

Optimal Resource Allocation for Type-II-HARQ-based OFDMA Ad Hoc Networks

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Abstract—We address the problem of multiuser power and bandwidth allocation for a general class of OFDMA-based wireless networks that employ a Type-II Hybrid Automatic Repeat reQuest (HARQ) transmission scheme. We assume the resource manager has only statistical knowledge of the Channel State Information (CSI) of the Rayleigh-distributed fast-fading links forming the network. We then propose the optimal algorithm minimizing the total transmit power required to satisfy per-user goodput constraints when practical Coding and Modulation Schemes (MCSs) are carried out.

I. INTRODUCTION

In modern wireless network standards, such as IEEE 802.16 and 3GPP LTE, HARQ has been included since it is a powerful link-layer mechanism that allows reliable communications over unknown varying channels. Among the different HARQ schemes, the so-called Type-II, which includes Chase Combining HARQ (CC-HARQ) and Incremental Redundancy HARQ (IR-HARQ) [1], is the most promising in terms of performance. In addition, in order to handle properly the multipath and the multi-user interference, Orthogonal Frequency Division Multiple Access (OFDMA) is now widely spread.

Aiming to **reduce energy consumption**, the paper deals with transmit power minimization for wireless networks using OFDMA at the physical layer and Type-II HARQ at the link layer.

To the best of our knowledge, only few works *e.g.*, [2]-[9], have addressed the problem of multiuser resource allocation for communication systems utilizing HARQ. In [2], [3], the system goodput is maximized with respect to user selection, and power and rate allocation for a system employing Type-II HARQ and outdated CSIT. However, due to the user selection, no more than one user can be scheduled at any given time, and the system goodput is computed using an information-theoretical approach that fails to account for practical MCSs. This last limitation also applies to [4]. In [5], perfect CSIT and Type-I HARQ are considered along with practical MCSs. Nevertheless, the proposed resource allocation is suboptimal since subchannel assignment and power allocation are not jointly optimized. A heuristic suboptimal resource allocation scheme is proposed in [6] for multiuser HARQ-based uplink communications in Single Carrier (SC)-FDMA systems. In the context of cognitive radio, some works are also devoted

to resource allocation of secondary users when HARQ are employed [7], [8]. Finally, in [9], transmit power minimization is done in presence of statistical CSIT and practical MCS but only for Type-I HARQ. *In this paper, our main contribution will be to extend [9] in the context of Type-II HARQ.* More precisely, we address the power and bandwidth allocation for **minimizing the total transmit power** under per-user goodput constraint in multi-user OFDMA-based networks using Type-II HARQ and practical MCSs when only statistical CSIT is available. Notice that Type-II HARQ is more appealing than Type-I HARQ from a practical point of view but the extension from Type-I HARQ to Type-II HARQ is not straightforward since the involved closed-form expressions for their performance metrics are much more complicated.

Before going further, we recall the justification of the assumption of only statistical CSIT. In wireless ad hoc networks, generally, a node, called “resource manager”, is elected to perform the resource allocation even if pairwise communications are allowed. The time delay between the initiation of a specific link and the reception by the resource manager of the CSI feedback associated with that link may last several frame periods [9]. As a consequence, the resource manager has only outdated CSI whereas its statistical knowledge of the channel is much more accurate due to its much larger coherence time.

The rest of the paper is organized as follows. The system model is depicted in Section II. In Section III, the resource allocation problem is mathematically formulated and its solution is analytically derived. Numerical results are presented in Section IV, while conclusions are provided in Section V.

II. SYSTEM MODEL

We focus on a network with K active point-to-point links. One node amongst all the nodes is the resource manager and will perform the proposed resource allocation algorithm. Each link k ($k \in \{1, 2, \dots, K\}$) is considered as a time-varying frequency-selective channel whose M time-domain taps are Rayleigh distributed. It is assumed that OFDM (with N subcarriers covering a total bandwidth of W Hz) is employed and that channels remain constant over one OFDM symbol but change independently between consecutive OFDM symbols.

Let $\mathbf{h}_k(i) = [h_k(i, 0), \dots, h_k(i, M - 1)]^T$ be the channel impulse response of link k associated with OFDM symbol i where the superscript $(\cdot)^T$ stands for the transposition operator.

The multi-variate complex-valued circular Gaussian distribution with mean \mathbf{a} and covariance matrix Σ is hereafter denoted by $\mathcal{CN}(\mathbf{a}, \Sigma)$. Let $\mathbf{H}_k(i) = [H_k(i, 0), \dots, H_k(i, N-1)]^T$ be the Fourier Transform of $\mathbf{h}_k(i)$. The received signal associated with OFDM symbol i for link k at subcarrier n is

$$Y_k(i, n) = H_k(i, n)X_k(i, n) + Z_k(i, n),$$

where $X_k(i, n)$ is the symbol transmitted at subcarrier n of OFDM symbol i on link k , and where $Z_k(i, n) \sim \mathcal{CN}(0, N_0 W/N)$ is an additive noise. It is assumed that the time-domain channel taps $\{h_k(i, m)\}_{i, 0 \leq m \leq M-1}$ are independent random variables with variances $\zeta_{k,m}^2$ that are constant with respect to the OFDM symbol index i but which possibly vary from tap to tap, *i.e.*, $\mathbf{h}_k(i) \sim \mathcal{CN}(0, \Sigma_k)$ with $\Sigma_k \stackrel{\text{def}}{=} \text{diag}_{M \times M}(\zeta_{k,0}^2, \dots, \zeta_{k,M-1}^2)$. Therefore, the subcarriers of a single user are identically distributed satisfying $H_k(i, n) \sim \mathcal{CN}(0, \zeta_k^2)$ where $\zeta_k^2 = \text{Tr}(\Sigma_k)$. Define the gain-to-noise ratio (GNR) associated with user k as

$$G_k \stackrel{\text{def}}{=} \frac{\mathbb{E}[|H_k(i, n)|^2]}{N_0} = \frac{\zeta_k^2}{N_0}. \quad (1)$$

The resource manager is assumed to only know the gains G_k for all active links. Since G_k is subcarrier-independent, we cannot decide which subset of subcarriers a link should use, but only how many. Let n_k be the number of subcarriers assigned to link k . Its bandwidth proportion is thus

$$\gamma_k \stackrel{\text{def}}{=} \frac{n_k}{N}.$$

For the same reason, it is natural to transmit with same average power $P_k \stackrel{\text{def}}{=} \mathbb{E}[|X_k(i, n)|^2]$ on all the n_k subcarriers. Let

$$E_k \stackrel{\text{def}}{=} \frac{P_k}{W/N}$$

be the energy consumed to transmit one symbol on one subcarrier of link k and define $\sigma_k^2 \stackrel{\text{def}}{=} N_0 W/N$ as the corresponding noise variance. As the average energy consumed on link k to send its OFDM symbol part is equal to $N\gamma_k E_k$, we would like to jointly optimize the power parameters $\{E_k\}_{1 \leq k \leq K}$ and the bandwidth parameters $\{\gamma_k\}_{1 \leq k \leq K}$. Each subcarrier of link k undergoes an average signal-to-noise ratio (SNR) given by

$$\text{SNR}_k \stackrel{\text{def}}{=} \frac{\zeta_k^2 P_k}{\sigma_k^2} = G_k E_k.$$

At the Medium Access Layer (MAC), each link arranges an infinite stream of information bits coming from the upper layer into packets of n_b bits each. A Type-II HARQ scheme is then used to transmit each information packet in at most L transmissions. The content of each one of these L transmissions is called a MAC Packet (MP). It depends on the particular Type-II scheme in use. We examine two such possible schemes: *i) CC-HARQ*: The MP is obtained by encoding the information packet with a Forward Error Correcting (FEC) code of rate R_k . At the end of each transmission l ($1 \leq l \leq L$), the receiver combines the so-far received l MPs according to the Maximum Ratio Combining (MRC) principle [10]. *ii) IR-HARQ*: The

information packet is firstly encoded by a FEC code of rate R_k/L (known as the mother code). The resulting codeword is then split into L MPs by following the *rate compatible coding* principle [11]. The length of the MPs can be made equal for some values of L , which is assumed from now on. After the reception of the l th MP, the receiver tries to decode the information packet by concatenating the current MP with the $l-1$ previously received MPs. Note that the l th step is equivalent to decoding a lR_k -rate code. We assume that the symbols transmitted on the subcarriers assigned to any link k are chosen from a 2^{m_k} -QAM constellation. The MCS associated with link k can thus be represented by the couple (m_k, R_k) where R_k , we recall, is the rate of the FEC code associated with the first transmission of the HARQ round.

Let $\mathcal{E}_{k,l}$ be the event that decoding of the information packet based on the combination/concatenation of the first l MPs results in an error and define $\pi_{k,l} \stackrel{\text{def}}{=} \mathbb{P}\{\mathcal{E}_{k,l}\}$. We assume that Bit-Interleaved Coded modulation (BICM) along with a random Subcarrier Assignment Scheme (SAS) are utilized by all the wireless links. Under this assumption, if the channel model is close enough to fast fading, the l th transmission of any HARQ round can achieve the maximum diversity gain of $d_{k,l}$ defined as follows. In the case of IR-HARQ, the non-decreasing sequence $d_{k,1}, \dots, d_{k,L}$ represent the minimal Hamming distances associated respectively with transmissions $l = 1, \dots, L$ and can be found in the tables provided in [11] for several coding rates R_k . As for CC-HARQ, $d_{k,l} = ld_{k,1}$. The results of [12], [13] can thus be applied to show that

$$\pi_{k,l} \approx \frac{g_{k,l}(m_k, R_k)}{\text{SNR}_k^{d_{k,l}(R_k)}}, \quad (2)$$

where $g_{k,l}$ is a constant depending on the selected MCS and designed to fit the simulated $\mathbb{P}\{\mathcal{E}_{k,l}\}$ curve.

III. OPTIMAL POWER AND BANDWIDTH ALLOCATION

We will consider that the MCSs of the different users are fixed in advance. The selection of these MCSs can be done using the near-optimal greedy algorithm provided in [9].

We remind that we would like to minimize the total transmit power, proportional to $\sum_{k=1}^K \gamma_k E_k$, while minimum per-user goodput for each link k , denoted by $\eta_k^{(0)}$, is guaranteed. The goodput corresponds to the number of successfully-decoded information bits per channel use and the goodput associated with the link k is denoted by $\eta_k(\gamma_k, E_k)$. Therefore the optimization issue of interest writes as follows:

Problem 1. *The general optimization problem is*

$$\min_{\gamma_1, \dots, \gamma_K, E_1, \dots, E_K} \sum_{k=1}^K \gamma_k E_k \quad \text{subject to} \quad (3)$$

$$\eta_k(\gamma_k, E_k) \geq \eta_k^{(0)}, \forall k \in \{1, \dots, K\}, \quad (4a)$$

$$\sum_{k=1}^K \gamma_k \leq 1, \quad (4b)$$

$$\gamma_k > 0, E_k > 0, \forall k \in \{1, \dots, K\}. \quad (4c)$$

In order to go further, we need to exhibit the relationship between η_k and (γ_k, E_k) . According to [10], we know that the goodput for any Type-II HARQ writes as follows

$$\eta_k(\gamma_k, E_k) = m_k R_k \gamma_k \frac{1 - q_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} q_{k,l}(G_k E_k)}. \quad (5)$$

where $q_{k,l}(\text{SNR}_k)$ is the probability that the first l transmissions of a HARQ round are all received in error *i.e.*,

$$q_{k,l}(\text{SNR}_k) = \mathbb{P}\{\mathcal{E}_{k,1}, \mathcal{E}_{k,2}, \dots, \mathcal{E}_{k,l}\}. \quad (6)$$

It is difficult to get $q_{k,l}$ in closed-form. However, as done in [10], $q_{k,l}$ can be upper-bounded by $\pi_{k,l}$ which can be analytically expressed thanks to Eq. (2). Moreover this bound is relatively tight for all practical values of L and the SNR [10]. Consequently, the goodput η_k can be lower-bounded as follows

$$\eta_k \geq m_k R_k \gamma_k \frac{1 - \pi_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(G_k E_k)}. \quad (7)$$

We thus can modify slightly Problem 1 by replacing the LHS of Eq. (7) with its RHS in the constraint related to the goodput.

Problem 2. *The considered optimization problem becomes*

$$\min_{\gamma_1 \dots \gamma_K, E_1 \dots E_K} \sum_{k=1}^K \gamma_k E_k \quad \text{subject to} \quad (8)$$

$$\gamma_k \frac{1 - \pi_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(G_k E_k)} \geq \frac{\eta_k^{(0)}}{m_k R_k}, \forall k \in \{1, \dots, K\} \quad (9a)$$

$$\sum_{k=1}^K \gamma_k \leq 1, \quad (9b)$$

$$\gamma_k > 0, E_k > 0, \forall k \in \{1, \dots, K\}. \quad (9c)$$

For the sake of tractability, we consider that γ_k can take any value in $(0, 1)$. The following lemma provides a sufficient and necessary feasibility condition for Problem 2.

Lemma 1. *Problem 2 is feasible if and only if,*

$$\sum_{k=1}^K \frac{\eta_k^{(0)}}{m_k R_k} < 1. \quad (10)$$

Moreover, Eq. (10) implies that Slater's condition holds.

In general, Problem 2 is not convex due to the constraint related to the goodput and given by Eq. (9a). Nevertheless, by assuming the specific expression for $\pi_{k,l}$ given by Eq. (2), we obtain a *geometric program* [14] which means that all the involved function (except Eq. (9c) considered as implicit constraint) are posynomials with respect to $\{\gamma_k\}_{k=1, \dots, K}$ and $\{E_k\}_{k=1, \dots, K}$. Eq. (8) and Eq. (9b) are straightforwardly posynomials. Plugging Eq. (2) into Eq.(9a) leads to the following new expression of the constraint related to the goodput

$$\frac{\eta_k^{(0)}}{m_k R_k} \gamma_k^{-1} + \sum_{l=1}^{L-1} \frac{\eta_k^{(0)} g_{k,l}}{m_k R_k G_k^{d_{k,l}}} \gamma_k^{-1} E_k^{-d_{k,l}} + \frac{g_{k,L}}{G_k^{d_{k,L}}} E_k^{-d_{k,L}} \leq 1,$$

$\forall k \in \{1, \dots, K\}$. Clearly, this previous constraint (which replaced Eq. (9a)) is also posynomial.

It is well-known that a *geometric program* can be transformed to a convex optimization problem thanks to the change of variables $\gamma_k = e^{x_k}$ and $E_k = e^{y_k}$ for $k \in \{1, \dots, K\}$ [14]. Consequently, Problem 2 can be viewed as a convex optimization problem whose KKT conditions provide thus a globally-optimal solution since Slater's condition holds due to Lemma 1. Let μ_k, λ be the non-negative Lagrangian multipliers associated with constraints (9a), (9b) respectively and define function $x \mapsto f_k(x)$ for any value $x \in \mathbb{R}_+^*$ of the SNR as

$$f_k(x) = \frac{1 + \sum_{l=1}^{L-1} \pi_{k,l}(x)}{1 - \pi_{k,L}(x)}. \quad (11)$$

Note that the LHS of Eq. (9a) is equal to $\gamma_k / f_k(G_k E_k)$ and that f_k is strictly decreasing on $(g_{k,L}^{1/d_{k,L}}, +\infty)$. Here, $g_{k,L}^{1/d_{k,L}}$ is the smallest value that $G_k E_k$ can take while the approximate goodput (the RHS of Eq. (7)) is non-negative. First derived in x_k, y_k then in γ_k, E_k , the KKT conditions boil down to

$$\gamma_k E_k - \mu_k \frac{\eta_k^{(0)}}{m_k R_k \gamma_k} \left(1 - \sum_{l=1}^{L-1} \frac{g_{k,l}}{G_k E_k^{d_{k,l}}} \right) + \lambda \gamma_k = 0 \quad (12)$$

$$\gamma_k E_k - \mu_k \left(\frac{\eta_k^{(0)}}{m_k R_k} \sum_{l=1}^{L-1} \frac{g_{k,l} d_{k,l}}{\gamma_k G_k E_k^{d_{k,l}}} - \frac{g_{k,L} d_{k,L}}{G_k E_k^{d_{k,L}}} \right) = 0 \quad (13)$$

$$\mu_k \left(\frac{\eta_k^{(0)}}{m_k R_k} - \frac{\gamma_k}{f_k(G_k E_k)} \right) = 0 \quad (14)$$

$$\lambda \left(\sum_{k=1}^K \gamma_k - 1 \right) = 0. \quad (15)$$

From Eq. (9c) and Eq. (13), $\mu_k \neq 0$. Eq. (14) thus yields

$$\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k). \quad (16)$$

Moreover, we can eliminate the non-zero multipliers μ_k by plugging Eqs. (13) and (16) into Eq. (12) to obtain

$$\lambda = \frac{E_k}{\frac{\sum_{l=1}^{L-1} d_{k,l} \pi_{k,l}(G_k E_k)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(G_k E_k)} + \frac{d_{k,L} \pi_{k,L}(G_k E_k)}{1 - \pi_{k,L}(G_k E_k)}} - E_k \stackrel{\text{def}}{=} \frac{1}{G_k} F_k(G_k E_k), \quad (17)$$

where we defined for any $x \in \mathbb{R}_+^*$,

$$F_k(x) = \frac{x}{\frac{\sum_{l=1}^{L-1} d_{k,l} \pi_{k,l}(x)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(x)} + \frac{d_{k,L} \pi_{k,L}(x)}{1 - \pi_{k,L}(x)}} - x. \quad (18)$$

After some tedious derivations, we show that the inverse function F_k^{-1} exists on $[0, +\infty)$ and that it is strictly increasing on its domain. Consequently, the following theorem holds.

Theorem 1. *Assume that Condition (10) holds. The optimal solution $(\gamma_1^* \dots \gamma_K^*, E_1^* \dots E_K^*)$ to Problem 2 is as follows. If $\sum_{k=1}^K \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(0)) \leq 1$, then*

$$E_k^* = \frac{1}{G_k} F_k^{-1}(0), \quad \gamma_k^* = \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(0)).$$

Else,

$$E_k^*(\lambda^*) = \frac{1}{G_k} F_k^{-1}(G_k \lambda^*), \quad \gamma_k^*(\lambda^*) = \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(G_k \lambda^*))$$

with $\lambda^* > 0$ the unique solution to $\sum_{k=1}^K \gamma_k^*(\lambda^*) = 1$.

The following relatively simple algorithm provides the optimal resource allocation parameters $\{\gamma_k^*, E_k^*\}_{k=1\dots K}$.

Algorithm 1 Optimal resource allocation for Problem 2

```

λ ← 0
repeat
  for all k ∈ {1, 2, ..., K} do
    γk ←  $\frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(G_k \lambda))$ 
    Ek ←  $\frac{1}{G_k} F_k^{-1}(G_k \lambda)$ 
  end for
  Increment λ
until  $\sum_{k=1}^K \gamma_k \leq 1$ 
return {γk* = γk}k=1...K, {Ek* = Ek}k=1...K

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IV. NUMERICAL RESULTS

A network with $K = 10$ active links is considered. The total bandwidth is assumed equal to $W = 5$ MHz centered around the carrier frequency $f_0 = 2400$ MHz. Each information packet is of length $n_b = 128$ bits. The distance D_k between the transmitter and the receiver of any link k is randomly drawn from a uniform distribution on $[D_{\min}, D_{\max}]$ with $D_{\min} = 100$ m and $D_{\max} = 1$ km. The parameter ς_k defined in Eq. (1) and corresponding to the pathloss, follows a free-space model so that $\varsigma_k(D_k) = 1/(4\pi f_0 D/c)^2$, where c is the speed of light in vacuum. For the sake of simplicity, each link has the same target efficiency $\eta^{(0)}$ so that the required sum rate is equal to $K\eta^{(0)}$. Finally, we fix $N_0 = -170$ dBm/Hz.

At the physical layer, QPSK is used on all the links. At the link layer, we have implemented 4 HARQ schemes: a Type-I HARQ based on the convolutional code of rate 1/2 given in [11], a CC-HARQ based on the same code, an IR-HARQ based on the RCPC codes of rates $\{4/5, 2/5, 4/15\}$ given in [11] and an IR-HARQ built with the nested codes of rates $\{1/2, 1/4, 1/6\}$ from [15]. The corresponding MCSs trivially satisfy Condition (10) for all the sum rates up to 5 Mbps. In Figure 1, we plot the corresponding sum transmit power $W \sum_{k=1}^K \gamma_k E_k$ as determined by Algorithm 1 vs the sum target rate $K\eta^{(0)}$. Each point of the plot was obtained using 200 Monte-Carlo runs. We note that the 1/2-rate nested-codes IR-HARQ results in the lowest power consumption. The 4/5-rate IR-HARQ outperforms the 1/2-rate Type-I and CC-HARQ schemes for all target rates up to 3.3 Mbps. However, it is worse for higher target rates since larger transmit powers are needed to compensate for its larger pre-HARQ performance $(\pi_{k,1})$. This motivates the need for optimizing the MCS selection, a problem that will be addressed in future works.

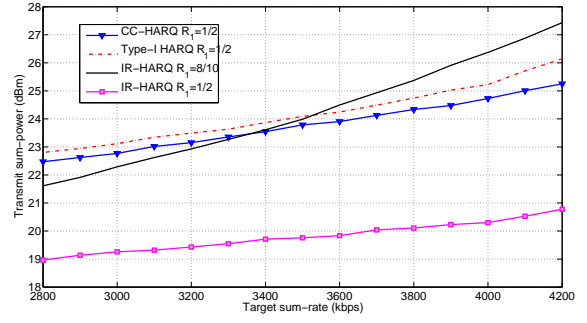


Fig. 1. Sum transmit power vs sum target rate

V. CONCLUSION

We provided an optimal algorithm to minimize sum-power consumption in OFDMA-based networks using Type-II HARQ subject to per-link goodput constraints. Under the assumption of statistical CSI and practical MCSs, our algorithm returns the optimal power and bandwidth parameters. Additional per-link and per-node transmit power constraints are dealt with in future works.

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