On OFDMA Resource Allocation for Delay Constrained HARQ Systems

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Abstract—**The paper addresses multiuser power and bandwidth allocation in an OFDMA system using HARQ for a Rayleigh channel. New algorithms for minimizing the total transmit power under individual rate and delay constraints are proposed.**

I. Introduction

We consider a Mobile Ad Hoc Network (MANET) for which pairwise communications are possible. In order to simplify the network management, a clustering structure is performed where, for each cluster, a cluster head is selected and has to assign the resource allocation but not necessarily to relay the information between two users. In this paper, we focus on the resource allocation optimization inside a single cluster (without inter-cluster interference).

As envisaged in the future wireless systems, our communication scheme is based on i) Orthogonal Frequency Division Multiple Access (OFDMA) for managing the frequency-selectivity of multipath channels and vanishing the multiple access interference (as in [1]), and ii) Hybrid Automatic Repeat reQuest (HARQ) for enforcing the quality of the link thanks to the packet retransmission. In wireless mobile environment, the users may not be able to provide instantaneous and perfect Channel State Information (CSI) at the cluster head due to fast channel variations. Moreover, the amount of CSI is huge in an ad hoc network since CSI for each pairwise link must be reported. Therefore, we will not consider the case of perfect CSI at the cluster head. As a consequence, each channel has to be modeled with a random variable assuming here Rayleigh distribution. As the statistics (mean, variance, etc) of each channel varies slowly, the cluster head is assumed to only know the channel statistics (but so not the realizations). In order to handle the diversity issue induced by the channel randomness, Frequency Hopping (FH) is performed.

In [1], the authors addressed the power minimization issue at the cluster head under user rate constraint. Assuming capacity-achieving coding, the rate was evaluated through the so-called ergodic capacity. In our paper, practical Modulation and Coding Schemes (MCS) will be considered. Unfortunately, the results in [1] cannot be extended to practical MCS since the notion of SNR gap (see [2] and references therein) cannot be applied to the ergodic capacity. Furthermore, since HARQ is used, the so-called goodput will be the relevant metric for characterizing the information rate.

In the literature [2]–[4], the goodput has moderately been used in multiuser resource allocation based on HARQ/OFDMA systems, but neither with statistical CSI at the transmitter, nor with delay constraints. Although constraining the packet errors at MAC level enables to roughly control maximum transmission delay, it is better to consider the HARQ based delay metric as the true delay constraint.

Therefore our purpose is to minimize the total transmit power under individual rate and delay constraints, when only statistical CSI is available at the cluster head. Power minimization is of great interest to increase the network lifetime and to mitigate the inter-cluster interference. So, the main novelty of our paper is to include delay constraints as well as statistical CSI into a resource allocation issue in HARQ/OFDMA scheme. Note that similar works in [5], [6] have been introduced but with different constraints.

II. System model and notations

Each link is considered as a (time-varying) frequency-selective channel, hence OFDM (with N subcarriers) is used to compensate for the frequency selectivity. It is assumed that the channel remains constant over one OFDM symbol but may change between two consecutive OFDM symbols. The channel corresponds to the link between the transmitting user k and its corresponding receiving node in the network. We assume that Fourier Transform of each channel taps for link k is identically distributed (i.d.) with variance ς_k^2 [1]. Finally, it is assumed the cluster head only knows the variance ς_k^2 of each active link k.

The mean Gain to Noise Ratio (GNR) between the transmitter and receiver of link k is $G_k := \varsigma_k^2/N_0$, where N_0 is the background noise power spectral density, and ς_k^2 depends on the path-loss only characterized by the length of the link D_k and the chosen model (free-space, for instance).

Assume that K users are active in the considered cluster. OFDMA is used to separate the users. Since the cluster head only knows statistical CSI through G_k, which does not depend on n, it cannot allocate which subcarriers user k will use, but only how many. The bandwidth proportion occupied by user k is

$$\gamma_k = n_k/N,$$

where n_k is the number of subcarriers assigned to user k. As the variance on each subcarrier is the same for the considered link k, user k will use the same average power $P_k = \mathbb{E}\left[|X_k(i,n)|^2\right]$ on each subcarrier. Let $E_k := P_k/(W/N)$ and $\sigma^2 := N_0(W/N)$ be the energy consumed to send one symbol on each subcarrier and the corresponding noise variance, respectively. Then, on each subcarrier, user k undergoes average signal-to-noise ratio (SNR) given by $\mathrm{SNR}_k = \varsigma_k^2 P_k/\sigma^2 = E_k G_k$. Finally, let Q_k be the average energy consumed to send the part of the OFDM symbol associated with user k. One can easily show that

$$Q_k = \frac{n_k P_k}{W} = \gamma_k E_k. \tag{1}$$

Each user employs Type-I HARQ for which an information packet can be sent at most L times. The user (successful) data rate ρ_k (in bits/s) is related to the HARQ goodput η_k (in bits/s/Hz) given $\rho_k = \eta_k W$. According to [7], the goodput is

$$\eta_k = \gamma_k m_k R_k (1 - \pi_k), \tag{2}$$

where π_k is the raw (information) Packet Error Probability (PEP) without considering the retransmission scheme, m_k is the number of bits per symbol, and R_k is the coding rate of the considered FEC in the Type-I HARQ scheme. Due to ARQ mechanism, the (successfully received) information packets associated with user k are received after $\delta(\pi_k)$ packet transmissions with $x \mapsto \delta(x) = 1/(1-x) - Lx^L/\left(1 - x^L\right)$ [7]. The term $\delta(\pi_k)$ corresponds to the so-called "delay" in HARQ literature. Actually, as a user does not occupy the entire bandwidth, the real delay is $\delta(\pi_k)$ divided by a factor associated with the bandwidth occupation rate, namely, γ_k. Therefore the delay for each (successfully received) information packet of user k, denoted by d_k, is given by

$$d_k = \frac{1}{\gamma_k}\delta(\pi_k), \tag{3}$$

Finally, we have $\pi_k = g_{m,R}(\mathrm{SNR}_k)$, where $g_{m,R}$ is a function depending on the code (of rate R) and modulation (of size 2^m). Some closed-form expressions for $g_{m,R}$ can be found in [8], [9] for the uncoded case and in [10] for the coded case.

III. Power and bandwidth allocation

Our objective is to minimize the total energy used for sending an OFDM symbol, *i.e.* to minimize $\mathcal{Q}_T = \sum_{k=1}^K Q_k$ with respect to the user energy Q_k, the bandwidth γ_k, and the MCS (driven by m_k and R_k). The choice of the best MCS can be done as in [9] and therefore it will not be discussed later. So, m_k and R_k are fixed.

Each user has to ensure minimum rate and maximum delay, *i.e.* there exist strictly positive constants $\rho_k^{(0)}$ and $d_k^{(0)}$ such that $\rho_k \geq \rho_k^{(0)}$ and $d_k \leq d_k^{(0)}$, respectively. The rate constraint can be translated into goodput constraint as $\eta_k \geq \eta_k^{(0)}$ with $\eta_k^{(0)} = \rho_k^{(0)}/W$. Thus, the optimization problem can be formalized as in Problem 1.

Problem 1. *Let us denote $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_K]^T$ and $\boldsymbol{Q} = [Q_1, \cdots, Q_K]^T$, where ^{T} stands for the transposition operator. The optimization problem boils down to*

$$(\boldsymbol{\gamma}^*, \boldsymbol{Q}^*) = \arg\min_{(\boldsymbol{\gamma}, \boldsymbol{Q})} \sum_{k=1}^K Q_k \tag{4}$$

subject to

(C1) $\eta_k(\gamma_k, Q_k) \geq \eta_k^{(0)}$, $\forall k$, (C3) $\sum_{k=1}^K \gamma_k \leq 1$,

(C2) $d_k(\gamma_k, Q_k) \leq d_k^{(0)}$, $\forall k$, (C4) $\gamma_k \geq 0, Q_k \geq 0$, $\forall k$.

We can easily obtain the following feasibility condition.

Theorem 1. *Problem 1 is feasible if, and only if,*

$$\sum_{k=1}^K \underbrace{\max\left(\eta_k^{(0)}/(m_k R_k), 1/d_k^{(0)}\right)}_{c_k} < 1 \tag{5}$$

is satisfied.

Proof: If Problem 1 is feasible, then there exists $(\boldsymbol{\gamma}, \boldsymbol{Q}) \in (0,1)^K \times \mathbb{R}_+^K$ such that for all $k \in \mathcal{K}$,

$$\begin{cases} \eta_k^{(0)} < \gamma_k m_k R_k (1 - \pi_k(G_k Q_k/\gamma_k)) \\ d_k^{(0)} > \frac{1}{\gamma_k}\delta(\pi_k(G_k Q_k/\gamma_k)) \\ \sum_k \gamma_k \leq 1 \end{cases} \Rightarrow \begin{cases} \eta_k^{(0)} < \gamma_k m_k R_k \\ d_k^{(0)} > \frac{1}{\gamma_k} \end{cases}$$

since $1 - \pi_k(G_k Q_k/\gamma_k) < 1$ and $\delta(\pi_k(G_k Q_k/\gamma_k)) > 1$. So we have $\gamma_k > \max(\eta_k^{(0)}/(m_k R_k), 1/d_k^{(0)})$ which concludes the first part of the proof.

Conversely, let us define the open set

$$\mathcal{O} = \left\{ (\boldsymbol{\eta}, \boldsymbol{d}) \in \mathbb{R}_{+*}^{2K} \mid \sum_{k=1}^K \max(\eta_k/(m_k R_k), 1/d_k) < 1 \right\},$$

and thus $\mathbf{t}_0 = (\eta_k^{(0)}, d_k^{(0)}) \in \mathcal{O}$. Therefore, there exists $\epsilon > 0$ such that the ball

$$B(\mathbf{t}_0, \epsilon) = \{\mathbf{t} \in \mathbb{R}_{+*}^{2K} \mid ||\mathbf{t} - \mathbf{t}_0|| < \epsilon\} \subset \mathcal{O}.$$

where $\|.\|$ is the L_∞-norm.

In particular, we have $\{(\boldsymbol{\eta}, \boldsymbol{d}) \in \mathbb{R}_{+*}^{2K} \mid \forall k \in \{1, \cdots, K\}, |\eta_k - \eta_k^{(0)}| < \epsilon$ and $|d_k - d_k^{(0)}| < \epsilon\} \subset B(\mathbf{t}_0, \epsilon)$.

Now let us consider, $\forall k \in \{1, \cdots, K\}$,

$$\gamma_k = \max\left(\frac{\eta_k^{(0)} + \epsilon/2}{m_k R_k}, \frac{1}{d_k^{(0)} - \epsilon/2}\right).$$

Since $(\eta_k^{(0)} + \epsilon/2, d_k^{(0)} - \epsilon/2) \in B(\mathbf{t}_0, \epsilon)$, we have $\sum_{k=1}^K \gamma_k < 1$.

Furthermore let $\eta_k = m_k R_k \gamma_k (1 - \pi_k(G_k Q_k/\gamma_k))$ and $d_k = \delta(\pi_k(G_k Q_k/\gamma_k))/\gamma_k$. When $Q_k \to \infty$, we obtain

$$\eta_k \longrightarrow m_k R_k \max\left(\frac{\eta_k^{(0)} + \frac{\epsilon}{2}}{m_k R_k}, \frac{1}{d_k^{(0)} - \frac{\epsilon}{2}}\right) \geq \eta_k^{(0)} + \frac{\epsilon}{2} > \eta_k^{(0)}$$

and

$$d_k \longrightarrow \min\left(\frac{m_k R_k}{\eta_k^{(0)} + \frac{\epsilon}{2}}, d_k^{(0)} - \frac{\epsilon}{2}\right) \leq d_k^{(0)} - \frac{\epsilon}{2} < d_k^{(0)},$$

which concludes the proof. ■

Let us go back to the analysis of Problem 1. Whereas constraints (C1), (C3), and (C4) are convex, the delay constraint (C3) is not convex. Problem 1 is thus not convex and seems difficult to solve efficiently. Hence we developed two (sub-optimal) algorithms to solve Problem 1. We hereafter carefully describe them and this corresponds to the main contributions of the paper.

A. Algorithm 1

By looking numerically, the bivariate function $(x,y) \mapsto d_k(x,y)$ seems very close to be a quasi-convex function. As a consequence, using the KKT conditions seem to be a relevant way even if we are not able to guarantee their optimality [11]. Therefore, after tedious algebraic manipulations (given below), we deduce a simple algorithm from the KKT conditions.

We denote by $\boldsymbol{x} = [\boldsymbol{\gamma}, \boldsymbol{Q}]^T$ any solution of the allocation problem. Let $\boldsymbol{\mu}, \boldsymbol{\theta}, \lambda, \boldsymbol{\alpha}, \boldsymbol{\beta}$ be the Lagrange multipliers associated with the $4K+1$ constraints (C1)-(C4), respectively. Then, a KKT point $(\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\theta}, \lambda, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by the conditions[1]

$$\nabla Q(\boldsymbol{x}) - \sum_{k=1}^{K} \mu_k \nabla \eta_k(\boldsymbol{x}) + \sum_{k=1}^{K} \theta_k \nabla d_k(\boldsymbol{x})$$

$$+\lambda \nabla \left(\sum_{k=1}^{K} \gamma_k \right) - \sum_{k=1}^{K} \alpha_k \nabla \gamma_k - \sum_{k=1}^{K} \beta_k \nabla Q_k = 0, \quad \text{(6a)}$$

$$\eta_k(\boldsymbol{x}) \geq \eta_k^{(0)},\ d_k(\boldsymbol{x}) \leq d_k^{(0)},\ \sum_{k=1}^{K} \gamma_k \leq 1,\ \boldsymbol{x} \succeq 0, \quad \text{(6b)}$$

$$\boldsymbol{\mu} \succeq 0,\ \boldsymbol{\theta} \succeq 0,\ \lambda \geq 0,\ \boldsymbol{\alpha} \succeq 0,,\ \boldsymbol{\beta} \succeq 0, \quad \text{(6c)}$$

$$-\mu_k(\eta_k(\boldsymbol{x}) - \eta_k^{(0)}) = 0,\ \theta_k(d_k(\boldsymbol{x}) - d_k^{(0)}) = 0,$$

$$\lambda \left(\sum_{k=1}^{K} \gamma_k - 1 \right) = 0,\ \alpha_k \gamma_k = 0,\ \beta_k Q_k = 0. \quad \text{(6d)}$$

Before working on the KKT equations, we compute the gradients:

$$\frac{\partial \pi_k}{\partial Q_k} = G_k \frac{1}{\gamma_k} \pi_k'(G_k Q_k/\gamma_k),$$

$$\frac{\partial \pi_k}{\partial \gamma_k} = -G_k \frac{Q_k}{\gamma_k^2} \pi_k'(G_k Q_k/\gamma_k).$$

We remark that the gradients of η_k and d_k can be expressed as:

$$\nabla \eta_k = -m_k R_k \gamma_k \nabla \pi_k + \begin{bmatrix} 0 & m_k R_k (1-\pi_k) \end{bmatrix}^T, \quad (7)$$

$$\nabla d_k = \frac{\delta'(\pi_k)}{\gamma_k} \nabla \pi_k + \begin{bmatrix} 0 & -\frac{d_k}{\gamma_k} \end{bmatrix}^T, \quad (8)$$

with the function $\delta' : x \mapsto 1/(1-x)^2 - L^2 x^{L-1}/(1-x^L)^2$ for $x \in [0,1]$.

As a consequence, Eq. (6a) leads to the following $2K$ scalar equalities:

$$1 + \left(m_k R_k \gamma_k \mu_k + \frac{\delta'(\pi_k)}{\gamma_k} \theta_k \right) \frac{\partial \pi_k}{\partial Q_k} - \beta_k = 0, \quad (9)$$

and

$$\left(m_k R_k \gamma_k \mu_k + \frac{\delta'(\pi_k)}{\gamma_k} \theta_k \right) \frac{\partial \pi_k}{\partial \gamma_k}$$

$$- \quad m_k R_k (1-\pi_k) \mu_k - \frac{d_k}{\gamma_k} \theta_k + \lambda - \alpha_k = 0. \quad (10)$$

[1]In all the paper, a vector $\boldsymbol{x} \succ 0$ is equivalent to $x_\ell > 0, \forall \ell$.

It is easy to prove the following result:

Lemma 1. *The optimal solution $\boldsymbol{x}^*$ is such that $\boldsymbol{\gamma}^* \succ 0$ and $\boldsymbol{Q}^* \succ 0$.*

Proof: If $\exists k$ such that $\gamma_k = 0$, then this user would have no way to satisfy his rate nor its delay requirements ($\eta_k = 0$ whereas $d_k \to \infty$), and such a point would be primal infeasible. Thus $\forall k \in \{1, \cdots, K\}, \gamma_k > 0$.

Now, since $Q_k = \gamma_k E_k$, we have $Q_k = 0 \Rightarrow E_k = 0$ and hence $\pi_k = 1$. Once again, this leads to a primal infeasible solution. ■

Hence, by complementarity slackness (cf. Eq. (6d)), the associated Lagrange multipliers $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ vanish, and the set of $2K$ equalities in Eqs. (9)-(10) are equivalent to the following K independent 2-by-2 matrix linear equalities on (μ_k, θ_k),

$$S \begin{bmatrix} \mu_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} 1 \\ \lambda \end{bmatrix} = 0, \quad (11)$$

where

$$S = \begin{bmatrix} m_k R_k \gamma_k \frac{\partial \pi_k}{\partial Q_k} & \frac{\delta'(\pi_k)}{\gamma_k} \frac{\partial \pi_k}{\partial Q_k} \\ m_k R_k \left(\gamma_k \frac{\partial \pi_k}{\partial \gamma_k} - (1-\pi_k) \right) & \frac{\delta'(\pi_k)}{\gamma_k} \frac{\partial \pi_k}{\partial \gamma_k} - \frac{d_k}{\gamma_k} \end{bmatrix}.$$

These matrix equalities can be easily solved iff $\det S \neq 0$. Such a property is ensured by the next result.

Lemma 2. $\det S > 0$.

Proof: By direct computation:

$$\det S = -m_k R_k d_k \frac{\partial \pi_k}{\partial Q_k} + m_k R_k (1-\pi_k) \frac{\delta'(\pi_k)}{\gamma_k} \frac{\partial \pi_k}{\partial Q_k}$$

$$= \frac{-m_k R_k G_k \pi_k'(G_k Q_k/\gamma_k)}{\gamma_k^2} \left(\delta(\pi_k) - (1-\pi_k)\delta'(\pi_k) \right).$$

Since the packet error rate is a decreasing function of SNR, $\pi_k'(G_k Q_k/\gamma_k) \leq 0$. Thus $\det S$ has the same sign than $\delta(\pi_k) - (1-\pi_k)\delta'(\pi_k)$:

$$\delta(x) - (1-x)\delta'(x) = \frac{-Lx^L}{1-x^L} + (1-x)L^2 \frac{x^{L-1}}{(1-x^L)^2}$$

$$= \frac{L^2(1-x)x^{L-1} - Lx^L(1-x^L)}{(1-x^L)^2}$$

$$= \frac{Lx^{L-1}}{(1-x^L)^2} \left(x^2 - (L+1)x + L \right).$$

Finally, since the polynomial $x^2 - (L+1)x + L = (x-1)(x-L) > 0$ for $0 < x < 1$ (remind that $\pi_k < 1$ from Lemma 1), then $\det S > 0$. ■

After some simple algebra, we obtain the following solutions for Eq. (11):

$$\begin{bmatrix} \mu_k \\ \theta_k \end{bmatrix} = -\frac{S'}{\det S}$$

with

$$S' = \begin{bmatrix} \frac{-1}{\gamma_k{}^2} \left(\delta'(\pi_k) \pi_k'(G_k Q_k/\gamma_k) G_k \left(\frac{Q_k}{\gamma_k} + \lambda \right) + \delta(\pi_k) \right) \\ m_k R_k \pi_k'(G_k Q_k/\gamma_k) G_k \left(\frac{Q_k}{\gamma_k} + \lambda \right) + m_k R_k (1-\pi_k) \end{bmatrix}.$$

Hence, the complementary slackness equations (cf. Eq. (6d)) are now equivalent to

$$(M(G_kQ_k/\gamma_k) - \lambda G_k) \times \Big(\eta_k^{(0)} - \gamma_k m_k R_k\big(1-\pi_k(G_kQ_k/\gamma_k)\big)\Big) = 0 \quad (12)$$

$$(\Theta(G_kQ_k/\gamma_k) - \lambda G_k) \times \left(\frac{\delta\big(\pi_k(G_kQ_k/\gamma_k)\big)}{\gamma_k} - d_k^{(0)}\right) = 0, \quad (13)$$

$$\lambda\left(\sum_{k=1}^{K}\gamma_k - 1\right) = 0 \quad (14)$$

where, $\forall x \in \mathbb{R}_*^+$,

$$M(x) = -\frac{x\delta'(\pi_k(x))\pi_k'(x) + \delta(\pi_k(x))}{\delta'(\pi_k(x))\pi_k'(x)} \quad (15)$$

$$\Theta(x) = -\frac{x\pi_k'(x) + 1 - \pi_k(x)}{\pi_k'(x)}. \quad (16)$$

We will denote by $f^{(-1)}$ the inverse function of any function f with respect to the composition.

It is worth to emphasize that if user k satisfies $d_k^{(0)} \geq (m_kR_k)/\eta_k^{(0)}$, then its delay constraint is inactive. As a consequence, the right term in the LHS of Eq. (12) is equal to zero while the left term in the LHS of Eq. (13) is equal to zero. These both equalities finally perfectly characterize the associated γ_k and Q_k (see Item 1. in Algorithm 1).

Otherwise (*i.e.*, $d_k^{(0)} < (m_kR_k)/\eta_k^{(0)}$), we have two cases:

Let us assume that the rate constraint is inactive, *i.e.*, $\eta_k^{(0)} < \gamma_k m_k R_k(1-\pi_k(G_kQ_k/\gamma_k))$. Then, by Eq. (12), we have $M(G_kQ_k/\gamma_k) = \lambda G_k$ which leads to $Q_k = \frac{\gamma_k}{G_k}M^{(-1)}(\lambda G_k)$. Then two cases are possible: the delay constraint is active or not.

- if the delay constraint is active, then $\gamma_k = \frac{\delta(\pi_k(M^{(-1)}(\lambda G_k))}{d_k^{(0)}}$.
- if the delay constraint is inactive, then $\Theta(G_kQ_k/\gamma_k) = \lambda G_k$ which implies that $M^{(-1)}(\lambda G_k)) = \Theta^{(-1)}(\lambda G_k)$. This is impossible.

Let us assume that the rate constraint is active, *i.e.*, $\eta_k^{(0)} = \gamma_k m_k R_k(1-\pi_k(G_kQ_k/\gamma_k))$. Once again, two cases are possible: the delay constraint is active or not.

- if the delay constraint is active, then $\delta(\pi_k(G_kQ_k/\gamma_k)) = \gamma_k d_k^{(0)}$ which implies that it exists π_k such that (due to the active rate constraint)

  $$m_kR_k(1-\pi_k)\delta(\pi_k) = \eta_k^{(0)}d_k^{(0)}.$$

  According to the closed-form expression of δ, the corresponding π_k (in $(0,1)$) is a root of the polynomial equation:

  $$Lx^{L+1} - (L+1-d_k^{(0)}\eta_k^{(0)}/m_k)x^L + 1 - d_k^{(0)}\eta_k^{(0)}/m_k = 0. \quad (17)$$

- if the delay constraint is inactive, then $\Theta(G_kQ_k/\gamma_k) = \lambda G_k$. As a consequence, $\gamma_k(\lambda) = \eta_k^{(0)}/(m_kR_k(1-\pi_k(\Theta^{(-1)}(\lambda G_k))))$ (thanks to the active rate constraint) and $Q_k(\lambda) = \frac{\gamma_k(\lambda)}{G_k}\Theta^{(-1)}(\lambda G_k)$ (thanks to the inactive delay constraint).

Notice that when the rate constraint is inactive and the delay constraint is active, we have

$$\delta(\pi_k(M^{(-1)}(\lambda G_k)))(1-\pi_k(M^{(-1)}(\lambda G_k))) > \frac{\eta_k^{(0)}d_k^{(0)}}{m_kR_k}.$$

Similarly, when the rate constraint is active and the delay constraint is inactive, we have

$$\delta(\pi_k(\Theta^{(-1)}(\lambda G_k)))(1-\pi_k(\Theta^{(-1)}(\lambda G_k))) < \frac{\eta_k^{(0)}d_k^{(0)}}{m_kR_k}.$$

Quite arbitrarily, the algorithm is initialized with $\lambda = 0$ and assuming that only the rate constraint is active. If the delay constraint is not satisfied, we choose a higher γ_k for satisfying this constraint even if the constraint (C3) does not hold anymore. Then we will increase λ until a feasible solution is found, especially, to satisfy (C3). We summarize the above-mentioned steps in Algorithm 1.

Algorithm 1: Type-I HARQ-based resource allocation.

Set $\lambda = 0$,
$\gamma_k(0) = \max\left\{\frac{\eta_k^{(0)}}{m_kR_k(1-\pi_k(\Theta^{(-1)}(0)))}, \frac{\delta(\pi_k(\Theta^{(-1)}(0)))}{d_k^{(0)}}\right\}$,
$Q_k(0) = \frac{\gamma_k}{G_k}\Theta^{(-1)}(0),\ \forall k$,
if $\sum_{k=1}^{K}\gamma_k(0) < 1$ **then**
  exit
else
  $\mathbb{K}_M = \{k \in \{1,\ldots,K\} \,|\, d_k^{(0)} \geq 1/\eta_k^{(0)}\}$ and
  $\mathbb{K}_\Theta = \{1,\ldots,K\}\backslash\mathbb{K}_M$
  while $\sum_{k=1}^{K}\gamma_k(\lambda) > 1$ **do**
    1. $\forall k \in \mathbb{K}_M$, compute
      $\gamma_k(\lambda) = \frac{\eta_k^{(0)}}{m_kR_k(1-\pi_k(\Theta^{(-1)}(\lambda G_k)))}$, and
      $Q_k(\lambda) = \frac{\gamma_k(\lambda)}{G_k}\Theta^{(-1)}(\lambda G_k)$.
    2. $\forall k \in \mathbb{K}_\Theta$, compute
      $m_\lambda = M^{(-1)}(\lambda G_k)$, $\theta_\lambda = \Theta^{(-1)}(\lambda G_k)$
    if $\delta(\pi_k(m_\lambda))(1-\pi_k(m_\lambda)) > \frac{\eta_k^{(0)}d_k^{(0)}}{m_kR_k}$ **then**
      $\gamma_k(\lambda) = \delta(\pi_k(m_\lambda))/d_k^{(0)}$, and
      $Q_k(\lambda) = \frac{\gamma_k(\lambda)}{G_k}m_\lambda$.
    else if $\delta(\pi_k(\theta_\lambda))(1-\pi_k(\theta_\lambda)) < \frac{\eta_k^{(0)}d_k^{(0)}}{m_kR_k}$ **then**
      $\gamma_k(\lambda) = \eta_k^{(0)}/(m_kR_k(1-\pi_k(\theta_\lambda)))$, and
      $Q_k(\lambda) = \frac{\gamma_k(\lambda)}{G_k}\theta_\lambda$.
    else
      Compute x^* the root in $(0,1)$ of Eq. (17).
      $\gamma_k(\lambda) = \delta(x^*)/d_k^{(0)}$, and
      $Q_k = \frac{\gamma_k(\lambda)}{G_k}\pi_k^{(-1)}(x^*)$
    end
    3. Increase λ
  end
end

B. Algorithm 2

The second algorithm consists in rewriting Problem 1 versus (γ_k, E_k) instead of (γ_k, Q_k). Then, the function to minimize becomes biconvex but the constraints are much simpler.

Problem 2. *Problem 1 is equivalent to*

$$(\boldsymbol{\gamma}^*, \boldsymbol{E}^*) = \arg \min_{(\boldsymbol{\gamma}, \boldsymbol{E})} \sum_{k=1}^{K} \gamma_k E_k \tag{18}$$

subject to (C3) and

(C1') $\gamma_k \geq \eta_k^{(0)} / \left(m_k R_k \left(1 - \pi_k(G_k E_k)\right)\right), \forall k,$
(C2') $E_k \geq \pi_k^{(-1)}(\delta^{(-1)}(\gamma_k d_k^{(0)}))/G_k, \forall k,$
(C4') $\gamma_k \geq 0, E_k \geq 0,\ \forall k.$

Indeed, assuming $\boldsymbol{\gamma}$ fixed, then the Problem in $\boldsymbol{E} = [E_1, \cdots, E_K]^T$ is linear and vice-versa. Consequently, we propose to start by optimizing $\boldsymbol{E}$ given $\gamma_k = c_k / (\sum_{k'=1}^{K} c_{k'})$, and then to optimizing γ given the value of $\boldsymbol{E}$ obtained in the previous step, and so on. Obviously, we are once again not able to guarantee the optimality.

To be more precise, we split Problem 2 into two subproblems which will be solved alternately.

Problem 2.a (on $\boldsymbol{E}$)**.** *For fixed $\boldsymbol{\gamma}$, the subproblem is*

$$\boldsymbol{E}^* = \arg \min_{\boldsymbol{E}} \sum_{k=1}^{K} \gamma_k E_k$$

subject to (C1', C2', C4').

Problem 2.b (on γ)**.** *For fixed $\boldsymbol{E}$, the subproblem is*

$$\boldsymbol{\gamma}^* = \arg \min_{\boldsymbol{\gamma}} \sum_{k=1}^{K} \gamma_k E_k$$

subject to (C1', C2', C3, C4').

The solution of the i-th iteration of Problem 2.a (with $\gamma_k^{(i-1)}$ the solution of the $(i-1)$-th iteration) is given by

$$\begin{aligned} E_k^{(i)} &= \max\big(\pi_k^{(-1)}(1 - \eta_k^{(0)} / (m_k R_k \gamma_k^{(i-1)}))/G_k, \\ &\quad \pi_k^{(-1)}(\delta^{(-1)}(\gamma_k^{(i-1)} d_k^{(0)}))/G_k\big),\ \forall k. \end{aligned}$$

The solution of Problem 2.b can be efficiently obtained by using linear programming tool, for instance, the Simplex method [12].

IV. Numerical Results

An uncoded ARQ scheme with $L = 3$ is considered for $K = 4$ links. Each user sends a data packet consisted of 32 uncoded bits within a bandwidth $W = 1$ MHz. BPSK is used, and the path-loss follows a free-space model. The distance D_k between both users associated with the k-th link is randomly chosen from a uniform distribution in $[D_m, D_M]$, with $D_m = 50$ m and $D_M = 1$ km. For the sake of simplicity, each link has the same target efficiency and delay constraints.

As a benchmark, we will also compute the following straightforward algorithm: $\gamma_k = c_k / (\sum_{k'=1}^{K} c_{k'})$, and $Q_k = P/K, \forall k$, where P is chosen such that (C1)-(C2) are satisfied.

In Fig. 1, we display the total transmit power versus the sum rate for two different delay constraints ($d^{(0)} = 8$, and $d^{(0)} = 20$). Both proposed algorithms outperform the straightforward one. Notice that when $\eta_k^{(0)} / (m_k R_k) \geq 1/d_k^{(0)}, \forall k$, one can prove that the delay constraint is never active, and so (C2) can be deleted in Problem 1. Consequently Problem 1 is convex and KKT becomes optimal. When $d^{(0)} = 8$, the KKT is optimal as soon as the sum rate is larger than $KW/8 = 500$ kbps as observed in Fig. 1. When $d^{(0)} = 20$, the optimality of the KKT is ensured if the sum rate is larger than 200 kbps. When the sum rate is small enough, the other proposed algorithm may become better as observed for $d^{(0)} = 8$.

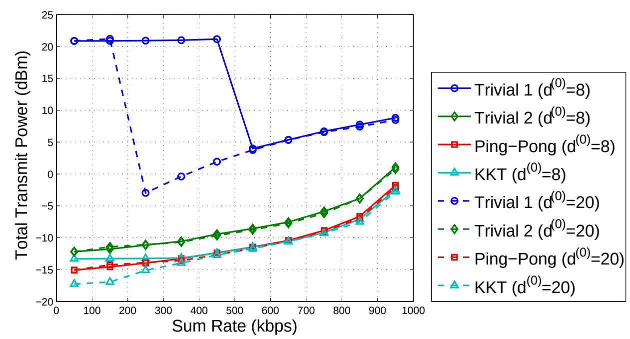

Figure 1. Total Transmit Power (in dBm) versus the sum rate.

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