About the outage probability optimization in MISO Rician channels

E. Bouton[(1)], N. Ksairi[(2)], P. Ciblat[(1)], P. Bianchi[(1)], W. Hachem[(1)]

[(1)] TELECOM ParisTech – CNRS/LTCI; 46, rue Barrault, 75013 Paris, France
Email: {bouton,ciblat,bianchi,hachem}@telecom-paristech.fr
[(2)] Supélec; Plateau du Moulon, 3 rue Joliot-Curie, 91192 Gif-sur-Yvette, France
Email: nassar.ksairi@supelec.fr

***Abstract*— We address the optimization issue of the outage probability in Rician block fading channels with N transmit antennas and 1 receive antenna. It is already known that the eigenvectors of the covariance matrix of the transmit signal that minimizes the outage probability are the normalized mean vector of the Rician channel and $N-1$ orthonormal vectors belonging to the space orthogonal to the range of the mean vector. Our contribution is to optimize the eigenvalues. At high SNR, we show that the part of the power allocated to the orthogonal space of the range of the mean vector should be uniformly distributed amongst its $N-1$ eigenvectors. In addition, thanks to an original approximation of the outage probability, we characterize the part of power associated with the mean vector. We numerically observe improvement achieved by the proposed allocation scheme compared to uniform power allocation.**

I. Introduction

Within the context of slow flat fading multiple antenna channels, the outage probability represents a fundamental limit on the system's performance. Its optimization is of particular interest both at the theoretical and practical levels. When partial channel state information is known at the transmitter, whether from a true Rician behaviour of the propagation channel or from partial feedback coming from the receiver, only a few results about the outage probability optimization with respect to the correlation matrix of the transmit signal can be found in the literature.

The correlation matrix optimization problem has been tackled intensively for fast flat fading and only partially for slow flat fading channels. In the case of fast flat fading, we remind that the relevant information-theoretic criterion is the ergodic capacity. All the following works focused on ergodic capcacity optimization. In [1], it has been conjectured that, for uncorrelated Rayleigh channels, the optimum power allocation would be to uniformly allocate the available power. Correlated Rayleigh channels were studied in [2], [3], where it is shown that the eigenvectors of the transmit covariance matrix match those of the channel's transmission correlation matrix. For uncorrelated Rician fading channels, [4] shows that the eigenvectors of the transmit signal covariance are the right singular vectors of the channel's mean matrix. The correlated case was studied in [5], where the power allocation can be obtained by a numerical stochastic method. [6] also focused on this problem, by giving an approximation of the optimal power allocation scheme through a simple iterative deterministic algorithm, based on the theory of large random matrices.

For the non-ergodic case, an analytical expression of the outage probability is only available for the MISO and SIMO cases. The MIMO case has been studied in [7], [8] for the correlated Rayleigh and white Rice cases. Therein, the first two moments of the mutual information are calculated and a Gaussian approximation is used to determine the outage probability. However, optimization with respect to the transmission covariance matrix seems very complex and only a few papers dealt with it [1], [9], [10], [11]. As the previous results will be needed throughout the paper, further explanations will be directly provided in the core of the paper.

Our work focuses on a Rician MISO block fading channel. Our main contribution is to find the structure and the value of the eigenvalues of covariance matrix optimizing the outage probability.

The paper is organized as follows: in Section II, we introduce the MISO based system as well as the problem statement. In Section III, we derive an achievable lower bound for the outage probability. Through an original approximation of the lower bound, we provide useful insight on the optimized power allocation with respect to the channel deterministic parameters. Section IV is devoted to numerical illustrations. Concluding remarks are drawn in Section V.

II. Channel model & problem statement

We consider here a system with N transmit antennas and 1 receive antenna. Therefore, the discrete-time receive scalar signal, denoted by y, can written in the following manner

$$y = \sqrt{P}\mathbf{h}^{\mathrm{H}}\mathbf{x} + b \tag{1}$$

where $\mathbf{x}$ is a $N \times 1$ unit-power and zero-mean transmit signal, b is a complex circular Gaussian noise with zero mean and variance $\sigma^2/2$ per real dimension and $\mathbf{h}$ is the $N \times 1$ vector representing the instantaneous impulse response. Finally P is the total transmit power. The superscript $(.)^{\mathrm{H}}$ stands for the complex-transposition operator.

As the channel is assumed to satisfy a Rician block flat fading model, $\mathbf{h}$ can be decomposed as follows

$$\mathbf{h} = \sqrt{\frac{K}{K+1}}\mathbf{h}_d + \sqrt{\frac{1}{K+1}}\mathbf{h}_r \quad (2)$$

where $\mathbf{h}_d$ is a deterministic unit-norm vector (*i.e.*, $\|\mathbf{h}_d\|^2 = 1$), and $\mathbf{h}_r$ is a complex circular Gaussian random vector with zero mean. The covariance matrix is chosen such that $\mathbb{E}[\mathbf{h}_r\mathbf{h}_r^{\mathrm{H}}] = (1/N)\mathbf{Id}_N$ which means that the components of $\mathbf{h}_r$ are i.i.d.. Coefficient K is the so-called Rician factor. In this paper, we assume that K, $\mathbf{h}_d$, and covariance matrix are known at the transmitter.

This channel model can be justified in the following two ways. Firstly, in most wireless applications, it is usual to consider that the channel can be decomposed into two parts: the first part refers to the Line of Sight (LOS), which corresponds to the first term of the righthand side of (2), and the second part is associated with the non-line of sight (NLOS) components of the channel, which corresponds to the second term of the righthand side of (2). Notice that, in the context of urban area wireless communications, the first part may be equal to zero which corresponds to $K = 0$. Secondly, even when the Rice model does not hold true, it is often possible to decompose $\mathbf{h}$ as depicted in Eq. (2). For instance, consider the case where the receiver estimates the channel impulse response and feedbacks these estimates to the transmitter. The latter therefore has a partial knowledge of the channel impulse response, which can be described as in Eq. (2) where the random part represents the uncertainties due to estimation/feedback errors and time-variations of the channel. Notice that the estimation errors are often well approximated by a Gaussian distribution which justifies the Rice model.

The outage probability of the link defined by Eq. (1) for a given transmission rate R (in bits per channel use) can be expressed as [12]

$$\mathbb{P}_{out}(\mathbf{Q}) = \mathbb{P}\{\log_2(1+\rho\mathbf{h}^H\mathbf{Q}\mathbf{h}) < R\} \quad (3)$$

where $\rho = P/\sigma^2$ is the Signal-to-Noise Ratio (SNR) and $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ is the covariance matrix of the transmitted signal $\mathbf{x}$.

In this paper, we focus on the minimization of the outage probability given in Eq. (3) with respect to $\mathbf{Q}$. Notice that $\mathbf{Q}$ is subject to two constraints: i) $\mathbf{Q}$ is a semi-definite Hermitian $N \times N$ matrix and ii) $\mathrm{Tr}(\mathbf{Q}) = 1$.

III. Optimization of $\mathbb{P}_{out}$

In order to optimize $\mathbb{P}_{out}(\mathbf{Q})$ versus $\mathbf{Q}$, we first perform the eigenvalue decomposition of $\mathbf{Q}$. Thus we have

$$\mathbf{Q} = \mathbf{U}\mathrm{diag}(\lambda_1 \ldots \lambda_N)\mathbf{U}^{\mathrm{H}}$$

where $\{\lambda_1, \cdots, \lambda_N\}$ are the N real-valued eigenvalues and $\mathbf{U}$ is the unitary matrix containing the associated eigenvectors. Our objective now is to find the vector $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]^{\mathrm{T}}$ and $\mathbf{U}$ optimizing the outage probability which actually can be written as

$$\mathbb{P}_{out}(\boldsymbol{\lambda}, \mathbf{U}) = \mathbb{P}\left\{\sum_{i=1}^{N} \lambda_i |\nu_i|^2 < \mu\right\}$$

with $\boldsymbol{\nu} = \mathbf{U}^{\mathrm{H}}\mathbf{h} = [\nu_1, \cdots, \nu_N]^{\mathrm{T}}$ and $\mu = (2^R - 1)/\rho$. The superscript $(.)^{\mathrm{T}}$ stands for the transposition operator. Due to the constraint on the trace of $\mathbf{Q}$, the eigenvalues have to satisfy the following constraint

$$\sum_{i=1}^{N} \lambda_i = 1.$$

The random vector $\boldsymbol{\nu}$ is a complex circular Gaussian vector with mean $\sqrt{K/(K+1)}\mathbf{U}^{\mathrm{H}}\mathbf{h}_d$ and covariance matrix $1/(N(K+1))\mathbf{Id}_N$.

Notice that finding the matrix $\mathbf{U}$ optimizing the outage probability has already been solved in [11]. The optimal matrix $\mathbf{U}$ contains N orthonormal column vectors. As $||\mathbf{h}_d|| = 1$, the N^{th} vector is equal to $\mathbf{h}_d$ and is thus associated with the Rice part of the channel. The first $(N-1)$ vectors are selected in order to form an orthonormal basis of the space orthogonal to the space spanned by $\mathbf{h}_d$. This $(N-1)$-dimensional space is thus associated with the pure Rayleigh part of the channel.

Our objective now boils down to finding the eigenvalues $\{\lambda_1, \cdots, \lambda_N\}$ minimizing the outage probability. Assuming the optimal choice of matrix $\mathbf{U}$, the outage probability can be rewritten as follows

$$\mathbb{P}_{out}(\boldsymbol{\lambda}) = \mathbb{P}\left\{\sum_{i=1}^{N-1} \lambda_i |X_i|^2 + \lambda_N |Y|^2 < \eta\right\} \quad (4)$$

where $\eta = 2N(K+1)\mu$, X_i are i.i.d. complex circular Gaussian variables with zero-mean and unit-variance per real dimension, and Y is an independent complex circular Gaussian scalar with mean $m = \sqrt{2NK}$ and unit-variance per real dimension. Notice that λ_N corresponds to the power allocated to the Rice part of the channel whereas the other λ_i (for $i = 1, \cdots, N-1$) represent the powers allocated to the Rayleigh part of the channel.

Before going further, it is very important to remark that each $|X_i|^2$ is an independent central chi-square random variable with two degrees of freedom. Thus one can write $|X_i|^2 = X_{i,1}^2 + X_{i,2}^2$ where $X_{i,1}$ and $X_{i,2}$ are independent **real-valued** Gaussian variables with zero mean and unit-variance. In addition, $|Y|^2$ is a non-central chi-square random variable with two degrees of freedom and non-centrality parameter equal to $2NK$. In the sequel, $\chi_d^2(s)$ denotes a chi-square distribution with d degrees of freedom and non-centrality parameter s. When $s = 0$, $\chi_d^2(0)$ simplifies to χ_d^2.

When the Rice part of the channel vanishes (i.e., $K = 0$), the optimal choice of the eigenvalues $\boldsymbol{\lambda}$ has been recently solved in [9]. Hereafter, we recall their main results. Let Z_i be a **real-valued** Gaussian variable with zero-mean and unit-variance. Let $\{\lambda_i\}_{i=1,\cdots,n}$ be a set of n positive terms such that $\sum_{i=1}^{n} \lambda_i = 1$. Theorem 1 in [9] proves that

$$I_n(t) = \inf_{\substack{\{\lambda_1, \cdots, \lambda_n\} \\ \sum_{i=1}^{n} \lambda_i = 1}} \mathbb{P}\left\{\sum_{i=1}^{n} \lambda_i Z_i^2 \leq t\right\}$$

is equal to

$$I_n(t) = \begin{cases} \mathbb{P}(d^{-1}\chi_d^2 \le t), & \forall t \in [t_d, t_{d-1}), \\ & d = 1, 2, \ldots, n-1, \\ \mathbb{P}(n^{-1}\chi_n^2 \le t), & \forall t \in [0, t_{n-1}), \end{cases} \quad (5)$$

where t_d is the unique intersection point of the curves of the cumulative distribution functions $\mathbb{P}(d^{-1}\chi_d^2 \le t)$ and $\mathbb{P}((d+1)^{-1}\chi_{d+1}^2 \le t)$. By convention, $t_0 \triangleq \infty$.

Eq. (5) indicates that when t is small, each term λ_i is non-zero and even identical, that is to say, all the directions have to be treated in the same way. In contrast, as soon as t becomes larger than a certain threshold, some directions have to be switched off. When inspecting Eq. (4) in a pure Rayleigh context (i.e., without the term Y), we remark that the result given in [9] can be directly applied. Actually, the authors in [9] prove the so-called Telatar conjecture [1] but were not aware of it. Notice also that, when η is small enough or equivalently when SNR is large enough, the uniform power distribution is optimal in a pure Rayleigh context.

Let us come back to our original problem dealing with the eigenvalues optimization in a Rice context. In order to partially reuse the result given in [9], we will rewrite our problem in the following manner. We rearrange the Left Hand Side (LHS) of inequality in Eq. (4) in order to keep only pure Rayleigh terms in LHS. After some other straightforward algebraic manipulations, we get

$$\mathbb{P}_{out}(\boldsymbol{\lambda}) = \mathbb{P}\left\{\sum_{i=1}^{2(N-1)} \ell_i Z_i^2 < \frac{\eta - \lambda_N |Y|^2}{1-\lambda_N}\right\} \quad (6)$$

where $\ell_{2j} = \ell_{2j-1} = \frac{\lambda_j}{2(1-\lambda_N)}$ for $j = 1, \cdots, N-1$ and we have $\sum_{i=1}^{2(N-1)} \ell_i = 1$. Moreover Z_i, defined as $Z_{2j} = X_{j,2}$ and $Z_{2j-1} = X_{j,1}$ for $j = 1, \cdots, N-1$, is a **real-valued** Gaussian variable with zero-mean and unit-variance.

In order to benefit from the result in [9], the Right Hand Side (RHS) of the inequality in Eq. (6) should be deterministic. But due to the presence of Y, the RHS is clearly random. To overcome this problem, we introduce the event $\mathcal{E}$ defined as

$$\mathcal{E} = \left\{ Y \;\Big|\; \frac{\eta - \lambda_N |Y|^2}{1-\lambda_N} \le \alpha \right\}$$

where α is the intersection point of the cumulative distribution functions $\mathbb{P}\left((2N-3)^{-1}\chi_{2N-3}^2 \le t\right)$ and $\mathbb{P}\left((2N-2)^{-1}\chi_{2N-2}^2 \le t\right)$. Notice that the specific choice of α will be justified in the next subsection.

Now, the outage probability takes the following form

$$\begin{aligned} \mathbb{P}_{out}(\boldsymbol{\lambda}) &= \mathbb{P}\left\{\sum_{i=1}^{2(N-1)} \ell_i Z_i^2 < \frac{\eta - \lambda_N |Y|^2}{1-\lambda_N}\Big|\mathcal{E}\right\} P_{\mathcal{E}} \\ &+ \mathbb{P}\left\{\sum_{i=1}^{2(N-1)} \lambda_i Z_i^2 < \frac{\eta - \lambda_N |Y|^2}{1-\lambda_N}\Big|\overline{\mathcal{E}}\right\} \overline{P_{\mathcal{E}}} \quad (7) \end{aligned}$$

where $P_{\mathcal{E}}$ is the probability of event $\mathcal{E}$ and $\overline{P_{\mathcal{E}}} = 1 - P_{\mathcal{E}}$.

A. *Optimal structure of the eigenvalues* $\boldsymbol{\lambda}$

From now on, in order to exhibit a closed-form solution to our optimization problem, we will analyze it for large SNR.

Due to the definition of event $\mathcal{E}$ and a direct application of the result in [9], the first term of the RHS of Eq. (7) can be minimized by choosing $\ell_1 = \cdots = \ell_{2(N-1)} = 1/2(N-1)$. Moreover, the second term of the RHS of Eq. (7) can also be easily bounded in such a way that

$$\begin{aligned} &\mathbb{P}\left\{\sum_{i=1}^{2(N-1)} \ell_i Z_i^2 < \frac{\eta - \lambda_N |Y|^2}{1-\lambda_N}\Big|\overline{\mathcal{E}}\right\} \\ &\ge \mathbb{P}\left\{\sum_{i=1}^{2(N-1)} \ell_i Z_i^2 < \alpha\right\} \ge \mathbb{P}\left\{\frac{\chi_{2(N-1)}^2}{2(N-1)} < \alpha\right\}. \end{aligned}$$

Hence,

$$\mathbb{P}_{out}(\boldsymbol{\lambda}) \ge \mathbb{P}_{\inf}(\lambda_N)$$

with

$$\begin{aligned} \mathbb{P}_{\inf}(\lambda_N) &= \mathbb{P}\left\{\frac{\chi_{2(N-1)}^2}{2(N-1)} < \frac{\eta - \lambda_N |Y|^2}{1-\lambda_N}\Big|\mathcal{E}\right\} P_{\mathcal{E}} \\ &+ \mathbb{P}\left\{\frac{\chi_{2(N-1)}^2}{2(N-1)} < \alpha\right\} \overline{P_{\mathcal{E}}}. \end{aligned}$$

Let λ_N^* be the value belonging to $[0,1]$ that minimizes the lower bound $t \mapsto \mathbb{P}_{\inf}(t)$. We will prove that at the point $\boldsymbol{\lambda} = [(1-\lambda_N^*)/(N-1), \cdots, (1-\lambda_N^*)/(N-1), \lambda_N^*]^{\mathrm{T}}$, the outage probability reaches the minimum of its lower bound.

In the sequel, for the sake of clarity, we substitute λ_N with the variable t. First of all, we remark that, when event $\mathcal{E}$ occurs, we have the following inequality for t

$$t \le \left(1 - \frac{\eta}{\alpha}\right) + t\frac{|Y|^2}{\alpha}.$$

Thus, since t and $|Y|^2$ are positive, if $t \in [0, 1-\eta/\alpha]$, then $P_{\mathcal{E}} = 1$.

Secondly, we need to prove the following lemma.

Lemma 1: Let $t \mapsto p_{\chi_2^2(s)}(t)$ be the density function of the distribution $\chi_2^2(s)$. Let $a_{\eta,m} = \inf_{t\in[0,\eta]} p_{\chi_2^2(m^2)}(t)$ and $b_{\eta,m} = \sup_{t\in[\eta,\eta/(1-\eta/\alpha)]} p_{\chi_2^2(m^2)}(t)$. Let $\mathcal{S}_N$ be the set of points (η, m) from $\mathbb{R}_+^2$ such that

$$\frac{a_{\eta,m}}{b_{\eta,m}} \ge \frac{\int_0^\eta (\eta - u) p_{\frac{\chi_{2(N-1)}^2}{2(N-1)}}(u)du}{\int_\eta^\alpha (u-\eta) p_{\frac{\chi_{2(N-1)}^2}{2(N-1)}}(u)du}. \quad (8)$$

If $\mathcal{S}_N$ is not empty, the minimum of $t \mapsto \mathbb{P}_{\inf}(t)$ is reached for $t \in [0, 1-\eta/\alpha]$ if $(\eta, m) \in \mathcal{S}_N$.
The proof is reported in the Appendix.

First of all, one can remark that if η is small enough or equivalently if the SNR is large enough, $a_{\eta,m} \approx b_{\eta,m} \approx p_{\chi_2^2(m^2)}(0)$. Then, one can check that Eq. (8) easily holds by choosing η once again small enough. So as soon as the SNR is large enough, the set $\mathcal{S}_N$ is clearly not empty and Lemma 1 is of interest.

Remind that

$$\mathbb{P}_{out}(\boldsymbol{\lambda}) \geq \mathbb{P}_{\inf}(\lambda_N^*) \tag{9}$$

where λ_N^* is the argument of the minimum of $t \mapsto \mathbb{P}_{\inf}(t)$. Under the assumption of large SNR, Lemma 1 enables us to deduce that λ_N^* belongs to $[0, 1 - \eta/\alpha]$ which implies that $P_{\mathcal{E}} = 1$. This leads to

$$\begin{aligned} \mathbb{P}_{\inf}(\lambda_N^*) &= \mathbb{P}\left\{ \frac{\chi^2_{2(N-1)}}{2(N-1)} < \frac{\eta - \lambda_N^* |Y|^2}{1 - \lambda_N^*} \right\} \\ &= \inf_{\lambda_N} \mathbb{P}\left\{ \frac{\chi^2_{2(N-1)}}{2(N-1)} < \frac{\eta - \lambda_N |Y|^2}{1 - \lambda_N} \right\} \end{aligned} \tag{10}$$

Due to Eqs. (9)-(10), we have proven the following proposition.

Proposition 1: If SNR is large enough, the outage probability for any vector $\boldsymbol{\lambda}$ is lower-bounded by the outage probability obtained for vector $\boldsymbol{\lambda}^* = [(1 - \lambda_N^*)/(N-1), \cdots, (1 - \lambda_N^*)/(N-1), \lambda_N^*]^{\mathrm{T}}$ where λ_N^* is defined as follows

$$\lambda_N^* = \arg \min_{t \in [0, 1 - \frac{\eta}{\alpha}]} \mathbb{P}\left\{ (1-t)\frac{\chi^2_{2(N-1)}}{2(N-1)} + t\chi^2_2(m^2) < \eta \right\}. \tag{11}$$

Proposition 1 corresponds to the first main contribution of the paper.

B. Optimal value of λ_N

In this section, we would like to provide a way for finding λ_N^* without computing Eq. (11) which unfortunately requires a Monte-Carlo based evaluation.

Let us define

$$\xi_t = (1-t)\frac{\chi^2_{2(N-1)}}{2(N-1)} + t\chi^2_2(m^2).$$

Clearly, ξ_t is distributed as a linear combination of independent non-central chi-square variables. Due to the complexity of the closed-form expression of the associated density function, the optimization with respect to t of the cumulative distribution function seems untractable analytically. To overcome this problem, it is usual to approximate the density function of the random variable ξ_t by a simpler function.

As done in [13] and references therein, ξ_t can be well aproximated by a Gamma distribution as soon as N is large enough. Actually, we will show, by simulation, that the approximation fits well even for small N. Let $p_G(t|p_1, p_2)$ be the Gamma probability density function with parameters (p_1, p_2). We have

$$p_G(t|p_1, p_2) = \frac{1}{p_2^{p_1}\Gamma(p_1)} t^{p_1 - 1} e^{-\frac{t}{p_2}}.$$

Hence, following the approach done in [13], we can approximate of ξ_t's density function by $p_G(t|p_1, p_2)$ where parameters p_1 and p_2 are chosen such that both density functions offer the same first two central moments. By definition of ξ_t and $p_G(t|p_1, p_2)$, this leads to

$$p_1 = \frac{\beta_1^2}{\beta_2} \quad \text{and} \quad p_2 = \frac{\beta_2}{\beta_1}$$

with

$$\beta_1 = t(1+m) \tag{12}$$

$$\beta_2 = \left(\frac{1}{N-1} + 4 + 4m\right)t^2 - \frac{2}{N-1}t + \frac{1}{N-1}. \tag{13}$$

By using the approximate distribution in Eq. (11), we get

$$\lambda_N^* \approx \arg \min_{t \in [0, 1 - \frac{\eta}{\alpha}]} \mathbb{P}\left\{ p_G(t|p_1, p_2) < \eta \right\}$$

with p_1 and p_2 defined in (12) and (13), respectively. As the cumulative distribution function of the Gamma distribution is expressed in a known closed-form, we obtain the following proposition

Proposition 2: Let $\Gamma(x)$ and $\gamma(x, y)$ be the Gamma function and the lower incomplete gamma function, respectively. We have

$$\lambda_N^* \approx \arg \min_{t \in [0, 1 - \frac{\eta}{\alpha}]} \frac{\gamma(p_1, \eta/p_2)}{\Gamma(p_1)}. \tag{14}$$

Proposition 2 corresponds to the second main contribution of the paper.

It is important to remember here that Eq. (14) can easily be computed. The value of λ_N^* can thus be found by a one dimensional search without any need for a Monte-Carlo simulation. Next section provides numerical results to illustrate, on the one hand, the precision of our approximation and, on the other hand, the improvement of the proposed power allocation scheme compared to uniform power allocation.

IV. Numerical illustrations

Let us consider a MISO system using a uniform linear array of N antennas at the transmitter and 1 antenna at the receiver. The propagation channel is assumed to a Rician one.

In order to check the accuracy of the approximation we derived in Eq. (14), we plot in Figure 1 the mean-square error (MSE) between the empirical histogram of the true cumulative distribution function of ξ_t (obtained by Monte-Carlo simulation) and the Gamma cumulative distribution function that approximates it.

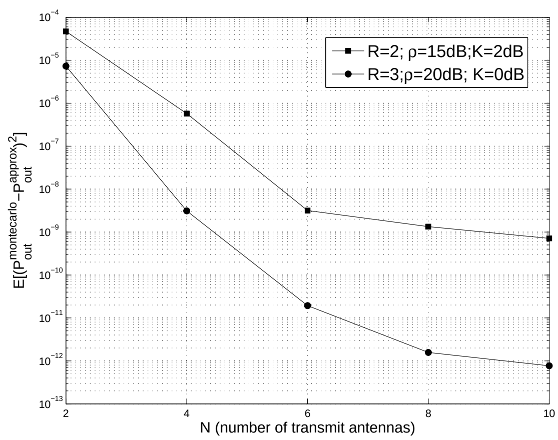

Fig. 1. MSE between empirical and approximate $\mathbb{P}_{out}$ vs. N

As expected, the MSE decreases as N increases. All the different system configurations (various SNR, K and R) that have been tested are specified on all the figures' legend. Also, for each empirical curve in this paper, points are determined by $10,000$ trials and $\mathbf{h}_d$ is randomly selected from a complex circular Gaussian distribution and then normalized.

Actually, to perfectly validate our approximation from Section III-B, we would like to inspect the gap between the optimal values of λ_N^* obtained via Eqs. (11) and (14). Therefore, we display on Figure 2 the MSE between the value of λ_N^* obtained via Eq. (14) and the actual value of λ_N^* obtained through Eq. (11), with respect to N. Similar system configurations to those in Figure 1 are used, as shown in the figure's legend. Once again, the MSE decreases as N increases and the MSE is small enough to consider our approximation.

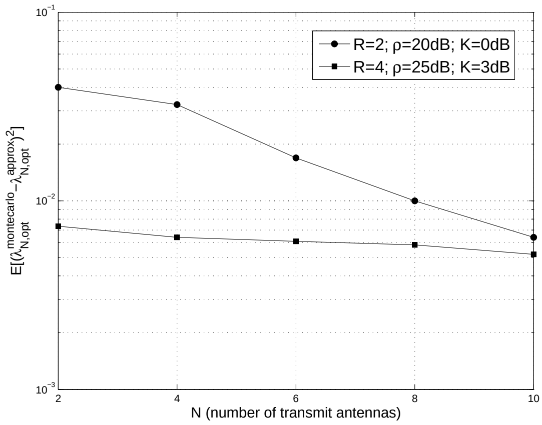

Fig. 2. MSE between empirical and approximate λ_N^* vs. N

Given these results, we will from now on only use the expressions for the outage probability and the optimal power allocation given by the Gamma approximation, thus avoiding any Monte-Carlo simulation. In Figure 3, we plot the outage probability in the case of eigenvalues $\{(1-\lambda_N)/(N-1), \cdots, (1-\lambda_N)/(N-1), \lambda_N\}$ versus λ_N.

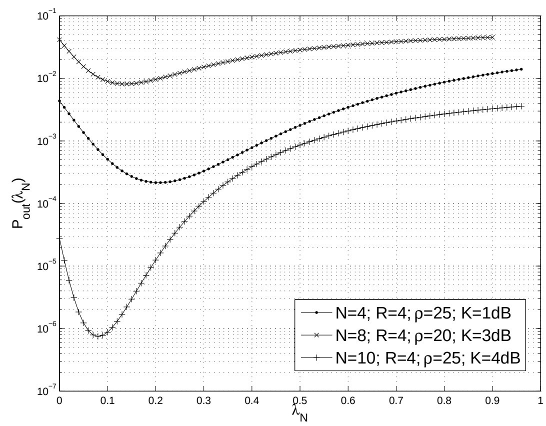

Fig. 3. $\mathbb{P}_{out}$ versus λ_N

We observe that if λ_N is close to λ_N^*, the outage probability only degrades very slightly. In contrast, when λ_N is far away from λ_N^*, the outage probability yields significant loss in performance. Therefore the optimization of λ_N as done in this paper is a crucial task.

In Figure 4, we plot the outage probability versus the SNR for the proposed power allocation scheme and uniform power allocation, for different values of N. We observe that gain in performance is achieved.

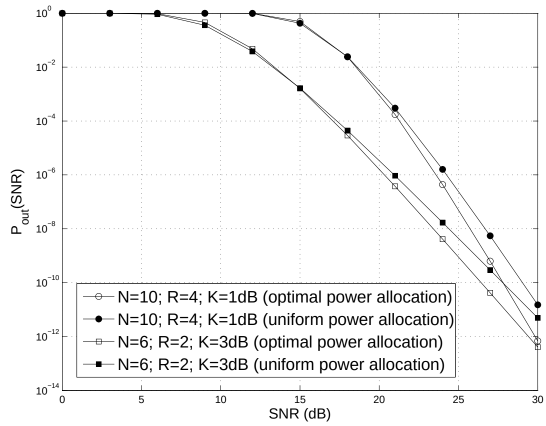

Fig. 4. $\mathbb{P}_{out}$ versus SNR

Finally, in Figure 5, we compare the outage probability obtained with the proposed power allocation and the non-trivial power allocation scheme suggested in [6] versus SNR. Notice that the authors in [6] propose the optimal matrix $\mathbf{Q}$ associated with the ergodic capacity optimization.

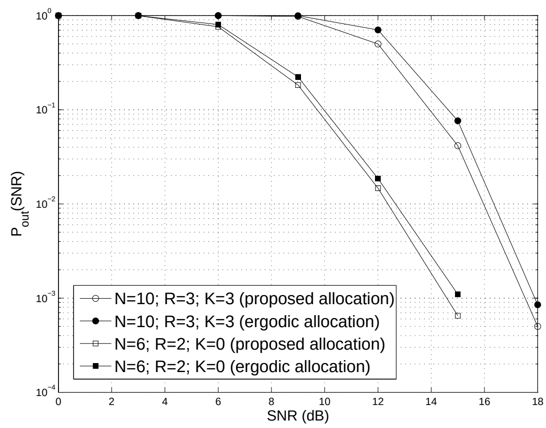

Fig. 5. $\mathbb{P}_{out}$ versus SNR, for different Q matrices.

V. Conclusions

In this paper, we derived an achievable lower bound for the outage probability for Rician block flat fading channels in the MISO context. We provide useful insights on the power allocation scheme achieving this bound. We also propose an original and accurate approximation of the outage probability, enabling an efficient characterization of the power proportion to be associated with the Rician part of the channel.

APPENDIX

If $t \in (1-\eta/\alpha, 1]$, $\mathbb{P}_{\inf}(t)$ can be rewritten as

$$\begin{aligned}\mathbb{P}_{\inf}(t) &= \mathbb{P}\Big\{(1-t)\frac{\chi^2_{2(N-1)}}{2(N-1)} + t\chi^2_2(m^2) < \eta, \\ &\quad \chi^2_2(m^2) \geq h(t,\alpha)\Big\} \\ &+ \mathbb{P}\left\{\frac{\chi^2_{2(N-1)}}{2(N-1)} < \alpha\right\}\mathbb{P}\left\{\chi^2_2(m^2) \leq h(t,\alpha)\right\}\end{aligned}$$

with $h(t,\alpha) = \alpha - (\alpha-\eta)/t$. Simplifying, we get

$$\begin{aligned}\mathbb{P}_{\inf}(t) &= \iint_{\Delta_t} p_{\chi^2_2(m^2)}(v) p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) dv du \\ &+ \mathbb{P}\left\{\frac{\chi^2_{2(N-1)}}{2(N-1)} < \alpha\right\}\mathbb{P}\left\{\chi^2_2(m^2) \leq h(t,\alpha)\right\}\end{aligned}$$

with

$$\begin{aligned}\Delta_t &= \{(u,v) | u \geq 0, v \geq 0, (1-t)u + tv < \eta, v \geq h(t,\alpha)\} \\ &= \{(u,v) | 0 \leq u \leq \alpha, h(t,\alpha) \leq v \leq h(t,u)\}.\end{aligned}$$

Notice that, as $h(t,\alpha) > 0$ for $t \in (1-\eta/\alpha, 1]$, the second definition of Δ_t makes sense. Using the definition of Δ_t in the previous equation leads to

$$\begin{aligned}\mathbb{P}_{\inf}(t) &= \int_0^\alpha \left[\int_{h(t,\alpha)}^{h(t,u)} p_{\chi^2_2(m^2)}(v) dv\right] p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) du \\ &+ \mathbb{P}\left\{\frac{\chi^2_{2(N-1)}}{2(N-1)} < \alpha\right\}\mathbb{P}\left\{\chi^2_2(m^2) \leq h(t,\alpha)\right\}\end{aligned}$$

In order to analyse the location of the minimum of $t \mapsto \mathbb{P}_{\inf}(t)$, we derive its derivative function. Simple algebraic manipulations enable us to obtain the following expression for $t \mapsto \mathbb{P}'_{\inf}(t)$.

$$\begin{aligned}\mathbb{P}'_{\inf}(t) &= \int_0^\alpha p_{\chi^2_2(m^2)}(h(t,u)) \frac{\partial h(t,u)}{\partial t} p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) du \\ &- \int_0^\alpha p_{\chi^2_2(m^2)}(h(t,\alpha)) \frac{\partial h(t,\alpha)}{\partial t} p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) du \\ &+ \mathbb{P}\left\{\frac{\chi^2_{2(N-1)}}{2(N-1)} < \alpha\right\} p_{\chi^2_2(m^2)}(h(t,\alpha)) \frac{\partial h(t,\alpha)}{\partial t}\end{aligned}$$

Due to the expression of $\mathbb{P}\left\{(2(N-1))^{-1}\chi^2_{2(N-1)} < \alpha\right\}$, the second and third terms of RHS of previous equation are identical and they cancel each others. Then, we get

$$\mathbb{P}'_{\inf}(t) = \int_0^\alpha p_{\chi^2_2(m^2)}(h(t,u)) \frac{\partial h(t,u)}{\partial t} p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) du.$$

Using the closed-form expression of $h(t,\alpha)$, we get

$$\begin{aligned}&\mathbb{P}'_{\inf}(t) = \frac{1}{t^2}\int_0^\alpha (u-\eta) p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) p_{\chi^2_2(m^2)}(u - \frac{u-\eta}{t}) du \\ &\geq \frac{1}{t^2}\int_\eta^\alpha (u-\eta) p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) \inf_{t\in\mathcal{I}}\left[p_{\chi^2_2(m^2)}(u - \frac{u-\eta}{t})\right] du \\ &- \frac{1}{t^2}\int_0^\eta (\eta-u) p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) \sup_{t\in\mathcal{I}}\left[p_{\chi^2_2(m^2)}(u - \frac{u-\eta}{t})\right] du.\end{aligned}$$

with $\mathcal{I} = (1-\eta/\alpha, 1]$. However,

$$\begin{aligned}\inf_{t\in\mathcal{I}}\left[p_{\chi^2_2(m^2)}(u - \frac{u-\eta}{t})\right] &\geq a_{\eta,m} \\ \sup_{t\in\mathcal{I}}\left[p_{\chi^2_2(m^2)}(u - \frac{u-\eta}{t})\right] &\leq b_{\eta,m}.\end{aligned}$$

Finally,

$$\begin{aligned}\mathbb{P}'_{\inf}(t) &\geq \frac{a_{\eta,m}}{t^2}\int_\eta^\alpha (u-\eta) p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) du \\ &- \frac{b_{\eta,m}}{t^2}\int_0^\eta (\eta-u) p_{\frac{\chi^2_{2(N-1)}}{2(N-1)}}(u) du.\end{aligned}$$

If η and m belong to $\mathcal{S}_N$ (defined in Lemma 1), then $\mathbb{P}'_{\inf}(t) \geq 0$ when $t \in (1-\eta/\alpha, 1]$ which implies that the argument minimizing $\mathbb{P}_{\inf}(t)$ does not lie on $(1-\eta/\alpha, 1]$. This concludes the proof.

REFERENCES

[1] E. Telatar, “Capacity of multi-antenna Gaussian channels”, *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–596, June 1999.

[2] A. Goldsmith, S. A. Jafar, N. Jindal and S. Vishwanath, “Capacity limits of MIMO channels”, *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, June 2003.

[3] A. Lozano, A. Tulino and S. Verdú, “Capacity-Achieving Input Covariance for Correlated Multi-Antenna Channels”, *Allerton Conference on Communication, Control, and Computing*, Monticello, USA, Oct. 2003.

[4] D. Hoesli, Y.-H. Kim and A. Lapidoth, “Monotonicity results for coherent MIMO Rician channels”, *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4334–4339, Dec. 2005.

[5] M. Vu and A. Paulraj, “Capacity optimization for Rician correlated MIMO wireless channels”, *Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, USA, Nov. 2005.

[6] J. Dumont, Ph. Loubaton and S. Lasaulce, “On the Capacity Achieving Transmit Covariance Matrices of MIMO Correlated Rician Channels: A Large System Approach”, *IEEE Global Communications Conference (GLOBECOM)*, San Francisco, USA, Nov. 2006.

[7] M. Chiani, , M. Z. Win and A. Zanella, “The distribution of eigenvalues of a Wishart matrix with correlation and application to MIMO capacity”, *IEEE Global Communications Conference (GLOBECOM)*, San Francisco, USA, Dec. 2003.

[8] M. Kang and M. S. Alouini, “Capacity of MIMO Rician channels”, *IEEE Transactions on Wireless Communications*, vol. 5, no. 1, pp. 112–122, Jan. 2006.

[9] G. J. Székely and N. K. Bakirov, “Extremal probabilities for Gaussian quadratic forms”, *Probability Theory and Related Fields*, vol. 126, no. 2, pp. 184–202, June 2003.

[10] A. L. Moustakas and S. H. Simon, “Optimizing multiple-input single-output (MISO) communication systems with general Gaussian channels: nontrivial covariance and nonzero mean”, *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2770–2780, Oct. 2003.

[11] Y. Xie, C. N. Georghiades and A. Arapostathis, “Minimum Outage Probability Transmission With Imperfect Feedback for MISO Fading Channels”, *IEEE Transactions on Wireless Communications*, vol. 4, no. 3, pp. 1084–1091, May 2005.

[12] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, New York: Cambridge University Press, 2005.

[13] Q. T. Zhang and D. P. Liu, “A Simple Capacity Formula for Correlated Diversity Rician Fading Channels”, *IEEE Communications Letters*, vol. 6, no. 11, pp. 481–483, Nov. 2002.