

Optimal Resource Allocation in HARQ-based OFDMA Wireless Networks

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Abstract—This paper deals with multiuser resource allocation (power, bandwidth, constellation size, and code rate) for an OFDMA system using HARQ in the context of Rayleigh distributed channel. We assume that the resource manager (base station or cluster head) only knows the channel statistics of the active links. Then, an optimal algorithm for minimizing the total transmitted power under per user goodput constraints is proposed. Extension to imperfect feedback on HARQ scheme is also performed. This algorithm can be especially applied to military ad hoc wireless networks.

I. INTRODUCTION

In the context of ad hoc wireless networks, which can be deployed on a military scene, we consider that pairwise communications are possible. In order to simplify the network management, a cluster-based structure is advocated where an elected cluster head will perform the resource allocation but not necessarily the relaying of the information between two users as illustrated in Fig. 1. In this paper, we focus on the resource allocation optimization inside a single cluster (without inter-cluster interference). Obviously, this work can also be applied to a traditional cellular network where the cluster (resp. cluster head) is replaced with the cell (resp. base station).

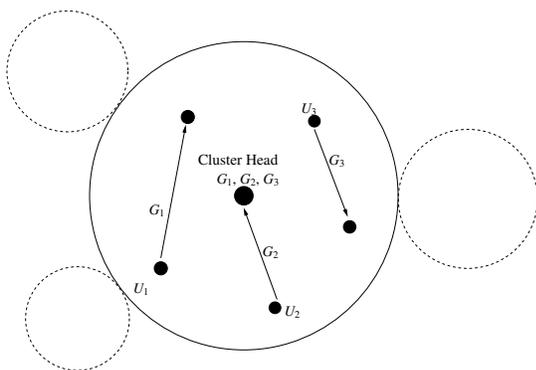


Figure 1. Clustered ad hoc wireless network.

As envisaged in the future wireless systems, our communication scheme (closely related to that developed in [1]) is based on i) Orthogonal Frequency Division Multiple Access (OFDMA) for managing the frequency-selectivity of multipath channels and vanishing the multiple access interference (as

in [1]), and ii) Hybrid Automatic Repeat reQuest (HARQ) for enforcing the quality of the link thanks to the packet retransmission. In wireless mobile environment, the channel impulse response may vary fast enough such that the users cannot provide instantaneous and perfect Channel State Information (CSI) at the resource allocation manager (*i.e.* the cluster head). Moreover, the amount of CSI is huge in an ad hoc network since CSI for each pairwise link has to be provided and thus this may flood the network. Therefore, we will consider that the cluster head does not have the knowledge of the channels realizations. As a consequence, the channels are random and follow a Rayleigh distribution. We assume that the cluster head knows the channels statistics, which makes sense since the channel statistics vary slowly and thus do not need to be updated frequently. In order to handle the diversity issue induced by the randomness of the channels, Frequency Hopping (FH) is performed. Furthermore, FH provides an interesting way to counteract eavesdroppers from a military point of view.

In [1], the authors addressed the power minimization issue at the cluster head under per user rate constraint. Assuming capacity-achieving coding, the rate is evaluated through the so-called ergodic capacity. In our paper, practical Modulation and Coding Schemes (MCS) will be considered. The notion of SNR gap, defined in [2] for the additive Gaussian channel (or equivalently perfect CSI at the transmitter), cannot be applied to the ergodic capacity. Thus the results in [1] cannot be extended to practical MCS. Furthermore, since HARQ is used the so-called goodput is a relevant metric for characterizing the information rate, as a natural trade-off between capacity (as an information reward) and QoS (through packet error penalty).

Therefore our purpose is to minimize the total power under per user goodput constraint when only statistical CSI at the cluster head is available. Power minimization is of great interest to increase the network lifetime, to mitigate the inter-cluster interference, and to provide low detection capability.

In the literature, the goodput has been already moderately used in multiuser resource allocation issue [3]–[6] based on HARQ/OFDMA systems. In [3], the authors focused on power and rate (involved in outage probability) allocation when deterministic CSI at the Transmitter (CSIT), *i.e.* associated with the channel realization, is delayed and so outdated. In [4], power and subcarrier allocation and bit-loading are performed

when practical MCS are considered but with perfect CSIT. In [5], the authors proposed a new algorithm for assigning slots (not subcarriers) to users as well as for finding power and MCS in order to maximize the system goodput, under partial, but deterministic, CSIT assumption. In [6], a heuristic algorithm for subcarrier assignment and power allocation is performed under perfect CSIT. As a summary, the main novelty of our paper is to consider statistical CSIT instead of deterministic CSIT.

Notice that several papers in the literature (see [7], [8] and references therein) also dealt with power optimization but in a strongly different context, since the objective was to adapt the power between each (H)ARQ retransmission. Although this issue is of great interest, it is not considered hereafter. We focus on power allocation (and some other parameters) among the users assuming a given HARQ scheme, rather than optimizing the power within a HARQ context under single user assumption.

More precisely, the purpose of our paper is to perform OFDMA resource allocation (power, bandwidth, MCS) when Stop-and-Wait Type-I HARQ is used, and when only statistical CSI is available at the cluster head. Our contributions are threefold: i) for a given MCS, the optimal algorithm for power and bandwidth allocation minimizing the sum-power under individual goodput constraints is derived, ii) a sub-optimal MCS allocation is then proposed, and iii) the case of imperfect feedback is eventually investigated. The rest of the paper is organized as follows. The system model is depicted in Section II. Section III is devoted to the power and bandwidth allocation. Section IV tackles the MCS selection problem. The impact of the imperfect feedback is analyzed in Section V. Numerical illustrations are provided in Section VI, and conclusions are drawn in Section VII.

II. SYSTEM MODEL

A. Channel model

Each link is considered as a (time-varying) frequency-selective channel. OFDM (with N subcarriers) is employed to compensate for the frequency selectivity. It is assumed that the channel remains constant over one OFDM symbol but may change between two consecutive OFDM symbols. The channel corresponds to the link between the transmitting user k and any receiving node in the network, including the cluster head. In all the paper, the superscript T stands for the transposition operator, and the (multi-variate) complex-valued circular Gaussian distribution with mean a and covariance matrix Σ is denoted $\mathcal{CN}(a, \Sigma)$.

Let $\mathbf{h}_k(i) = [h_k(i, 0), \dots, h_k(i, M-1)]^{\text{T}}$ be the channel impulse response of user k associated with OFDM symbol i , where M is the number of taps. Let us denote by $\mathbf{H}_k(i) = [H_k(i, 0), \dots, H_k(i, N-1)]^{\text{T}}$ the Fourier Transform of $\mathbf{h}_k(i)$. Assuming OFDM (with well-designed cyclic prefix), the received signal at OFDM symbol i and subcarrier n for user k is

$$Y_k(i, n) = H_k(i, n)X_k(i, n) + Z_k(i, n), \quad (1)$$

where $X_k(i, n)$ is the transmitted symbol by user k at subcarrier n of OFDM symbol i , and the additive noise $Z_k(i, n) \sim \mathcal{CN}(0, N_0 W/N)$ where N_0 is the noise power spectral density and W is the total bandwidth. It is assumed that each channel is an independent random process with possibly different variances $\zeta_{k,m}^2$ for each tap, *i.e.* $\mathbf{h}_k(i) \sim \mathcal{CN}(0, \Sigma_k)$ with $\Sigma_k := \text{diag}_{M \times M}(\zeta_{k,m}^2)$. Thus, the Fourier Transform vector is an independent random process $\mathbf{H}_k(i) \sim \mathcal{CN}(0, \zeta_k^2 \mathbf{I}_N)$ with $\zeta_k^2 := \text{Tr}(\Sigma_k)$. Therefore, the subcarriers of a single user are identically distributed [1].

Let $g_k(i, n) = |H_k(i, n)|^2/N_0$ be the instantaneous gain-to-noise ratio (GNR) for user k at subcarrier n and OFDM symbol i . It is exponentially distributed, with a mean independent of n , given by

$$G_k := \mathbb{E}[g_k(i, n)] = \frac{\zeta_k^2}{N_0}. \quad (2)$$

In this paper, we assume that the cluster head only knows the terms G_k (*i.e.*, the average GNR instead of the instantaneous one) for all active links. Since G_k is independent of n , the resource allocation algorithm will not distinguish between subcarriers for a given user. On the other hand, as G_k depends on k , the users will obviously be treated differently. We assume that the behavior of G_k is driven by the so-called path-loss. Let D_k be the distance between user k and corresponding receiver. Then

$$G_k = \frac{\ell(D_k)}{N_0}$$

where $\ell(D_k)$ depends on the path-loss model.

B. Power/bandwidth parameters

Assume that K users are active in the considered cluster. OFDMA is employed to separate the users. We remind that the cluster head does not have instantaneous CSI but only statistical CSI through G_k , for each user/link k . As G_k is not subcarrier-dependent, the cluster head cannot allocate *which* subcarriers user k will use, but only *how many*. Let n_k be the number of subcarriers assigned to user k . So the bandwidth proportion occupied by user k is

$$\gamma_k = \frac{n_k}{N} \quad (3)$$

and corresponds to the bandwidth parameter to be optimized.

Due to the independence of G_k with respect to the subcarrier n , it is natural for user k to use the same average power $P_k = \mathbb{E}[|X_k(i, n)|^2]$ on each subcarrier. Let $E_k := P_k/(W/N)$ and $\sigma_k^2 := N_0(W/N)$ be the energy consumed to send one symbol on each subcarrier and the corresponding noise variance, respectively. Then, on each subcarrier user k undergoes average signal-to-noise ratio (SNR) given by

$$\text{SNR}_k = \frac{\zeta_k^2 P_k}{\sigma_k^2} = E_k G_k. \quad (4)$$

Finally, let Q_k be the average energy consumed to send the part of the OFDM symbol associated with user k , which

corresponds to the power parameter to be optimized. It can be easily shown that

$$Q_k = \frac{n_k P_k}{W} = \gamma_k E_k. \quad (5)$$

C. HARQ and relevant metric definitions

At MAC layer, the users employ Type-I HARQ for which a single information packet can be sent at most L times. The information packet can be built according to the two following ways: i) the uncoded case where the users choose the size of their own 2^{m_k} -QAM constellation ($m_k \in \mathcal{M}$), and ii) the coded case where the users choose their code rate $R_k \in \mathcal{R}$ in addition to their modulation.

The user (successful) data rate ρ_k (in bits/s) is proportional to its HARQ goodput η_k (in bits/s/Hz), *i.e.* $\rho_k = \eta_k W$. According to [9], the goodput is equal to

$$\eta_k = \gamma_k m_k R_k f(\pi_k), \quad (6)$$

where $f : x \mapsto 1 - x$ and π_k is the (information) Packet Error Probability (PEP).

We would like to express the PEP π_k as a function of SNR_k . A well-designed FH pattern is assumed in order to recover completely the diversity offered by the channel, *i.e.* at least M , leading to a fast-fading channel model.

In the uncoded case, assuming fast-fading channel and that information packets contain n_s symbols, the PEP can be written with respect to SNR as [10], [11]

$$\pi_k(\text{SNR}) \approx \frac{n_s a_{m_k}}{1 + \frac{g_{m_k}}{2^{m_k-1}} \text{SNR}}, \quad (7)$$

where a_{m_k} and g_{m_k} are constants related to the chosen constellation.

In the coded case, *Bit Interleaved Coded Modulation* (BICM) is carried out in order to retrieve the entire diversity offered by the code. For a fast-fading channel, the PEP can be written with respect to SNR as [12]

$$\pi_k(\text{SNR}) \approx \left(\frac{4}{d_h^2(m_k)} \right)^{d_f(R_k)} \frac{g_c(m_k, R_k)}{\text{SNR}^{d_f(R_k)}} \quad (8)$$

where $d_f(R_k)$ is the minimal (Hamming or free) distance of the code of rate R_k , $d_h(m_k)$ is the harmonic distance related to the modulation, and where $g_c(m_k, R_k)$ is a coding gain.

III. OPTIMAL POWER/BANDWIDTH ALLOCATION

A. Optimization problem

In order to mitigate the power radiated by a single cluster, our objective is to minimize the total energy used for sending an OFDM symbol, *i.e.*, we minimize $Q_T = \sum_{k=1}^K Q_k$ which is also equal to $\sum_{k=1}^K \gamma_k E_k$ through Eq. (5). To do that, one can adjust relevantly the user energy Q_k , the bandwidth γ_k , and the MCS (driven by m_k and R_k). From Eq. (3), γ_k are rational numbers, however for tractability purposes, we take $\gamma_k \in [0, 1]$ to make the problem continuous. For the sake of clarity, we assume in this Section that each user takes a given modulation and code rate, *i.e.*, m_k and R_k are fixed. The choice of the MCS will be discussed in Section IV. Besides, each user has

to ensure a minimum rate, *i.e.*, there exists strictly positive constants ρ_k^0 such that $\rho_k \geq \rho_k^0$. In order to remain bandwidth-independent, this means that the goodput $\eta_k \geq \eta_k^0$. Therefore, the optimization problem is now formalized in Problem 1:

Problem 1. Let us denote $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_K]^T$ and $\mathbf{Q} = [Q_1, \dots, Q_K]^T$. The optimization problem boils down to

$$(\boldsymbol{\gamma}^*, \mathbf{Q}^*) = \arg \min_{(\boldsymbol{\gamma}, \mathbf{Q})} \sum_{k=1}^K Q_k \quad (9)$$

subject to

$$(C1) \quad \eta_k(\gamma_k, Q_k) \geq \eta_k^0, \quad \forall k, \quad (C3) \quad \gamma_k \geq 0, \quad \forall k,$$

$$(C2) \quad \sum_{k=1}^K \gamma_k \leq 1, \quad (C4) \quad Q_k \geq 0, \quad \forall k.$$

Before going further, we check the problem feasibility. The next condition provides the inequality that the constellation size, the code rate and the target goodput have to satisfy. In the rest of the paper, we assume Condition 1 holds. The proof of this condition is actually a special case of that of Condition 2 reported in Section V and so is omitted here.

Condition 1. Problem 1 is feasible if, and only if,

$$\sum_{k=1}^K \frac{\eta_k^0}{m_k R_k} < 1. \quad (10)$$

In Lemma 1, we show that Problem 1 is convex as soon as the function $\text{SNR} \mapsto \pi_k(\text{SNR})$ is convex. The proof is straightforward and thus is omitted due to page limitation. Notice that π_k defined in Eqs. (7)-(8) satisfies the convexity property.

Lemma 1. The constraint function defined by

$$\eta_k(\gamma_k, Q_k) = \gamma_k m_k R_k f(\pi_k(G_k Q_k / \gamma_k)) \quad (11)$$

on $[0, 1] \times \mathbb{R}_+$, is concave as long as $\pi_k : \mathbb{R}_+ \rightarrow [0, 1]$ is a convex function.

B. Optimal algorithm

Assuming Problem 1 is feasible (cf. Condition 1), the Karush-Kuhn-Tucker (KKT) conditions enable us to exhibit the optimal solution $(\boldsymbol{\gamma}^*, \mathbf{Q}^*)$ to Problem 1 since it is convex [13]. Let $(\boldsymbol{\mu}, \lambda, \boldsymbol{\alpha}, \boldsymbol{\beta})$ be the positive Lagrangian multipliers associated with (C1, C2, C3, C4) respectively. The KKT conditions lead to the following equalities

$$\begin{aligned} \nabla \left(\sum_{k=1}^K Q_k \right) - \sum_{k=1}^K \mu_k \nabla \eta_k(\gamma_k, Q_k) + \lambda \nabla \left(\sum_{k=1}^K \gamma_k - 1 \right) \\ - \sum_{k=1}^K \alpha_k \nabla \gamma_k - \sum_{k=1}^K \beta_k \nabla Q_k = 0, \end{aligned} \quad (12a)$$

$$\mu_k (\eta_k(\gamma_k, Q_k) - \eta_k^0) = 0, \quad \lambda \left(\sum_{k=1}^K \gamma_k - 1 \right) = 0, \quad (12b)$$

$$\alpha_k \gamma_k = 0, \quad \beta_k Q_k = 0. \quad (12c)$$

where ∇ stands for the gradient operator.

In order to guarantee their minimum goodput, all the users have non-zero power and non-zero bandwidth. As a consequence, we have the following result.

Lemma 2. *The optimal solution (γ^*, \mathbf{Q}^*) is such that $\gamma_k^* > 0$ and $Q_k^* > 0, \forall k$.*

Hence, according to Eq. (12c), the multipliers associated with γ_k and Q_k vanish, i.e., $\alpha_k = 0$ and $\beta_k = 0, \forall k$. After some tedious algebraic manipulations and using Eqs (12a)-(12b), we obtain that the optimal power and bandwidth sharing have to satisfy the following equations

$$F(G_k Q_k^* / \gamma_k^*) = \lambda G_k \quad (13)$$

$$\gamma_k^* m_k R_k f(\pi_k(G_k Q_k^* / \gamma_k^*)) = \eta_k^0 \quad (14)$$

with $F : x \mapsto -(1 - \pi_k(x))^2 / (f(\pi_k(x)) \pi_k'(x)) - x$.

Now, the optimal Lagrangian multiplier λ^* is found. First of all, it is straightforward to show that F is strictly increasing over \mathbb{R}^+ , hence the inverse function F^{-1} of F with respect to the composition exists. Let **(P1)** be the property on π_k such that π_k is between 0 and 1, and $\pi_k(x)$ goes to 1 when x goes to 0. Thus, **(P1)** implies that $F(0) = 0$ and F is strictly positive on \mathbb{R}_*^+ . From this, $\lambda = 0$ implies that Q_k / γ_k is equal to 0 which is impossible. As a consequence, and due to the slackness condition (cf. Eq. (12b)), the bandwidth constraint (C2) is active. Therefore, by rewriting Eqs. (13)-(14), we have the following theorem which is our main contribution.

Theorem 1. *Under **(P1)**, the optimal power allocation (γ^*, \mathbf{Q}^*) is given by*

$$Q_k^* = \frac{\gamma_k^*}{G_k} F^{-1}(\lambda^* G_k) \quad (15)$$

$$\gamma_k^* = \frac{\eta_k^0}{m_k R_k f(\pi_k(F^{-1}(\lambda^* G_k)))}, \quad (16)$$

with $\lambda^* > 0$ chosen such that

$$\sum_{k=1}^K \frac{\eta_k^0}{m_k R_k f(\pi_k(F^{-1}(\lambda^* G_k)))} = 1. \quad (17)$$

Obviously, the property **(P1)** is checked for true PEP or empirical PEP curves. However, when approximate expressions are considered for PEP as in Eqs (7)-(8), $\pi_k(x)$ is larger than 1 when x becomes small, and then $F^{-1}(0) < 0$. Let $x_0 > 0$ be the point such that $\pi_k(x_0) = 1$, then one can prove that there exists $x_1 > x_0$ such that $F^{-1}(0) = x_1$. Then, two configurations occur:

- i) If $\sum_{k=1}^K \frac{\eta_k^0}{m_k R_k f(\pi_k(x_1))} < 1$, then the constraint on the total occupied bandwidth is never active. Indeed, for $\lambda = 0$, $\sum_{k=1}^K \gamma_k < 1$ and so cannot reach 1 since it is a decreasing function with respect to λ . As a consequence, due to the slackness condition, $\lambda^* = 0$ and Theorem 1 has to be applied by considering only Eqs. (15)-(16) and by discarding Eq. (17).
- ii) If $\sum_{k=1}^K \frac{\eta_k^0}{m_k R_k f(\pi_k(x_1))} > 1$, then $\lambda = 0$ is not a possible optimal Lagrangian multiplier since the constraint is not satisfied. Since the constraint decreases when λ increases, we have to increase λ until Eq. (17) is met.

Notice that when π_k follows property **(P1)**, we have $x_0 = x_1 = 0$ and configuration i) cannot occur.

IV. SUBOPTIMAL MCS SELECTION

The algorithm related to Theorem 1 leads to the minimal transmit power in the considered cluster when the modulation size and the code rate are given. Now, we solve the problem of MCS selection.

Problem 2. *Let us denote $\mathbf{m} = [m_1, \dots, m_K]^T$ and $\mathbf{R} = [R_1, \dots, R_K]^T$. Given (\mathbf{m}, \mathbf{R}) , Theorem 1 provides the minimum total energy*

$$\mathcal{Q}_T^*(\mathbf{m}, \mathbf{R}) = \sum_{k=1}^K \frac{F_k^{-1}(\lambda^* G_k) \eta_k^0}{m_k R_k G_k f(\pi_k(F_k^{-1}(\lambda^* G_k)))}. \quad (18)$$

Then, the MCS selection problem boils down to

$$(\mathbf{m}^*, \mathbf{R}^*) = \arg \inf_{(\mathbf{m}, \mathbf{R}) \in \mathcal{M}^K \times \mathcal{R}^K} \mathcal{Q}_T^*(\mathbf{m}, \mathbf{R}). \quad (19)$$

The optimal choice of modulation/code is a combinatorial problem and cannot be solved in an exhaustive way due to the high computational load. We hereafter propose a suboptimal solution inspired by [11]. Due to page limitation, we only describe (and not justify) the proposed method. The available modulations and code rates are described by the sets $\mathcal{M} = \{m_1, \dots, m_{|\mathcal{M}|}\}$ with $m_1 < \dots < m_{|\mathcal{M}|}$, and $\mathcal{R} = \{R_1, \dots, R_{|\mathcal{R}|}\}$ with $R_1 < \dots < R_{|\mathcal{R}|}$, respectively, where $|A|$ denotes the cardinal of a set A . Let $\mathbf{m}^{(0)} = [m_{i_1}^{(0)}, \dots, m_{i_K}^{(0)}]^T \in \mathcal{M}^K$ such that $\forall k, m_{i_k}^{(0)} = m_1$, and $\mathbf{R}^{(0)} = [R_{i_1}^{(0)}, \dots, R_{i_K}^{(0)}]^T \in \mathcal{R}^K$ such that $\forall k, R_{i_k}^{(0)} = R_1$. The algorithm, called "Greedy MCS selection" is as follows: Set $\mathcal{Q}_T^* = \infty$ and $(\mathbf{m}, \mathbf{R}) = (\mathbf{m}^{(0)}, \mathbf{R}^{(0)})$

1. for $k = 1$ to K do

Let $\mathbf{m}^{(k)} \in \mathcal{M}^K$ with $m_{i_k} \leftarrow m_{i_{k+1}}$ and $\mathbf{R}^{(k)} \in \mathcal{R}^K$ where $R_{i_k} \leftarrow R_{i_{k+1}}$. Compute $\mathcal{Q}_T^*(\mathbf{m}^{(k)}, \mathbf{R})$, $\mathcal{Q}_T^*(\mathbf{m}, \mathbf{R}^{(k)})$, and $\mathcal{Q}_T^*(\mathbf{m}^{(k)}, \mathbf{R}^{(k)})$ according to Eq. (18). Then select the MCS minimizing $\mathcal{Q}_T^*(\dots)$. The corresponding power will be denoted by $\mathcal{Q}_T^{(k)}$.

end

2. Select $k^* = \arg \inf_k \mathcal{Q}_T^{(k)}$.

3. if $\mathcal{Q}_T^* > \mathcal{Q}_T^{(k^*)}$ or Condition 1 (or 2 in imperfect feedback context) does not hold then

$\mathcal{Q}_T^* \leftarrow \mathcal{Q}_T^{(k^*)}$, $\mathbf{m} \leftarrow \mathbf{m}^{(k^*)}$, and $\mathbf{R} \leftarrow \mathbf{R}^{(k^*)}$ and go back to step 1.

else

Exit.

V. EXTENSION TO IMPERFECT FEEDBACK

We now assume that the feedback is degraded due to some impairments as in [14]. As the power dedicated to the direct link does not influence the SNR of the reverse channel devoted to the acknowledgment, we assume erroneous feedback with constant probability p_{fb} . In this section, we will extend the previous work in the case $p_{fb} \neq 0$. For the sake of simplicity, for now we assume infinite retransmissions ($L = \infty$). Then,

the Type-I HARQ efficiency (see [14, Eq. (8) and (10)]) is given by

$$\eta_k = \gamma_k m_k R_k f_{\text{fb}}(\pi_k), \quad (20)$$

with $f_{\text{fb}} : x \mapsto 1/(1/(1-x) + p_{\text{fb}}/(1-p_{\text{fb}}))$. For $L < \infty$, Eq. (20) is much more complicated and this scenario is left for future work.

Condition 2. *Problem 1 (when f is replaced with f_{fb}) is feasible if, and only if,*

$$\sum_{k=1}^K \left(1 + \frac{p_{\text{fb}}}{1-p_{\text{fb}}}\right) \frac{\eta_k^0}{m_k R_k} < 1. \quad (21)$$

Sketch of proof: If Problem 1 is feasible, then there exists a sequence (γ, \mathbf{Q}) such that $\forall k, \eta_k^0 \leq \eta_k(\gamma_k, Q_k)$ and $\sum_k \gamma_k \leq 1$. This implies that $\eta_k^0 \leq \gamma_k m_k R_k f_{\text{fb}}(\pi_k(G_k Q_k / \gamma_k)) < \frac{\gamma_k m_k R_k}{1 + \frac{p_{\text{fb}}}{1-p_{\text{fb}}}}$. So we have

$$1 \geq \sum_k \gamma_k > \left(1 + \frac{p_{\text{fb}}}{1-p_{\text{fb}}}\right) \sum_k \frac{\eta_k^0}{m_k R_k}. \quad (22)$$

Conversely, assume that Eq. (21) holds. Then, for some sufficiently small $\epsilon > 0$, the problem is feasible by considering $Q_k \rightarrow \infty$ and $\gamma_k = (1 + p_{\text{fb}}/(1-p_{\text{fb}}))(\eta_k^0 + \epsilon)/(m_k R_k)$. ■

Condition 2 can be written in the following alternative way:

Corollary 1. *Assuming Condition 1, Problem 1 is feasible if, and only if, $p_{\text{fb}} < p_{\text{fb}}^t$ with*

$$p_{\text{fb}}^t = 1 - \sum_{k=1}^K \frac{\eta_k^0}{m_k R_k}. \quad (23)$$

One can remark that, under Condition 1, the threshold is well defined since the inequalities $0 < p_{\text{fb}} < 1$ hold. The proof of Corollary 1 is simple since Eq. (21) holds if and only if Eq. (23) holds. Therefore, we have characterized the maximum value of p_{fb} for the system to work.

VI. NUMERICAL RESULTS

An uncoded ARQ scheme is employed for $K = 4$ links. Each user sends a data packet composed by 32 uncoded bits within a bandwidth $W = 1$ MHz. The path loss follows the free-space model $\ell(D) = 1/((4\pi f_0/c)^2 D^2)$ where c is the light celerity and f_0 the carrier frequency. We put $f_0 = 400$ MHz and the noise density power is fixed to $N_0 = -170$ dBm/Hz. The distance D_k between both users associated with the k -th link is randomly drawn from a uniform distribution in $[D_m, D_M]$. We have considered $D_m = 50$ m and $D_M = 1$ km. Each simulated point is obtained via 500 Monte-Carlo runs. For the sake of simplicity, each link has the same target efficiency.

In order to evaluate the optimal power/bandwidth algorithm, we propose to compare it with respect to two suboptimal (quite naive) algorithms described below:

Suboptimal 1: $\gamma_k = (\eta_k^0/m_k R_k)/(\sum_{k'=1}^K \eta_{k'}^0/(m_{k'} R_{k'}))$ and $Q_k = Q/K$ with Q chosen such that all the constraints are satisfied.

Suboptimal 2: $\gamma_k = (\eta_k^0/m_k R_k)/(\sum_{k'=1}^K \eta_{k'}^0/(m_{k'} R_{k'}))$ and $Q_k = (\gamma_k/G_k)\pi_k^{-1}f_{\text{fb}}^{-1}(\eta_k^0/(\gamma_k m_k R_k))$ where f_{fb}^{-1} is the inverse function of f_{fb} with respect to the composition. When $p_{\text{fb}} = 0$, f_{fb} boils down to f .

Fig. 2 displays the total transmit power versus the sum rate when BPSK is used over each link. Clearly, the optimal

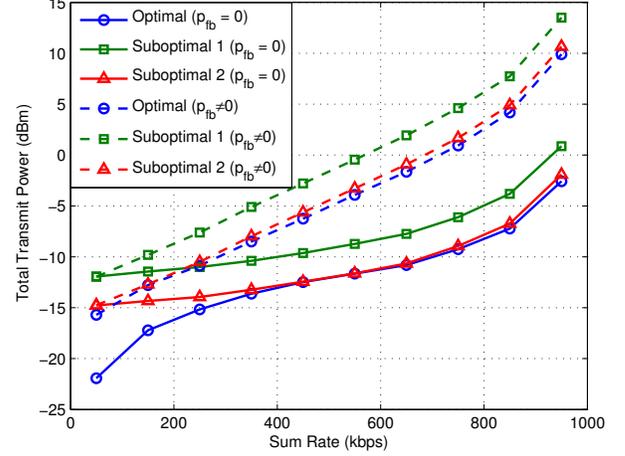


Figure 2. Total Transmit Power (in dBm) versus the sum rate.

algorithm outperforms the suboptimal ones (especially at low data rates) whatever the quality of the feedback. When $p_{\text{fb}} \neq 0$ we have fixed $p_{\text{fb}} = 0.95p_{\text{fb}}^t$. To explain the behavior at low data rates, let us inspect Fig. 3 in which we plot the occupied bandwidth proportion versus the sum rate. At low

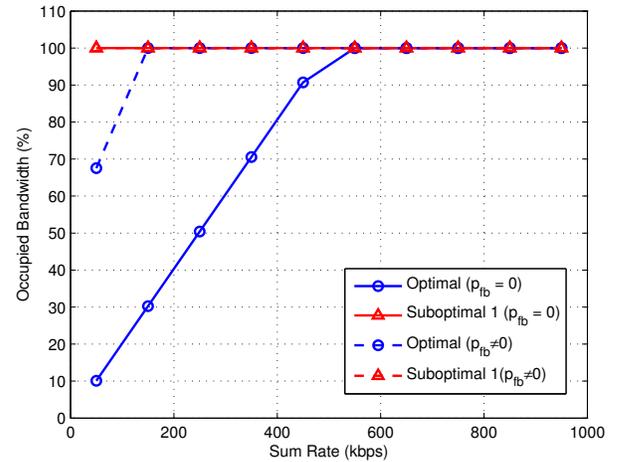


Figure 3. Occupied bandwidth proportion (in %) versus the sum rate.

data rates, as expected (see configuration i) in Section III-B), the optimal algorithm does not occupy the entire bandwidth and so $\sum_k \gamma_k < 1$, whereas the suboptimal ones do. Therefore these algorithms have different performance. On the other hand, at high data rates, Suboptimal 2 stays close to the

optimal algorithm. Actually, when $G_k = G, \forall k$, one can prove that both algorithms are identical.

Let us now consider the MCS selection algorithm. In Fig. 4, we display the total transmit power versus the sum rate when we combine the optimal bandwidth/power allocation algorithm i) with an exhaustive search for the best MCS associated with Problem 2, ii) with the proposed MCS selection algorithm or iii) with modulation size only adapted to the required sum rate (for instance, if the users require 2 bits/s/Hz, then the modulation is QPSK). In simulation, QAM constellations are considered with $\mathcal{M} = \{1, 2, 4, 6\}$. The exhaustive algorithm and the proposed one for MCS selection offer almost the same performance. On the other hand, the MCS solution based only on the rate constraint performs quite poorly especially when the efficiency is close to the value of an element of \mathcal{M} . The high peaks around 1, 2, 4 and 6 Mbps are explained by Corollary 1, *i.e.*, the power must be strongly increased if the normalized sum rate is close to 1. Actually, this work proves that the MCS has to be changed much before being close to the value of an element of \mathcal{M} .

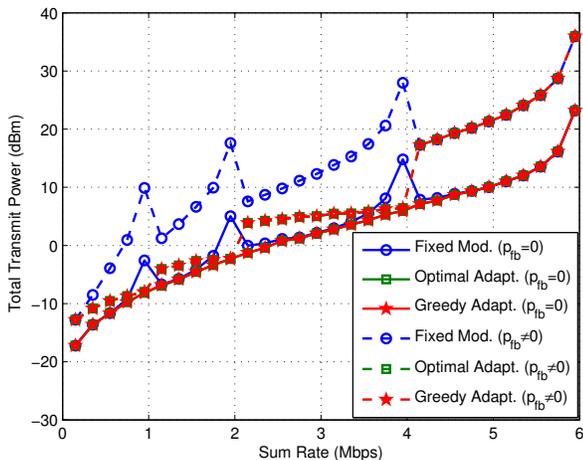


Figure 4. Total Transmit Power (in dBm) versus Sum Rate (in Mbps).

In order to analyze the influence of p_{fb} , we define the power loss (in dB) as $10 \log_{10}(\mathcal{Q}_T^*(p_{fb})/\mathcal{Q}_T^*(0))$ where $\mathcal{Q}_T^*(p_{fb})$ is the optimal total transmit power when the feedback error probability is p_{fb} . In Fig. 5, we plot the power loss versus p_{fb} . In this simulation, we have considered $\mathbf{D} = [50, 100, 500, 700]$ m and $\boldsymbol{\eta}^0 = [0.2, 0.2, 0.4, 0.1]$ bit/s/Hz, and BPSK is used. According to $\boldsymbol{\eta}^0$, we know that $p_{fb}^t = 0.1$. We observe that the power loss grows exponentially and becomes too huge when p_{fb} is close to p_{fb}^t .

VII. CONCLUSION

This contribution provided a solution for resource allocation in a clustered ad hoc wireless network. Assuming a multi-user HARQ/OFDMA system, we provided a low-complexity optimal power/bandwidth allocation algorithm for total transmit power minimization when HARQ-related metric (actually the

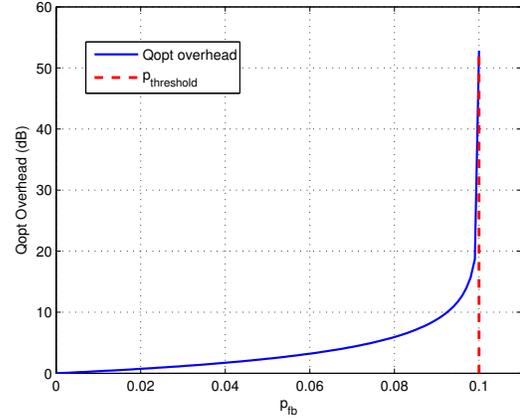


Figure 5. Power loss (in dB) versus the feedback error probability.

goodput) is considered and only statistical CSI at the cluster head is available. In addition, an efficient MCS selection algorithm for transmit power minimization has been proposed. Finally, the effect of non-ideal feedback has been studied.

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