\lambda)$ to any value $\Lambda \geq 0$: $\mathcal{A}(\Lambda) \stackrel{\text{def}}{=} \left\{ k \mid G_k \Lambda < F_k \left(\left(g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}} \right) \right\}$. Note that $\mathcal{A}(\Lambda)$ has a “physical” meaning *only when* $\Lambda = \lambda$ and that $\mathcal{A}(\Lambda) = \{1, \dots, K\}$ for Λ small enough. Now define the following function on \mathbb{R}_+ :

$$\Gamma(\Lambda) \stackrel{\text{def}}{=} \sum_{k \in \mathcal{A}(\Lambda)} \frac{\eta_k^{(0)}}{m_k R_k} f_k \left(\left(g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}} \right) + \sum_{k \in \overline{\mathcal{A}(\Lambda)}} \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(G_k \Lambda)). \quad (22)$$

Note that when $\Lambda = \lambda$, $\Gamma(\lambda)$ is the sum of the optimal bandwidth sharing factors. One can prove that function Γ is continuous and non-increasing. Putting all pieces together, we obtain the following theorem and optimal allocation algorithm.

Theorem 1. *Let condition (8) hold. The optimal solution to Problem 1 is as follows.*

If $\Gamma(0) \leq 1$, then $\forall k \in \mathcal{A}(0)$: $E_k = E_k^{(0)}$, $\gamma_k = \gamma_k^{(0)}$, and $\forall k \in \overline{\mathcal{A}(0)}$: $E_k = \frac{1}{G_k} F_k^{-1}(0)$, $\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k)$.

Else, $\forall k \in \mathcal{A}(\lambda)$: $E_k = E_k^{(0)}$, $\gamma_k = \gamma_k^{(0)}$, and $\forall k \in \overline{\mathcal{A}(\lambda)}$: $E_k = \frac{1}{G_k} F_k^{-1}(G_k \lambda)$, $\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k)$, with λ the unique solution in \mathbb{R}_+ to $\Gamma(\lambda) = 1$.

Algorithm 1 Optimal resource allocation for Problem 1

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 $\Lambda \leftarrow 0$ 
for all  $k \in \{1, \dots, K\}$  do
   $E_k^{(0)} \leftarrow$  RHS of Eq. (18),  $\gamma_k^{(0)} \leftarrow$  RHS of Eq. (19)
end for
repeat
   $\mathcal{A}(\Lambda) \leftarrow \left\{ k \mid G_k \Lambda < F_k \left( \left( g_{k,L} / \pi_k^{(0)} \right)^{1/d_{k,L}} \right) \right\}$ 
  for all  $k \in \mathcal{A}(\Lambda)$  do
     $E_k \leftarrow E_k^{(0)}$ ,  $\gamma_k \leftarrow \gamma_k^{(0)}$ 
  end for
  for all  $k \in \overline{\mathcal{A}(\Lambda)}$  do
     $E_k \leftarrow \frac{1}{G_k} F_k^{-1}(G_k \Lambda)$ ,  $\gamma_k \leftarrow \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k)$ 
  end for
  increment  $\Lambda$ 
until  $\sum_{k=1}^K \gamma_k \leq 1$ 
return  $\{\gamma_k, E_k\}_{k=1 \dots K}$ 

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IV. MCS SELECTION

Let \mathcal{M} designate the set of available modulation schemes and \mathcal{R} the set of available codes. Fixing $\mathbf{m} \stackrel{\text{def}}{=} [m_1, \dots, m_K]^T$ and $\mathbf{R} \stackrel{\text{def}}{=} [R_1, \dots, R_K]^T$, Algorithm 1 returns the optimal parameters $\gamma_1 \dots \gamma_K, E_1 \dots E_K$. Define $\mathcal{Q}_T^*(\mathbf{m}, \mathbf{R}) \stackrel{\text{def}}{=} \sum_{k=1}^K \gamma_k E_k$ as the minimal total transmit power when the corresponding optimization problem is feasible. Otherwise, set $\mathcal{Q}_T^*(\mathbf{m}, \mathbf{R}) = +\infty$. The optimal selection of the MCSs is the solution to the combinatorial problem $(\mathbf{m}^*, \mathbf{R}^*) = \arg \min_{(\mathbf{m}, \mathbf{R}) \in \mathcal{M}^K \times \mathcal{R}^K} \mathcal{Q}_T^*(\mathbf{m}, \mathbf{R})$. It can be found by an exhaustive search that becomes prohibitively costly in computations even for moderate numbers of links. Instead, we resort to the suboptimal but computationally-efficient greedy MCS selection algorithm used in [2] and inspired by [14]. Let \mathcal{M} and \mathcal{R} be sorted such that $\mathcal{M} = \{m_1, \dots, m_{|\mathcal{M}|}\}$ and $\mathcal{R} = \{R_1, \dots, R_{|\mathcal{R}|}\}$ with $m_1 \leq \dots \leq m_{|\mathcal{M}|}$ and $R_1 \leq \dots \leq R_{|\mathcal{R}|}$. The idea behind the algorithm is to perform MCS selection iteratively by changing the MCS of only one link per iteration. This is done by assigning to each link the next MCS in the ordered set $\mathcal{M} \times \mathcal{R}$ and by selecting the link whose MCS modification results in the lowest total power. This approach is greedy in the sense that it continues while the so-obtained transmit power decreases, and stops otherwise.

V. NUMERICAL RESULTS

We consider a network with $K = 10$ links with a bandwidth $W = 5$ MHz centered around $f_0 = 2400$ MHz. Each information packet is $n_b = 128$ bits long. The distance D_k associated with any link k is randomly drawn from a uniform distribution on $[0.1, 1]$ km. The path-loss parameter ς_k^2 (c.f. Eq. (1)) follows a free-space model so that $\varsigma_k^2(D_k) = 1 / (4\pi f_0 D / c)^2$. For the sake of simplicity, each link has the same target efficiency $\eta^{(0)}$ so that the required sum rate is equal to $K\eta^{(0)}$, while the PER constraint is fixed either to $\pi_k^{(0)} = 10^{-3}$ or to $\pi_k^{(0)} = 10^{-4}$. Finally, we fix $N_0 = -170$ dBm/Hz.

We first assume that each link uses QPSK and a CC-HARQ based on the 1/2-rate convolutional code from [15]. In Figure 1, we plot the sum power $W \sum_{k=1}^K \gamma_k E_k$ obtained using 200 Monte-Carlo runs of Algorithm 1. Note that for target sum-rates larger than 4.5 Mbps, the PER constraints have negligible effect on the value of the optimal total transmit power. As for target sum-rates larger than 5 Mbps, neither the constrained nor the unconstrained problems can be feasible. For the sake of comparison, we also plot the sum transmit power resulting from the sub-optimal resource allocation scheme that consists in fixing $\gamma_k = \frac{\eta_k^{(0)} / (m_k R_k)}{\sum_{l=1}^K \eta_l^{(0)} / (m_l R_l)}$ (which trivially satisfies constraint (7c)) and in choosing E_k to be equal to the minimal value such that both constraints (7a) and (7b) are respected. The advantage of using our optimal resource allocation algorithm over this sub-optimal scheme is clear from Figure 1.

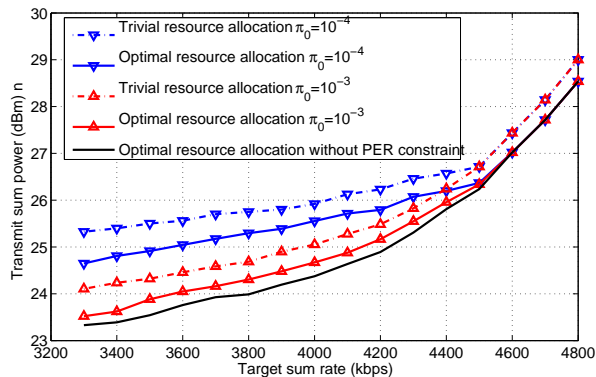


Fig. 1. Sum power vs. sum rate with/without PER constraints

We now assume that the available modulation schemes are the QAM constellations $\mathcal{M} = \{1, 2, 4, 6\}$ and that the available coding rates are $\mathcal{R} = \{1/2, 1\}$. In Figure 2, we plot the sum transmit-power resulting (when $\pi_k^{(0)} = 10^{-4}$) from applying the proposed MCS selection method with the following schemes: IR-HARQ based on the two nested convolutional codes from [16] with an initial rate equal to 1/2 and 1, CC-HARQ and Type-I HARQ based on the 1/2-rate convolutional code from [15]. We also compare these different sum-transmit powers to the ideal lower bound associated with the considered optimization problem. Since the wireless channels in our system model are fast fading, this lower bound is reached by endowing the links of the network with the possibility of achieving their *ergodic capacity*. Figure 2 shows that the proposed greedy MCS selection significantly reduces power consumption and makes it closer to the ergodic lower bound as opposed to Figure 1 where the MCS is fixed trivially.

VI. CONCLUSION

We proposed an algorithm to compute the optimal power and bandwidth parameters that minimize sum-power consumption in OFDMA-based wireless networks which use Type-II

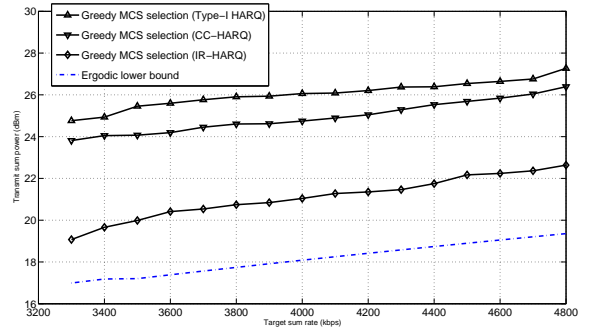


Fig. 2. Sum transmit power of the proposed MCS selection

HARQ. The minimization was done subject to per-link goodput and packet error rate constraints and under the assumption of statistical CSI and practical MCSs that are fixed in advance. We then showed that the total transmit power associated with the proposed resource allocation algorithm can be significantly reduced if the MCSs of the different links in the network are allowed to vary based on a greedy MCS-selection.

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