Non-data-aided carrier frequency offset estimation for OFDM and downlink DS-CDMA systems

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Abstract— **We address the problem of non-data aided frequency offset estimation for non-circular transmissions over frequency-selective channels in a downlink DS-CDMA system or an OFDM communications context. We observe that twice the frequency offset is a cyclic frequency of the received signal. We thus introduce an estimator relying on the maximisation of the empirical cyclocorrelations. We analyse its asymptotic behaviour and obtain a closed-form expression for the asymptotic covariance. This enables us to design relevant system parameters. Simulations are provided and confirm our assertions.**

I. Introduction

For several years, the Orthogonal Frequency Division Multiplexing (OFDM) based multi-carrier communication systems and the Direct-Sequence Code Division Multiple Access (DS-CDMA) based multi-user communications have received increasing attention. For instance, the OFDM technique is used in the European digital broadcast radio system. DS-CDMA is used in the third generation mobile communication network. Although used with different goals in mind, these two systems can be described by means of a general formalism based on linear precoders ([1], [2]). At the receiver, the transmitted signal is disturbed by two kinds of impairments : on the one hand, the selectivity of the propagation channel. On the other hand, the carrier frequency offset caused by a mismatch between the receive and transmit carrier frequencies. The mismatch is likely due to Doppler effect or local oscillator drift.

For a DS-CDMA system, the carrier frequency offset leads to an additional (if the channel is dispersive) loss of orthogonality between the different users. In an OFDM system, an inter-carrier interference (ICI) remains after the discrete Fourier transform ([3]). It consequently makes sense to remove the carrier frequency offset before any other processing (i.e., before compensating for the channel dispersion). We therefore focus on the carrier frequency offset estimation based on the received signal still corrupted by the channel.

Sending a training sequence leads traditionally to accurate estimates of the carrier frequency offset and the filter corresponding to the channel. Nevertheless this method reduces the effective information rate. It also cannot be carried out in some applications (e.g., passive listening in a military context). Therefore we only treat non-data-aided (NDA) estimation methods.

Several works have been reported about this problem. Subspace method based estimators have been recently developed for an OFDM system as well as for a DS-CDMA system ([4], [5], [6]). Nevertheless their validity domain is quite restrictive : in an OFDM case, the presence of virtual sub-carriers has to be assumed ; in a DS-CDMA case, the number of users has to be smaller than a certain bound depending on the spreading factor and the channel length. A maximum likelihood based estimator has been introduced in the context of an OFDM system ([7]). However this was done only under the strong assumption of the absence of dispersive channel. This restrictive assumption is also required for the estimator introduced in [8] which relies on an angle of a correlation product.

A new estimator has been recently introduced by [9] for a single-carrier and single-user transmission. It relies on the cyclostationary property of the received signal. Indeed, if the symbol constellation is non-circular (i.e., the mathematical expectation of the square symbol is not null), then twice the carrier frequency offset is the sole cyclic frequency of the baud-rate sampled received signal with respect to its conjugate correlation function. The estimator is obtained by maximising a weighted sum of conjugate cyclocorrelations, in the cyclic frequency domain. Its asymptotic behaviour has been analysed. The analysis showed that the performance is almost independent of the frequency-selective channel if the number of cyclocorrelation coefficients taken into account is large enough.

Our aim is to extend such an approach to multi-carrier (OFDM) and multi-user (downlink DS-CDMA) communication schemes.

The paper is organised as follows : In Section II, we briefly review the transmission using linear precoders. More precisely, we mention the general formalism putting together the OFDM and downlink DS-CDMA systems in unique framework. In Section III, we express the new estimator in the unifying framework. In Section IV, we analyse the asymptotic behaviour of the estimator. We obtain a closed-form expression for the asymptotic covariance. In a DS-CDMA system case, we conclude that the performance of the proposed estimator is almost insensitive to the intersymbol interference (ISI). This property holds if each user code and the symbol constellation are real-valued. The

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Section V is devoted to numerical illustrations.

II. Problem statement

At first, we consider the single-carrier and single-user communications framework. A linearly modulated symbol sequence, denoted by $\{s_n\}_{n\in\mathbb{Z}}$, is transmitted. This sequence is assumed to be zero-mean, i.i.d., non-circular and unit-variance. The transmitted signal passes through a multi-path propagation channel. Hence, the continuous-time baseband received signal $y_a(t)$ can be written as follows :

$$y_a(t) = \left(\sum_{n\in\mathbb{Z}} s_n h_a(t-nT_s)\right) e^{2i\pi\delta f_0 t} + w_a(t) \quad (1)$$

where T_s and δf_0 are the symbol period and the carrier frequency offset respectively. The unknown filter $h_a(t)$ arises from the convolution of the shaping filter and the propagation channel. Without restriction, one can assume that the mapping $t \mapsto h_a(t)$ is time-limited and causal. $w_a(t)$ denotes an additive zero-mean Gaussian noise.

As the transmitter uses a linear precoder, it sends the so-called "pseudo-symbol" sequence [1] $\{v_n\}_{n\in\mathbb{Z}}$ defined by Equation (2) instead of the usual symbol sequence [2] $\{s_n\}_{n\in\mathbb{Z}}$. We put

$$V_Q(n) = \mathbf{K} S_P(n), \quad (2)$$

with $V_Q(n) = [v_{nQ}, \cdots, v_{nQ+Q-1}]^{\mathrm{T}}$. $S_P(n)$ can be expressed in a similar way ([2]). $\mathbf{K}$ denotes a full-rank matrix and is called "precoding matrix". The previous general formalism encompasses the cyclic-prefix OFDM systems and the downlink DS-CDMA systems. Indeed, matrix $\mathbf{K}$ is equal to a particular Vandermonde matrix for a cyclic-prefix OFDM system. For a DS-CDMA system, each input of the vector $S_P(n)$ corresponds to one user and then each column of $\mathbf{K}$ corresponds to one user spreading code. The single-carrier and single-user system is obtained by setting $P = Q = 1$ and $\mathbf{K} = 1$.

In order to treat OFDM and DS-CDMA systems simultaneously, we now consider the unifying framework. We assume that the receiver knows perfectly the precoding matrix. We wish to estimate the carrier frequency offset from the sole knowledge of the "pseudo-symbol" baud-rate sampled received signal $y(n) = y_a(nT_v)$. According to Equation (1), the discrete-time signal $y(n)$ can be written in the following form

$$y(n) = a(n) e^{2i\pi\Delta f_0 n} + w(n) \quad (3)$$

with $a(n) = [h(z)].v_n$ and $w(n) = w_a(nT_v)$. $h(z)$ represents the "pseudo-symbol" baud-rate sampled version of the continuous-time filter $h_a(t)$. $h(z)$ is a causal FIR filter of degree M. We also denote $\Delta f_0 = (\delta f_0 T_v \bmod 1)$ [3].

[1] at baud rate $1/T_v = Q/PT_s$ where P and Q are integer such that $P \leq Q$.

[2] at baud rate $1/T_s$.

[3] $(a \bmod b)$ stands for a modulo b. By convention, $(a \bmod b)$ belongs to $[0, b[$.

Equation (3) shows that estimating the carrier frequency offset is equivalent to the harmonic retrieval in multiplicative and additive noise. In a single-carrier and single-user system, the multiplicative noise $a(n)$ is stationary whereas it becomes cyclostationary in a multi-carrier or multi-user system. Indeed the "pseudo-symbol" sequence is cyclostationary because it obeys to the structure defined by Equation (2). More precisely, as the symbol sequence is non-circular, the "pseudo-symbol" sequence and the process $a(n)$ are cyclostationary with respect to their autocorrelation and conjugate autocorrelation functions. The set of their cyclic frequencies is as follows : $\{k/Q \mid 0 \leq k \leq Q-1\}$. We recall that a zero-mean discrete-time stochastic process $p(n)$ is said cyclostationary if the correlation coefficients based sequence $\{\mathbb{E}[p(m+n)\overline{p(n)}]\}_{m\in\mathbb{Z}}$ [4] (or the conjugate correlation coefficients based sequence $\{\mathbb{E}[p(m+n)p(n)]\}_{m\in\mathbb{Z}}$) is "almost-periodic", which means that

$$\mathbb{E}[p(m+n)\overline{p(n)}] = \sum_{k=0}^{\infty} r^{(\alpha_k)}(m) e^{2i\pi\alpha_k n}$$

where $\{\alpha_k\}_{k\geq 0}$ are the so-called cyclic frequencies of $p(n)$. The sequence $\{r^{(\alpha_k)}(m)\}_{m\in\mathbb{Z}}$ denotes the cyclocorrelation sequence at cyclic frequency α_k of $p(n)$. The following Fourier expansion

$$S_p^{(\alpha_k)}(e^{2i\pi f}) = \sum_{m\in\mathbb{Z}} r^{(\alpha_k)}(m) e^{-2i\pi m f}$$

represents the cyclospectrum at cyclic frequency α_k of $p(n)$.

On account of the multiplicative noise cyclostationary, we can not directly apply the estimator introduced by [9] in the context of a single-carrier and single-user transmission to our more general context. The next section therefore addresses suitable modifications and extensions.

III. New Estimator

Let $r_c(n,\tau) = \mathbb{E}[y(n+\tau)y(n)]$ be the conjugate correlation at lag τ of $y(n)$. We put $\alpha_0 = (2\Delta f_0 \bmod 1)$. As

$$r_c(n,\tau) = \sum_{l=0}^{Q-1} r_c^{(\alpha_0+l/Q)}(\tau) e^{2i\pi(\alpha_0+l/Q)n}, \quad (4)$$

$y(n)$ is cyclostationary with respect to its conjugate correlation coefficients. Since Q is known, if $|\Delta f_0| < \min(1/4, 1/2Q)$, then one can check that the knowledge of the cyclic frequencies of $y(n)$ provides the value of α_0. Let $\mathcal{A}_0$ be a compact set included in $]0, \min(1/2, 1/Q)[$. Then

$$\forall\alpha \in \mathcal{A}_0, \alpha \neq \alpha_0, \forall\tau, \forall l, \quad r_c^{(\alpha+l/Q)}(\tau) = 0.$$

α_0 thus satisfies the following equality

$$\alpha_0 = \arg\max_{\alpha\in\mathcal{A}_0} J(\alpha), \quad J(\alpha) = \sum_{l=0}^{Q-1} \left\|\mathbf{r}_c^{(\alpha+l/Q)}\right\|_{W_l}^2$$

[4] Let $\mathbf{x}$ be a scalar or a vector. The overline $\overline{x}$ stands for the complex-conjugate of $\mathbf{x}$. The superscripts $\mathbf{x}^{\mathrm{T}}$ and $\mathbf{x}^{\mathrm{H}}$ denote its transpose and transpose-conjugate respectively.

with $\mathbf{r}_c^{(\alpha)} = [r_c^{(\alpha)}(-T), \cdots, r_c^{(\alpha)}(T)]^{\mathrm{T}}$ where T is an integer and $\{W_l\}_{l=0,Q-1}$ is a set of positive Hermitian weighting matrices [5]. In practice, the cyclocorrelations vector $\mathbf{r}_c^{(\alpha)}$ has to be estimated because only N observations are available. We thus consider the empirical estimate given by

$$\hat{\mathbf{r}}_{c,N}^{(\alpha)} = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{y}_2(n) e^{-2i\pi\alpha n}$$

with $\mathbf{y}_2(n) = [y(n-T)y(n), \cdots, y(n+T)y(n)]^{\mathrm{T}}$. Then the α_0 estimate, denoted by $\hat{\alpha}_N$, is defined by

$$\hat{\alpha}_N = \arg \max_{\alpha \in \mathcal{A}_0} J_N(\alpha), \quad J_N(\alpha) = \sum_{l=0}^{Q-1} \left\| \hat{\mathbf{r}}_{c,N}^{(\alpha+l/Q)} \right\|_{W_l}^2 .$$

This estimator extends the one introduced in [9] to a more general scheme including OFDM and DS-CDMA systems. Considering $Q = 1$ leads to the single-carrier and single-user case. This estimator has been partially introduced for a system using non-redundant precoding dedicated to a particular transmitter induced cyclostationary scheme ([10]). Nevertheless these authors only consider the cyclo-correlation coefficient at lag 0 (i.e., $T = 0$) and the identity weighting matrices (i.e., $W_l = 1$, for each l). In the sequel, we analyse the asymptotic behaviour of this estimator. This enables us to forecast the influence of the weighting matrices and the number of considered cyclocorrelation coefficients on the performance.

IV. Asymptotic Analysis

We consider the following zero-mean process : $\mathbf{e}(n) = \mathbf{y}_2(n) - \mathbb{E}[\mathbf{y}_2(n)]$. Equation (4) thus leads to

$$\mathbf{y}_2(n) = \sum_{l=0}^{Q-1} \mathbf{r}_c^{(\alpha_0+l/Q)} e^{2i\pi(\alpha_0+l/Q)n} + \mathbf{e}(n).$$

So $\mathbf{y}_2(n)$ corresponds to a sum of sinusoids with multivariate amplitudes disturbed by the additive noise $\mathbf{e}(n)$. The cost function $J_N(\alpha)$ represents a sum of weighted periodograms.

This link with harmonic retrieval has been successfully exploited in [9] in the context of single-carrier and single-user communications. Therefore we consider the same approach as that of [9]. Due to the lack of space, we only describe the main steps. For more details, see [9] and also [11].

Under weak assumptions on the noise $w(n)$, $\mathbf{e}(n)$ satisfies the following condition as $h(z)$ represents a FIR filter.

Condition 1: Let $\mathbf{e}^{(0)}(n) = \mathbf{e}(n)$ *and* $\mathbf{e}^{(1)}(n) = \overline{\mathbf{e}(n)}$.

$$\forall L, \exists B_L < \infty, \forall(\nu_1, \ldots, \nu_L) \in \{0,1\}^L, \forall(N, N')$$

$$\sup_{n_1 \in \mathbb{Z}} \sum_{n_2,\ldots,n_L=N'}^{N} ||cum_L(\mathbf{e}^{(\nu_1)}(n_1), \cdots, \mathbf{e}^{(\nu_L)}(n_L))|| \leq B_L$$

[5] Let $\mathbf{x}$ and W be a vector and a positive Hermitian matrix, respectively. Then $||\mathbf{x}||_W^2 = \mathbf{x}^{\mathrm{H}} W \mathbf{x}$.

Then one can prove the following lemma.

Lemma 1: Under condition 1, for each $K \in \mathbb{N}$, *we get*

$$\lim_{N\to\infty} \sup_{\alpha\in[0,1]} ||\frac{1}{N^{K+1}} \sum_{n=0}^{N-1} n^K \mathbf{e}(n) e^{2i\pi\alpha n}|| \overset{a.s}{=} 0$$

a.s. stands for "almost surely".

Finally, one can deduce the consistence and the asymptotic normality of the proposed estimator.

Theorem 1: As $N \to \infty$, *we get*

$$N(\hat{\alpha}_N - \alpha_0) \xrightarrow{a.s.} 0 \quad \textit{and} \quad N^{3/2}(\hat{\alpha}_N - \alpha_0) \xrightarrow{\mathcal{L}} \mathcal{N}(0,\gamma)$$

$\mathcal{L}$ *stands for a distribution convergence.* $\mathcal{N}(0,\gamma)$ *is a zero-mean Gaussian distribution with variance* γ.

We notice that the convergence rate is proportional to $N^{3/2}$. We obtain the following closed-form expression for the asymptotic covariance by means of a first order Taylor series expansion of the derivative of the cost function $J_N(\alpha)$ around the true point α_0,

$$\gamma = \frac{3}{\pi^2} \frac{\sum_{l,l'=0}^{Q-1} \mathbf{R}_l^{\mathrm{H}} \mathbf{W}_l \mathbf{G}_{l,l'} \mathbf{W}_{l'} \mathbf{R}_{l'}}{(\sum_{l=0}^{Q-1} \mathbf{R}_l^{\mathrm{H}} \mathbf{W}_l \mathbf{R}_l)^2}$$

with

$$\mathbf{R}_l = \begin{bmatrix} \mathbf{r}_c^{(\alpha_0+l/Q)} \\ \overline{\mathbf{r}_c^{(\alpha_0+l/Q)}} \end{bmatrix}, \mathbf{W}_l = \begin{bmatrix} W_l & 0 \\ 0 & \overline{W_l} \end{bmatrix}$$

and

$$\mathbf{G}_{l,l'} = \begin{bmatrix} \Gamma_{l,l'} & -\Gamma_{l,l'}^c \\ -\overline{\Gamma_{l,l'}^c} & \overline{\Gamma_{l,l'}} \end{bmatrix}$$

where

$$\Gamma_{l,l'} = \lim_{N\to\infty} N\mathbb{E}[\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l/Q)} . \delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l'/Q)^{\mathrm{H}}}]$$

$$\Gamma_{l,l'}^c = \lim_{N\to\infty} N\mathbb{E}[\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l/Q)} . \delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l'/Q)^{\mathrm{T}}}]$$

and

$$\delta\hat{\mathbf{r}}_{c,N}^{(\alpha)} = (\hat{\mathbf{r}}_{c,N}^{(\alpha)} - \mathbf{r}_c^{(\alpha)}).$$

The previous expressions are not computable as they stand. Indeed, we need more explicit expressions for Γ and Γ^c, the asymptotic covariance of the cyclocorrelation vector empirical estimate. In fact, we have obtained a closed-form expression for Γ and Γ^c with respect to the filter, the precoding matrix, and the statistics of the transmitted symbol sequence. Due to the lack of space, we do not address their derivations and final expressions which we leave to the readers. Nevertheless we provide the main derivation steps.

At first we show that these matrices can be derived in terms of the cyclospectra of the disturbance occurring in the equivalent harmonic retrieval problem, i.e., $\mathbf{e}(n)$. Then we determine the second order cyclic statistics of the disturbance $\mathbf{e}(n)$ with respect to the second and fourth order statistics of the received signal. It now remains to derive the expressions of these received signal cyclospectra in terms of the filter $h(z)$ and the second and high order cyclic statistics of the process $v(n)$. Lastly we have to obtain the cyclic second and fourth order cyclospectra of $v(n)$ in terms of the elements of matrix $\mathbf{K}$.

Then one can check that γ splits into two terms :

$$\gamma = \gamma_0 + \mathcal{O}(\sigma^2)$$

where γ_0 is a term independent of the noise $w(n)$, and σ^2 is the variance of the noise $w(n)$ ($w(n)$ need not to be white). After some obvious but tedious derivations, we obtain the following result.

Theorem 2: Let δ and Id be the Kronecker index and the identity matrix respectively.
We assume that the symbols $\{s_n\}_{n\in\mathbb{Z}}$ belong to a real-valued constellation. We also consider that the entries of $\mathbf{K}$ are real-valued.
If $T \geq M + Q$ and $W_l = \delta_{0,l} Id$, then $\gamma_0 = 0$.
As the most popular non-circular constellation is the BPSK real-valued constellation, the assumption on the constellation characteristic is not restrictive. Unfortunately the other assumption on the precoding matrix is more restrictive. Indeed the OFDM systems do not satisfy such a condition since the associated precoding matrix is complex-valued. On the contrary, the condition holds for the DS-CDMA systems since the associated precoding matrices (e.g., Walsh-Hadamard or Gold sequences) are real-valued generally.

In the sequel, the criterion associated with the following weighting matrices set $\{W_l = \delta_{0,l} Id \mid 0 \leq l \leq Q-1\}$ will be called *reduced*. In fact, it only takes advantage of the cyclocorrelations vector around the cyclic frequency α_0. The *complete* criterion is obtained with the following weighting matrices set $\{W_l = Id \mid 0 \leq l \leq Q-1\}$. It is based on the sum of cyclocorrelation vectors around all the cyclic frequencies.

Theorem 2 shows that the asymptotic covariance of the estimator associated with the reduced criterion is proportional to the noise variance if we take enough cyclocorrelation coefficients. This covariance then becomes null in the noiseless case. Moreover this implies that this estimator is almost insensitive to an inter-symbol interference effect. Theorem 2 also shows that we should consider the reduced criterion rather than the complete criterion. An explanation is that the cyclocorrelation coefficients at the cyclic frequencies different from α_0 are numerically weak.

V. Numerical illustrations

In this section, we shall only focus on a DS-CDMA system since the better performances are obtained for such a system. The precoding matrix $\mathbf{K}$ is obtained from a Walsh-Hadamard sequence. The spreading factor and the number of users are $Q = 4$ et $P = 3$ respectively. The symbol stream $\{s_n\}_{n\in\mathbb{Z}}$ is drawn from a BPSK constellation. We denote N_s the number of transmitted symbols s_n. N_s is equal to N/Q, with N the number of available received observations. The shaping filter is a square-root raised cosine at "pseudo-symbols" baud rate $1/T_v$ with the roll-off $\rho = 0.2$. The propagation multi-path channel is modelled by five paths with maximal delay $3T_s$. The noise $w(n)$ is assumed to be white. In each simulation, we average the result over 50 Monte-Carlo symbol sequence trials. Except where otherwise stated, we put $N_s = 200$, $T = M + Q$, and the Signal-to-Noise Ratio (SNR) is fixed to be 50 dB. Finally, the sought cyclic frequency α_0 is equal to 0.15.

In Figure 1, we have plotted the reduced cost function versus the cyclic frequency α. We observe that the peak around α_0 is much larger than those around the other cyclic frequencies (i.e., $\alpha_0 + l/Q$ with $l \neq 0$). The complete cost function takes *a priori* advantage of the peaks at all the cyclic frequencies for estimating α_0. As the power of these peaks is numerically weak, the resulting estimation is unreliable. Therefore considering all the peaks leads to degrading the estimation instead of improving it.

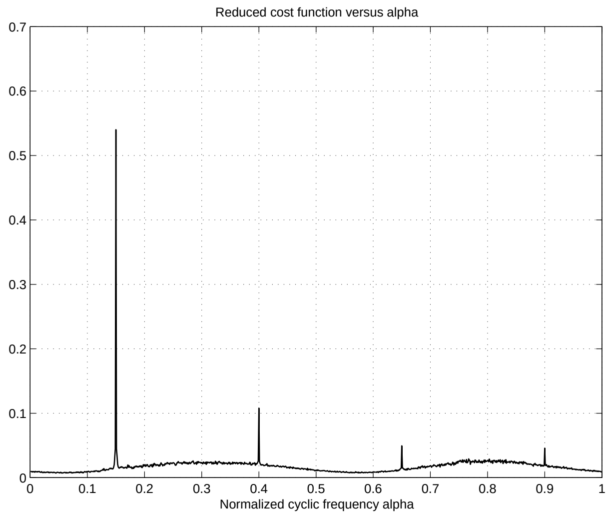

Fig. 1. The cyclocorrelation vector square norm $||\hat{\mathbf{r}}_{c,N}^{(\alpha)}||_{Id}^2$ versus α

Figure 2 depicts the theoretical and empirical mean square error (MSE) versus the SNR of the proposed estimator for different values of the design parameters. The theoretical MSE is computing by means of the term $\gamma/(Q{*}N_s)^3$. The empirical MSE is obtained by computing a gradient algorithm initialised at the true point α_0, in order to avoid local minima. As expected, the theoretical and empirical performances are better for the reduced cost function based estimator than for the complete cost function based estimator. For the reduced cost function based estimator, the theoretical performance fits in with that one obtained for a transmission over a flat fading channel (i.e., in a ISI-less case). Nevertheless there is a gap between the theoretical and empirical performances for high SNRs. It is due to the unreliable estimation of the cyclocorrelation coefficients for high lags : indeed, an accurate estimation of these cyclocorrelation coefficients requires many samples and, in particular, more than $N_s = 200$ ([9]).

Figure 3 shows that the gap between theoretical and empirical performances thus decreases as N_s increases.

Figure 4 represents the theoretical and empirical MSE versus T. We put SNR= 30 dB. The number T of considered conjugate cyclocorrelation lags varies from 0 to $(M + Q)$. We observe that the theoretical and empirical

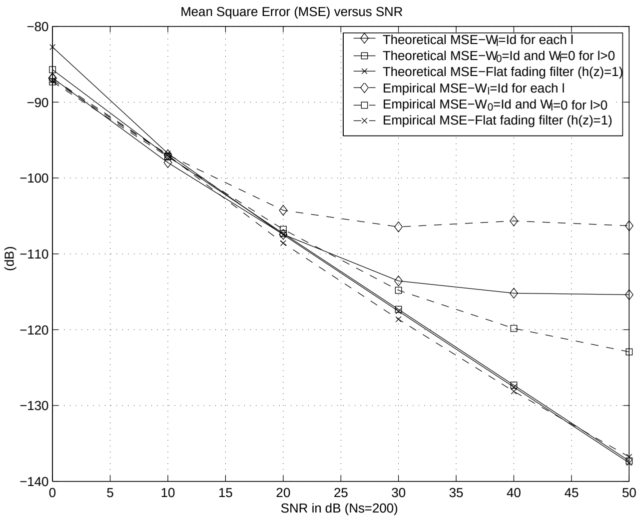

Fig. 2. Theoretical and empirical MSE versus SNR

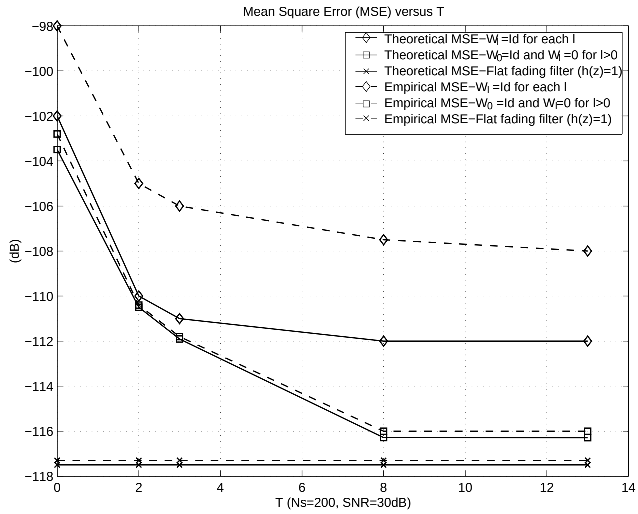

Fig. 4. Theoretical and empirical MSE versus T

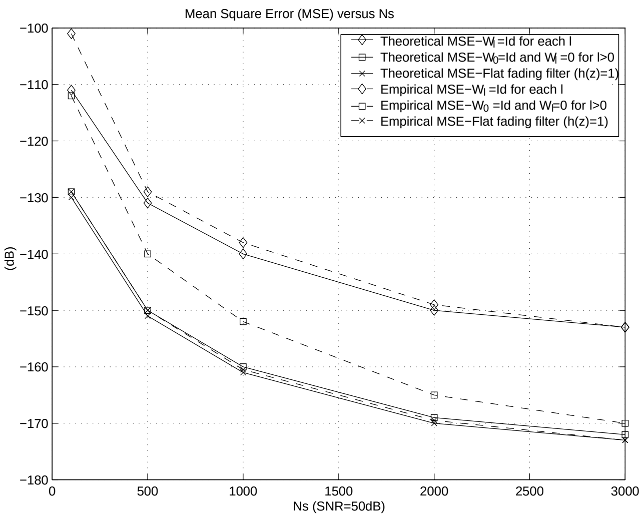

Fig. 3. Theoretical and empirical MSE versus N_s

MSE rapidly decrease. For the reduced criterion based estimator, the optimal performance is reached before taking into account all the cyclocorrelation coefficients. It is due to the filter impulse response weak tails. The weak tails can be neglected and the effective channel length then is smaller than M. For the complete criterion based estimator, the corresponding γ_0 converges to a non-zero term. Therefore, as observed, the influence of T on the performance is less strong.

These curves are in good agreement with the theoretical asymptotic analysis. Furthermore, they confirm the parameter choices.

VI. Conclusion

We have investigated a new estimator in an OFDM or downlink DS-CDMA context, assuming a frequency-selective channel and a non-circularly distributed symbol stream. We have rigorously analysed its asymptotic behaviour. According to this analysis, we have identified relevant design parameters. In a DS-CDMA context, if the user codes and the constellation are real-valued, then we have proved that the well-designed estimator is powerful even in the presence of inter-symbol interference.

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