Power Allocation for Multibeam Satellite Communications with Nonlinear Impairments

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***Abstract*—In the context of multibeam satellite uplink communications, we derive a closed-form expression for the sum rate when nonlinearity related to the high power amplifier is taken into account. We then propose a per-user power allocation for maximizing this sum rate. We resort to *Signomial Programming*. We show significant performance robustness compared to the allocation done by using linear regime only.**

Satellite communications are a candidate for routing traffic between two terrestrial points in order to handle the exponential increase of data traffic. Therefore satellite uplink/return link is of great interest where terrestrial users transmit data to the satellite, which then acts as a relay and sends it to a terrestrial gateway. The advocated bandwidth is the so-called Ka-band. Unfortunately, when operating close to their saturation regime, High Power Amplifiers (HPA) on satellite board may exhibit nonlinearity and communication may operate on this non-linear regime in order to ensure large Signal-to-Noise Ratio (SNR) enough. In addition, each satellite operates on various earth areas, called beams which may overlap. Consequently, the performance of the satellite system is affected by the nonlinear HPA which leads to several kinds of interference, especially, the inter-beam nonlinear interference.

The objective of this paper is twofold:

- Deriving in closed-form the per-user date rate by taking into account the nonlinearity and the multibeam context. Note that the single beam case has been addressed in [1].
- Optimizing the per-user power regarding several metrics: (a) minimization of the sum-power, (b) maximization of the minimum per-user rate, and (c) maximization of the sum-rate. The proposed solutions are valid for any HPA's operating mode.

The paper is organized as follows: in Section I, we introduce the system model. In Section II, we provide the data rate in closed-form and especially the non linear interference level. In Section III, we solve our resource allocation problem related to the the expressions exhibited in Section II. In Section IV, numerical evaluations are provided. Concluding remarks are drawn in Section V.

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I. System model

We assume B beams and each beam assigns M users. Frequency-Division Multiple-Access (FDMA) is considered for multiple access per beam. The user belonging to beam b and using subband m transmits a symbol sequence $\{a_{b,m,n}\}_n$. Let $P_{b,m} := \mathbb{E}\left[|a_{b,m,n}|^2\right]$ and $R_{b,m}$ be the power and the data rate for this user, respectively.

The baseband signal emitted by the user using subband m in beam b, denoted by $x_{b,m}(t)$, is

$$x_{b,m}(t) = \sum_{n\in\mathbb{Z}} a_{b,m,n} p_T(t - nT_s), \tag{1}$$

where $p_T(t)$ is a square-root Nyquist shaping filter, assumed to be the same for all users, and T_s is the symbol rate .

Each signal $x_{b,m}(t)$ is transposed around the frequency f_m. The difference between two adjacent frequencies is ΔF.

The antenna b at the satellite side associated with beam b receives the sum of the M transposed signals of this beam and the inter-beam interference. This received signal is denoted by $x_c^{(b)}(t)$,

$$x_c^{(b)}(t) = \sum_{m=1}^{M} \sqrt{G_m^{(b)}} x_{b,m}(t) e^{2i\pi f_m t} + x_{\text{IB}}^{(b)}(t) \tag{2}$$

where $x_{\text{IB}}^{(b)}(t)$ is the inter-beam interference,

$$x_{\text{IB}}^{(b)}(t) = \sum_{\substack{b'=1\\ b'\neq b}}^{B} \sum_{m=1}^{M} \sqrt{G_m^{(b',b)}} x_{b',m}(t) e^{2i\pi f_m t}, \tag{3}$$

with

- $G_m^{(b)}$ the channel gain on subband m between the user of beam b and antenna b,
- $G_m^{(b',b)}$ the channel gain on subband m between the user of beam b' and antenna b.

Let $y_c^{(b)}(t)$ be the received signal at the gateway coming from the antenna b. Due to the HPA on satellite board, modeled by a third-order power series, we get

$$y_c^{(b)}(t) = \omega_1 x_c^{(b)}(t) + \omega_3 x_c^{(b)}(t) x_c^{(b)}(t) \overline{x_c}^{(b)}(t) + w(t). \tag{4}$$

where $\overline{\cdot}$ stands for the complex-conjugate, and $w(t)$ is a complex-valued circularly-symmetric zero-mean AWGN. The coefficients ω_1 and ω_3 are positive parameters and characterize the nonlinear distortion of the HPA [2].

Let us now consider the demodulation for user belonging to beam b using subband m. We first go back in baseband,

$$y_{b,m}(t) = y_c^{(b)}(t)e^{-2i\pi f_m t}, \tag{5}$$

we then apply the matched filter $p_R(t) := \overline{p_T(-t)}$,

$$z_{b,m}(t) = \int_{\mathbb{R}} p_R(\tau) y_{b,m}(t-\tau)d\tau. \tag{6}$$

Assuming perfect synchronization between beams, which is realistic since the beams are collocated at the satellite, and after some straightforward derivations, we obtain

$$\begin{aligned}
&z_{b,m}(t) = \omega_1 \sum_{m'=1}^{M} \sum_{n'\in\mathbb{Z}} \sqrt{G_{m'}^{(b)}} a_{b,m',n'} e^{2i\pi(f_{m'}-f_m)t} \\
&\times h_1(t-n'T_s, m'-m) \\
&+ \omega_1 \sum_{\substack{b'=1\\ b'\neq b}}^{B} \sum_{m'=1}^{M} \sum_{n'\in\mathbb{Z}} \sqrt{G_{m'}^{(b',b)}} a_{b',m',n'} e^{2i\pi(f_{m'}-f_m)t} \\
&\times h_1(t-n'T_s, m'-m) \\
&+ \omega_3 \sum_{b_1,b_2,b_3=1}^{B} \sum_{m_1,m_2,m_3=1}^{M} \sum_{n_1,n_2,n_3\in\mathbb{Z}} \sqrt{G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)}} \\
&\times a_{b_1,m_1,n_1} a_{b_2,m_2,n_2} a_{b_3,m_3,n_3}^{*} e^{2i\pi(m_1+m_2-m_3-m)\Delta F t} \\
&\times h_3(t-n_1T_s, t-n_2T_s, t-n_3T_s, m_1+m_2-m_3-m) \\
&+ \int_{\mathbb{R}} p_R(\tau) w_c(t-\tau) e^{-2i\pi f_m(t-\tau)} d\tau,
\end{aligned} \tag{7}$$

with the following two Volterra kernels of first-order and third-order respectively

$$h_1(t_1,\ell) = \int_{\mathbb{R}} p_T(t_1-\tau)p_R(\tau)e^{-2i\pi\ell\Delta F\tau}d\tau,$$

$$h_3(t_1,t_2,t_3,\ell) = \int_{\mathbb{R}} \prod_{j=1}^{3} \left[p_T(t_j-\tau)\right] p_R(\tau)e^{-2i\pi\ell\Delta F\tau}d\tau.$$

Finally, the signal is sampled at the symbol rate T_s, resulting in the sequence $z_{b,m,n} = z_{b,m}(nT_s)$. This term can be decomposed into four parts:

$$z_{b,m,n} = z_{b,m,n}^{(\mathrm{L})} + z_{b,m,n}^{(\mathrm{I})} + z_{b,m,n}^{(\mathrm{NL})} + w_{b,m,n}, \tag{8}$$

with $z_{b,m,n}^{(\mathrm{L})}$ the part depending on the current symbol, $z_{b,m,n}^{(\mathrm{I})}$ the part depending linearly on the symbols $\{a_{b,m,n}\}$ except the current one, and $z_{b,m,n}^{(\mathrm{NL})}$ the part depending non-linearly on the symbols $\{a_{b,m,n}\}$.

As $h_1(nT_s, m)$ is zero for any $n \neq 0$ or any $m \neq 0$ (orthogonality in time and frequency between users), and one otherwise, we force $m' = m$ and $n' = n$ to obtain the linear part as follows

$$z_{b,m,n}^{(\mathrm{L})} = \omega_1 \sqrt{G_m^{(b)}} a_{b,m,n}, \tag{9}$$

$$z_{b,m,n}^{(\mathrm{I})} = \omega_1 \sum_{\substack{b'=1\\ b'\neq b}}^{B} \sqrt{G_m^{(b',b)}} a_{b',m,n}. \tag{10}$$

The non-linear part takes the following form

$$\begin{aligned}
z_{b,m,n}^{(\mathrm{NL})} =& \omega_3 \sum_{b_1,b_2,b_3=1}^{B} \sum_{m_1,m_2,m_3=1}^{M} \sum_{n_1,n_2,n_3\in\mathbb{Z}} \\
& e^{2i\pi(m_1+m_2-m_3-m)\Delta F n T_s} \\
& \times \sqrt{G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)}} \\
& \times a_{b_1,m_1,n-n_1} a_{b_2,m_2,n-n_2} a_{b_3,m_3,n-n_3}^{*} \\
& \times h_3(n_1T_s, n_2T_s, n_3T_s, m_1+m_2-m_3-m).
\end{aligned} \tag{11}$$

As currently done in most receivers, we consider the case of *nonlinearity-agnostic receiver*, i.e., a receiver which sees the nonlinear effects just as an additional noise. As a consequence, the data rate of user belonging to beam b using subband m becomes

$$R_{b,m} = \log_2\left(1 + Q_{b,m}\right) \tag{12}$$

with the following SINR,

$$Q_{b,m} = \frac{\mathcal{P}_{b,m}^{(\mathrm{L})}}{\mathcal{P}_{b,m}^{(\mathrm{I})} + \mathcal{P}_{b,m}^{(\mathrm{NL})} + \mathcal{P}_{\mathrm{W}}}, \tag{13}$$

where $\mathcal{P}_{b,m}^{(\mathrm{L})} := \mathbb{E}[|z_{b,m,n}^{(\mathrm{L})}|^2]$, $\mathcal{P}_{b,m}^{(\mathrm{I})} := \mathbb{E}[|z_{b,m,n}^{(\mathrm{I})}|^2]$, $\mathcal{P}_{b,m}^{(\mathrm{NL})} := \mathbb{E}[|z_{b,m,n}^{(\mathrm{NL})}|^2]$ are the power of the useful signal, the linear interference, and the nonlinear interference, respectively. The power of the AWGN is $\mathcal{P}_{\mathrm{W}} := \mathbb{E}[|w_{b,m,n}|^2]$.

II. Closed-form expression of the involved terms in data rate

The purpose of this Section is to express in closed-form the terms involved in (12), and emphasize their properties. To this aim, let us consider the two definitions below [3].

Definition 1. *A monomial function takes the following form:*

$$m(\mathbf{P}) = c \prod_{b} \prod_{m} P_{b,m}^{\alpha_{b,m}}$$

with $c \in \mathbb{R}^+$ and $\alpha_{b,m} \in \mathbb{R}$.

Definition 2. *A posynomial function takes the following form:*

$$p(\mathbf{P}) = \sum_{n=1}^{N} m_n(\mathbf{P})$$

where $\{m_n\}_{n=1,\cdots,N}$ are monomial functions.

A. Expression for the power of the useful signal

According to (9), the power of the useful signal is

$$\mathcal{P}_{b,m}^{(\mathrm{L})} = \omega_1^2 G_m^{(b)} P_{b,m} \tag{14}$$

where $P_{b,m} = \mathbb{E}[|a_{b,m,n}|^2]$ is the transmit power of user belonging to beam b using subband m.

Result 1. *The term $\mathcal{P}_{b,m}^{(\mathrm{L})}$ is a monomial function in $\mathbf{P}$.*

B. Expression for the power of the linear interference

According to (10), the power of the linear interference is

$$\mathcal{P}^{(\mathrm{I})}_{b,m} = \omega_1^2 \sum_{\substack{b'=1\\ b'\neq b}}^{B} G_m^{(b',b)} P_{b',m} \tag{15}$$

Result 2. *The term $\mathcal{P}^{(\mathrm{I})}_{b,m}$ is a posynomial function in $\mathbf{P}$.*

C. Expression for the power of the nonlinear interference

Since (11), the power of the nonlinear interference is

$$\begin{aligned}\mathcal{P}^{(\mathrm{NL})}_{b,m} = \omega_3^2 &\sum_{b_1,b_2,b_3=1}^{B} \sum_{b'_1,b'_2,b'_3=1}^{B} \sum_{m_1,m_2,m_3=1}^{M} \sum_{m'_1,m'_2,m'_3=1}^{M} \\ &\sum_{n_1,n_2,n_3\in\mathbb{Z}} \sum_{n'_1,n'_2,n'_3\in\mathbb{Z}} e^{2i\pi(m_1+m_2-m_3-m)\Delta F n T_s} \\ &\times e^{-2i\pi(m'_1+m'_2-m'_3-m)\Delta F n T_s} \\ &\times \sqrt{G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)}} \sqrt{G_{m'_1}^{(b'_1,b)} G_{m'_2}^{(b'_2,b)} G_{m'_3}^{(b'_3,b)}} \\ &\times \mathbb{E}[a_{b_1,m_1,n-n_1} a_{b_2,m_2,n-n_2} \overline{a_{b_3,m_3,n-n_3}} \\ &\qquad \overline{a_{b'_1,m'_1,n-n'_1} a_{b'_2,m'_2,n-n'_2}} a_{b'_3,m'_3,n-n'_3}] \\ &\times h_3(n_1T_s, n_2T_s, n_3T_s, m_1+m_2-m_3-m) \\ &\times \overline{h_3(n'_1T_s, n'_2T_s, n'_3T_s, m'_1+m'_2-m'_3-m)}. \end{aligned} \tag{16}$$

This term is already involved in the derivations done in [4] for single beam satellite. We easily extend this work in case of multibeam satellite.

Finally, we have

$$\begin{aligned}\mathcal{P}^{(\mathrm{NL})}_{b,m} = {}& 4\omega_3^2\theta_0^{(1)} \sum_{b_1,b_2,b_3=1}^{B} \sum_{m',m''=1}^{M} G_m^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} \\ &\times P_{b_1,m} P_{b_2,m'} P_{b_3,m''} \\ &+ 4\tilde{\delta}_{m,M}\omega_3^2\theta_1^{(1)} \sum_{b_1,b_2,b_3=1}^{B} \sum_{m',m''=1}^{M} G_{m+1}^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} \\ &\times P_{b_1,m+1} P_{b_2,m'} P_{b_3,m''} \\ &+ 4\tilde{\delta}_{m,1}\omega_3^2\theta_1^{(1)} \sum_{b_1,b_2,b_3=1}^{B} \sum_{m',m''=1}^{M} G_{m-1}^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} \\ &\times P_{b_1,m-1} P_{b_2,m'} P_{b_3,m''} \\ &+ 2\omega_3^2\theta_0^{(2)} \sum_{b_1,b_2,b_3=1}^{B} \sum_{\substack{m_1,m_2,m_3=1\\ m=m_1+m_2-m_3}}^{M} G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)} \\ &\times P_{b_1,m_1} P_{b_2,m_2} P_{b_3,m_3} \\ &+ 2\omega_3^2\theta_1^{(2)} \sum_{b_1,b_2,b_3=1}^{B} \sum_{\substack{m_1,m_2,m_3=1\\ m=m_1+m_2-m_3\pm1}}^{M} G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)} \\ &\times P_{b_1,m_1} P_{b_2,m_2} P_{b_3,m_3}. \end{aligned} \tag{17}$$

where $\tilde{\delta}_{m,m'} = 1 - \delta_{m,m'}$ with $\delta_{m,m'}$ being the Kronecker index. The coefficients $\theta_\ell^{(1)}$ and $\theta_\ell^{(2)}$ are positive and given by

$$\begin{aligned}\theta_\ell^{(1)} &= \sum_{n'\in\mathbb{Z}} \left| \sum_{n''\in\mathbb{Z}} h_3(n'T_s, n''T_s, n''T_s, \ell) \right|^2. \\ \theta_\ell^{(2)} &= \sum_{n_1,n_2,n_3\in\mathbb{Z}} |h_3(n_1T_s, n_2T_s, n_3T_s, \ell)|^2. \end{aligned}$$

Result 3. *The term $\mathcal{P}^{(\mathrm{NL})}_{b,m}$ is a posynomial function in $\mathbf{P}$.*

III. Power allocation strategies

So far, we have an expression of the data rate in closed-form as well as the mathematical properties of the involved terms. We now are interested in three optimization problems. For all the studied problems, the users undergo a maximum transmission power constraint, which is expressed by the following relationship,

$$0 \leq P_{b,m} \leq P_{\max} \quad \forall b, m. \tag{18}$$

Before starting to study the problems, we can notice that the term (13) is a ratio of a monomial function over a posynomial function. As a result, the expression of the data rate (12) is the logarithm of ratio of posynomial functions.

Optimization problems containing posynomial ratios boil down to *Signomial Programming* (SP) [5], which are non-convex and can be solved by using alternatively Successive Convex Approach (SCA) and Geometric Programming (GP). Consequently, we are able to obtain practical algorithms for solving SP optimization problems.

A. Minimization of the sum-power

In this Section, the problem of minimization of the sum-power is formulated.

Problem 1.

$$\begin{aligned}\mathbf{P}^\star = \arg\min_{\mathbf{P}} &\sum_{b=1}^{B}\sum_{m=1}^{M} P_{b,m} \\ \text{s.t. } &(18), \\ &\log_2(1+Q_{b,m}) \geq R^t_{b,m} \quad \forall b, m \end{aligned} \tag{19}$$

The formulation of Problem 1 is neither concave nor of the GP form. It is therefore impossible to use tools to solve it analytically or numerically with acceptable complexity, i.e., in polynomial time. The idea is to rewrite this problem in a standard GP form in order to be able to apply the standard tools of GP optimization.

The difficulty is located in the constraint (19), where we have a ratio of posynomial functions. However, this equation can be rewritten as:

$$\left(2^{R^t_{b,m}} - 1\right) {\mathcal{P}^{(\mathrm{L})}_{b,m}}^{-1} \left(\mathcal{P}^{(\mathrm{I})}_{b,m} + \mathcal{P}^{(\mathrm{NL})}_{b,m} + \mathcal{P}_{\mathrm{W}}\right) \leq 1 \tag{20}$$

Lemma 1. *The constraint* (20) *has the form of a posynomial less than or equal to one, leading to a valid constraint for GP.*

Proof. The term $2^{R^t_{b,m}} - 1$ is a positive scalar, ${\mathcal{P}^{(\mathrm{L})}_{b,m}}^{-1}$ is a monomial function. Thanks to the addition rule of posynomial, the rest of the Left Hand Side (LHS) is a posynomial function. Finally, thanks to multiplication rule between a monomial and a posynomial function, the LHS is a posynomial function. □

The resulting optimization problem writes as:

Problem 2.

$$\mathbf{P}^{\star} = \arg\min_{\mathbf{P}} \sum_{b=1}^{B}\sum_{m=1}^{M} P_{b,m}$$
$$\text{s.t. (18) and (20).}$$

Result 4. *Problem 2 is GP since it minimizes of posynomial over a set in GP form and can be efficiently solved by numerical algorithms [3].*

B. Maximization of the minimum per-user rate

In this Section, we formulate the problem of maximization of the minimum per-user rate.

Problem 3.

$$\mathbf{P}^{\star} = \arg\max_{\mathbf{P}} \min_{b,m} \log_2\left(1+Q_{b,m}\right) \text{ s.t.} \qquad (18).$$

Thanks to the monotonic growth of the logarithm function, Problem 3 is equivalent to the following one.

$$\mathbf{P}^{\star} = \arg\max_{\mathbf{P}} \min_{b,m} Q_{b,m}$$
$$\text{s.t. (18).}$$

The difficulty is located in the objective function, which is not concave nor a posynomial function because of the minimum operator and the ratio of posynomial functions. First, we introduce a new variable $t \in \mathbb{R}^{+*}$ in order to remove the minimum operator. The problem becomes

$$\{\mathbf{P}^{\star}, t^{\star}\} = \arg\max_{\mathbf{P},t} t$$
$$\text{s.t. (18),}$$
$$Q_{b,m} \geq t \quad \forall b,m \tag{21}$$

Remark 1. *We notice that* $\max t$ *is equivalent to* $\min t^{-1}$

Moreover, the constraint (21) can be rewritten as:

$$\mathcal{P}_{b,m}^{(\mathrm{L})\,-1} t\left(\mathcal{P}_{b,m}^{(\mathrm{I})} + \mathcal{P}_{b,m}^{(\mathrm{NL})} + \mathcal{P}_{\mathrm{W}}\right) \leq 1 \quad \forall b,m \tag{22}$$

Lemma 2. *The constraint* (22) *has the form of a posynomial less than or equal to one, leading to a valid constraint for GP.*

Proof. The term $\mathcal{P}_{b,m}^{(\mathrm{L})\,-1}$ is a monomial function. By recalling that posynomial are closed to addition and multiplication, the LHS is a posynomial function. □

The resulting optimization problem writes as:

Problem 4.

$$\{\mathbf{P}^{\star}, t^{\star}\} = \arg\min_{\mathbf{P},t} t^{-1}$$
$$\text{s.t. (18) and (22).}$$

Result 5. *Problem 4 is GP since it minimizes of monomial (which is a posynomial term) over a set in GP form and can be efficiently solved by numerical algorithms [3].*

C. Maximization of the sum-rate

In this Section, the problem of maximization of the sum-rate is formulated.

Problem 5.

$$\mathbf{P}^{\star} = \arg\max_{\mathbf{P}} \sum_{b=1}^{B}\sum_{m=1}^{M} \log_2\left(1+Q_{b,m}\right)$$
$$\text{s.t. (18).}$$

Thanks to the monotonic growth of the logarithm function, Problem 5 is equivalent to the following one.

$$\mathbf{P}^{\star} = \arg\min_{\mathbf{P}} \prod_{b=1}^{B}\prod_{m=1}^{M} \frac{\mathcal{P}_{b,m}^{(\mathrm{I})} + \mathcal{P}_{b,m}^{(\mathrm{NL})} + \mathcal{P}_{\mathrm{W}}}{\mathcal{P}_{b,m}^{(\mathrm{L})} + \mathcal{P}_{b,m}^{(\mathrm{I})} + \mathcal{P}_{b,m}^{(\mathrm{NL})} + \mathcal{P}_{\mathrm{W}}}$$
$$\text{s.t. (18).}$$

The difficulty is located in the objective function, where we have a ratio of posynomial functions, so the problem boils down to SP. Since the objective function of the problem is already a ratio of posynomials, we can directly use the successive monomial approximation of the denominator [5], [6]. In [6], the authors cover the case of linear interference and apply the trick of monomial approximation since its denominator is also a posynomial function. Here, our denominator includes the nonlinear interference terms and remains a posynomial function. We denote it by

$$D_{b,m} = \mathcal{P}_{b,m}^{(\mathrm{L})} + \mathcal{P}_{b,m}^{(\mathrm{I})} + \mathcal{P}_{b,m}^{(\mathrm{NL})} + \mathcal{P}_{\mathrm{W}} \tag{23}$$

The monomial approximation at the point $\mathbf{P}(i)$ is denoted by $\widetilde{D}_{b,m}^{(i)}(\mathbf{P})$. This approximation (25) is built in such way that it satisfies the SCA condition.

The coefficients introduced in (25) are given by

$$\lambda_{b,m}^{(\mathrm{L})} = \frac{\omega_1^2 G_m^{(b)} P_{b,m}^{(i)}}{D_{b,m}(\mathbf{P}(i))}, \lambda_{b,m}^{(\mathrm{I})}(b') = \frac{\omega_1^2 G_m^{(b',b)} P_{b',m}^{(i)}}{D_{b,m}(\mathbf{P}(i))}$$

$$\lambda_{b,m}^{(\mathrm{NL},1)}(b_1,b_2,b_3,m',m'') = \frac{4\omega_3^2\theta_0^{(1)} G_m^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} P_{b_1,m}^{(i)} P_{b_2,m'}^{(i)} P_{b_3,m''}^{(i)}}{D_{b,m}(\mathbf{P}(i))}$$

$$\lambda_{b,m}^{(\mathrm{NL},2)}(b_1,b_2,b_3,m',m'') = \frac{4\tilde{\delta}_{m,M}\omega_3^2\theta_1^{(1)} G_{m+1}^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} P_{b_1,m+1}^{(i)} P_{b_2,m'}^{(i)} P_{b_3,m''}^{(i)}}{D_{b,m}(\mathbf{P}(i))}$$

$$\lambda_{b,m}^{(\mathrm{NL},3)}(b_1,b_2,b_3,m',m'') = \frac{4\tilde{\delta}_{m,1}\omega_3^2\theta_1^{(1)} G_{m-1}^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} P_{b_1,m-1}^{(i)} P_{b_2,m'}^{(i)} P_{b_3,m''}^{(i)}}{D_{b,m}(\mathbf{P}(i))}$$

$$\lambda_{b,m}^{(\mathrm{NL},4)}(b_1,b_2,b_3,m_1,m_2,m_3) = \frac{2\omega_3^2\theta_0^{(2)} G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)} P_{b_1,m_1}^{(i)} P_{b_2,m_2}^{(i)} P_{b_3,m_3}^{(i)}}{D_{b,m}(\mathbf{P}(i))}$$

$$
\begin{aligned}
\widetilde{D}_{b,m}^{(i)}(\mathbf{P}) = {} & \left(\frac{\omega_1^2 G_m^{(b)} P_{b,m}}{\lambda_{b,m}^{(\mathrm{L})}}\right)^{\lambda_{b,m}^{(\mathrm{L})}} \left(\frac{\mathcal{P}_{\mathrm{W}}}{\lambda_{b,m}^{(\mathrm{W})}}\right)^{\lambda_{b,m}^{(\mathrm{W})}} \prod_{\substack{b'=1\\ b'\neq b}}^{B} \left(\frac{\omega_1^2 G_m^{(b',b)} P_{b',m}}{\lambda_{b,m}^{(\mathrm{I})}(b')}\right)^{\lambda_{b,m}^{(\mathrm{I})}(b')} \\
& \times \prod_{b_1,b_2,b_3}^{B} \prod_{m',m''=1}^{M} \left(\frac{4\omega_3^2 \theta_0^{(1)} G_m^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} P_{b_1,m} P_{b_2,m'} P_{b_3,m''}}{\lambda_{b,m}^{(\mathrm{NL},1)}(b_1,b_2,b_3,m',m'')}\right)^{\lambda_{b,m}^{(\mathrm{NL},1)}(b_1,b_2,b_3,m',m'')} \\
& \times \prod_{b_1,b_2,b_3=1}^{B} \prod_{m',m''=1}^{M} \left(\frac{4\tilde{\delta}_{m,M}\omega_3^2 \theta_1^{(1)} G_{m+1}^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} P_{b_1,m+1} P_{b_2,m'} P_{b_3,m''}}{\lambda_{b,m}^{(\mathrm{NL},2)}(b_1,b_2,b_3,m',m'')}\right)^{\lambda_{b,m}^{(\mathrm{NL},2)}(b_1,b_2,b_3,m',m'')} \\
& \times \prod_{b_1,b_2,b_3=1}^{B} \prod_{m',m''=1}^{M} \left(\frac{4\tilde{\delta}_{m,1}\omega_3^2 \theta_1^{(1)} G_{m-1}^{(b_1,b)} G_{m'}^{(b_2,b)} G_{m''}^{(b_3,b)} P_{b_1,m-1} P_{b_2,m'} P_{b_3,m''}}{\lambda_{b,m}^{(\mathrm{NL},3)}(b_1,b_2,b_3,m',m'')}\right)^{\lambda_{b,m}^{(\mathrm{NL},3)}(b_1,b_2,b_3,m',m'')} \\
& \times \prod_{b_1,b_2,b_3}^{B} \prod_{\substack{m_1,m_2,m_3=1\\ m=m_1+m_2-m_3}}^{M} \left(\frac{2\omega_3^2 \theta_0^{(2)} G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)} P_{b_1,m_1} P_{b_2,m_2} P_{b_3,m_3}}{\lambda_{b,m}^{(\mathrm{NL},4)}(b_1,b_2,b_3,m_1,m_2,m_3)}\right)^{\lambda_{b,m}^{(\mathrm{NL},4)}(b_1,b_2,b_3,m_1,m_2,m_3)} \\
& \times \prod_{b_1,b_2,b_3=1}^{B} \prod_{\substack{m_1,m_2,m_3=1\\ m=m_1+m_2-m_3\pm 1}}^{M} \left(\frac{2\omega_3^2 \theta_1^{(2)} G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)} P_{b_1,m_1} P_{b_2,m_2} P_{b_3,m_3}}{\lambda_{b,m}^{(\mathrm{NL},5)}(b_1,b_2,b_3,m_1,m_2,m_3)}\right)^{\lambda_{b,m}^{(\mathrm{NL},5)}(b_1,b_2,b_3,m_1,m_2,m_3)} \qquad (25)
\end{aligned}
$$

$$
\lambda_{b,m}^{(\mathrm{NL},5)}(b_1,b_2,b_3,m_1,m_2,m_3) = \frac{2\omega_3^2 \theta_1^{(2)} G_{m_1}^{(b_1,b)} G_{m_2}^{(b_2,b)} G_{m_3}^{(b_3,b)} P_{b_1,m_1}^{(i)} P_{b_2,m_2}^{(i)} P_{b_3,m_3}^{(i)}}{D_{b,m}(\mathbf{P}(i))}
$$

$$
\lambda_{b,m}^{(\mathrm{W})} = \frac{\mathcal{P}_{\mathrm{W}}}{D_{b,m}(\mathbf{P}(i))}
$$

The resulting approximated optimization problem is

Problem 6.

$$
\mathbf{P}^{\star}(i) = \arg\min_{\mathbf{P}} \prod_{b=1}^{B} \prod_{m=1}^{M} \left(\widetilde{D}_{b,m}^{(i-1)}\right)^{-1} \left(\mathcal{P}_{b,m}^{(\mathrm{I})} + \mathcal{P}_{b,m}^{(\mathrm{NL})} + \mathcal{P}_{\mathrm{W}}\right)
$$

s.t. (18).

Result 6. *Problem 6 is GP since it minimizes a posynomial function over a set in GP form and can be efficiently solved by numerical algorithms [3].*

Since the monomial approximation satisfies the SCA condition, the SCA procedure depicted in Algorithm 1 converges towards a stationary point. The obtained solution is sub-optimal.

Algorithm 1 SCA based procedure for solving Problem 5

Set $\epsilon > 0$, $E = \epsilon + 1$, $i = 0$
Find $\mathbf{P}(0)$ a feasible solution of Problem 5
Compute the sum-rate R_0 using (12)
while $E > \epsilon$ **do**
  $i = i + 1$
  Compute the monomial approximation $\widetilde{D}_{b,m}^{(i-1)}$ around the point $\mathbf{P}(i-1)$, using (25)
  Find $\mathbf{P}^{\star}(i)$ the optimal solution of Problem 6
  Compute the sum-rate R_i and $E = |R_i - R_{i-1}|$
end while
return $\mathbf{P}^{\star} = \mathbf{P}^{\star}(i)$

IV. NUMERICAL RESULTS

We consider the uplink of a multibeam multiband satellite communication system, where each beam utilizes the 27.5 - 29.5GHz band. We assume that the assignement of users in the beams and the subbands is done [7], [8]. We set $B = 2$ beams and $M = 6$ subbands (so we have 6 users per beam). The users use the same shaping filter, which is a SRRC with a roll-off factor of 0.25. The maximum transmit power of terrestrial user is $P_{\max} = 50$W. The channel gains are computed according to [8]. Notice that the channel gain values within a same beam $\{G_m^{(b)}\}_m$ are close to each others. The HPA distorsion coefficient ω_1 and ω_3 are 1 and 0.05 respectively. In addition, we add a variable gain pre-amplifier just before the HPA. This device allows to set the HPA regime and changes the channel gains uniformly for incoming signal of the same antenna. For simplicity, we assume that the gains of the pre-amplifiers are identical for all HPAs. The CVX toolbox is used to solve GP problem [9].

For each figure, we display three power allocations related to the considered problem:

- *naive1* heuristic allocation, where the users transmit at the same power, i.e. $P_{b,m} = P$, and line-search is performed to find the optimal allocation for the considered problem.
- *naive2* heuristic allocation, where the received power at the satellite level is the same, i.e. $G_m^{(b)} P_{b,m} = P_R$, and line-search is performed to find the optimal allocation for

the considered problem.

- the solution proposed by this paper, denoted $P^{\star}$.

In addition, we display the solution obtained when the considered problem does not take into account the nonlinear interference, i.e. when $\mathcal{P}_{b,m}^{(\mathrm{NL})} = 0$ in (13). This solution is denoted by P^{li} and the data rate is then evaluated with (12). In dotted line, we draw the data rate evaluated without the nonlinear interference.

In Fig. 1, we plot the sum-rate versus the pre-amplifier gain obtained for the four above mentioned resource allocations. We remark that taking into account the nonlinear interference in the optimization problem is relevant since it increases the sum-rate, and enables us to keep the same performance whatever the nonlinear regime.

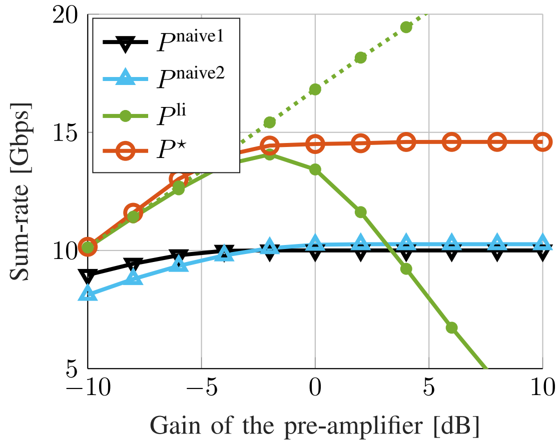

Fig. 1. Sum rate vs. pre-amplifier gain G_{amp}.

In Fig. 2, we plot the minimum per-user data rate versus the pre-amplifier gain for the four above mentioned resource allocations. We observe that the proposed solution is better when the HPA operates in nonlinear regime. It can be interesting to use this regime when the channel gains are very different.

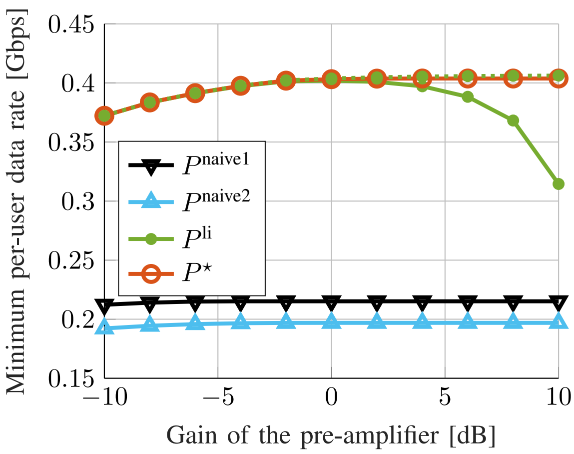

Fig. 2. Minimum per-user data rate vs. pre-amplifier gain G_{amp}.

In Fig. 3, we plot the sum-power versus the target data rate for the four above mentioned resource allocations. We assume that the target rate is the same for the users and we focus on two values of pre-amplifier gain. Once again, the proposed solution is interesting for the nonlinear regime of the HPA.

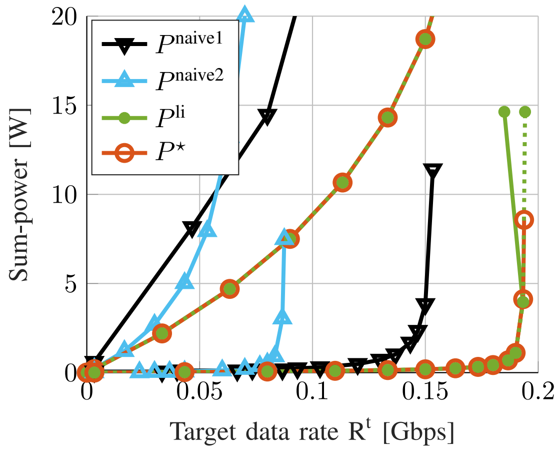

Fig. 3. Sum-power vs. target data rate $R_{b,m}^{t} = R^{t}$ for $G_{\mathrm{amp}} =\in \{-10, 10\}$.

V. Conclusion

The proposed allocation based on Signomial Programming (more precisely, Complementary Geometric Programming) outperforms the naive policies as well as the resource allocation obtained by taking into account only the inter-beam linear interference. For future work, we will focus on the case where the receiver exploits the nonlinear effect which leads to new data rate closed-form expressions and new resource allocations optimization problems.

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