Performance Analysis of Blind Carrier Phase Estimators for General QAM Constellations

E. Serpedin[1] (contact author), P. Ciblat[2], G. B. Giannakis[3], and P. Loubaton[2]

[1] Dept. of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128, Tel.: (979) 458 2287 Fax: (979) 862 4630 email: serpedin@ee.tamu.edu

[2] Université de Marne-la-Vallée, Laboratoire "Systèmes de Communication", 5 Bd. Descartes, 77454 Marne-la-Vallée cedex 2, France

[3] Dept. of Electrical and Computer Engr., University of Minnesota, 200 Union St. SE, Minneapolis, MN 55455

Abstract— **Large quadrature amplitude modulation (QAM) constellations are currently used in throughput efficient high speed communication applications such as digital TV. For such large signal constellations, carrier phase synchronization is a crucial problem because for efficiency reasons the carrier acquisition must often be performed blindly, without the use of training or pilot sequences. The goal of the present paper is to provide thorough performance analysis of the blind carrier phase estimators that have been proposed in the literature and to assess their relative merits.**

I. Introduction

Fast acquisition of the carrier phase is a crucial issue in high-speed communication systems that employ large QAM modulation schemes. One of the challenges associated with large QAM constellations is the blind carrier acquisition, which is often required in large and heavily loaded multipoint networks for bandwidth efficiency and little effort involved in network monitoring. It is known that for large QAM constellations, the conventional carrier tracking schemes frequently fail to converge and result in "spinning" [8], [10]. Therefore, developing computationally simple blind carrier phase estimators with guaranteed convergence and good statistical properties is well-motivated.

Recently, a number of blind carrier phase estimators have been proposed [1], [2], [3], [4], [6], [11, p. 266-277], [12], but thorough performance analysis of all these algorithms has not been performed. In order to quantify the performance of these estimators, the large sample (asymptotic) performance analysis of these phase estimators will be established and compared with the stochastic (modified) Cramér-Rao bound [11, Section 2.4]. It is shown that the seemingly different estimators [1], [2], [3], [5], [11, p. 266-277], [12], are the same, while the estimator proposed in [4] has a larger asymptotic variance than the power-law estimator [3], [6], [12]. It is also shown that by exploiting the additional samples acquired through oversampling the received continuous-time waveform does not improve the performance of the power-law estimator in [3], [6], [12]. Finally, computer simulations are presented to corroborate the theoretical developments and to compare the performance of the investigated phase estimators.

II. Problem Statement

We consider the baseband QAM communication system where the received signal $Y(n) = Y_r(n) + jY_i(n)$ is given by

$$Y(n) = e^{j\theta}X(n) + N(n) \ , \qquad (1)$$

where $Y_r(n)$ and $Y_i(n)$ denote the in-phase and quadrature components of $Y(n)$, $X(n)$ stands for the independent and identically distributed (i.i.d.) input QAM symbol stream, $N(n)$ is the circularly distributed Gaussian noise, assumed to be independent of $X(n)$, and θ denotes the unknown carrier phase offset. The problem of blind carrier phase estimation consists of recovering the phase error θ only from knowledge of the received data $Y(n)$. Because the input QAM constellation has quadrant $(\pi/2)$ symmetry, it follows that it is possible to recover the unknown phase θ only modulo a $\pi/2-$phase ambiguity. This ambiguity can be further eliminated through the use of appropriate coding schemes. Therefore, without any loss of generality, we can assume that the unknown phase θ lies the interval $(-\pi/4,\ \pi/4)$. In the next section, we briefly outline the blind phase estimators [1], [2], [3], [4], [5], [11, p. 266-277], [12], and establish their exact large sample performance.

III. Blind Carrier Phase Estimators

A. Approximate Maximum Likelihood Estimator: Fourth-Power Estimator

The maximum likelihood (ML) estimator of θ can be theoretically derived by maximizing a stochastic likelihood function, obtained by averaging the conditional probability density function of the received data with respect to the unknown data stream $X(n)$. However, for high order QAM constellations, the computational complexity involved in calculating the likelihood function and more importantly the resulting nonlinear optimization problem render the ML-estimator impractical for most high-speed applications. The need for computationally simple estimators with guaranteed convergence calls for alternative (possibly suboptimal, but computationally feasible) phase estimators.

Moeneclaey and de Jonghe have shown in [12] that for any arbitrary 2-dimensional rotationally symmetric constellations (such as square or cross QAM constellations) the fourth-power (or power-law) estimator can be obtained as an approximate ML-estimator in the limit of small Signal-to-Noise Ratio (SNR:= $10\log E|X(n)|^2/E|N(n)|^2$, where := stands for "is defined as"). The power-law estimator and its sampled version are defined as:

$$\theta := \frac{1}{4}\arg\left[(EX^{*4}(n))\ EY^4(n)\right] \ , \qquad (2)$$

$$\hat{\theta} := \frac{1}{4}\arg\left[E(X^{*4}(n))\frac{\sum_{n=1}^{N} Y^4(n)}{N}\right] \ , \qquad (3)$$

where the superscript $*$ stands for complex conjugation and the operator $E(\cdot)$ denotes the expectation operator. The fourth-power estimator does not require any complex nonlinear optimizations, but it requires a-priori knowledge of the input constellation $E(X^{*4}(n))$. However, this is not a restrictive assumption since for most QAM constellations, $EX^{*4}(n)$ is a negative real-valued number, whose effect can be easily accounted for. Using standard convergence results [9] it can be checked that asymptotically (3) is[1] w.p. 1 a consistent estimator ($\hat{\theta} \to \theta$ as $N \to \infty$) for any SNR range. An explanation can be obtained by observing that, in the presence of circularly and normally distributed noise $N(n)$, the following relation holds:

$$\frac{1}{N}\sum_{n=1}^{N} Y^4(n) \xrightarrow{\text{w.p.1}} EY^4(n) = e^{j4\theta}EX^4(n) \;, \qquad (4)$$

where the second equality in (4) is obtained by expanding $EY^4(n) = E(\exp(j\theta)X(n) + N(n))^4$, taking into account the independence between $X(n)$ and $N(n)$, and $EN^k(n) = 0$, for any positive integer k. Hence, (3) recovers the carrier phase from the phase of the fourth-order moment of the received data.

Cartwright has proposed estimating the unknown phase θ using a different set of fourth-order statistics [3]. Define the following fourth-order moments and cumulants:

$$\begin{aligned}
\gamma &:= E[Y_r^4] + E[Y_i^4] - 6E[Y_r^2 Y_i^2] \;, & (5)\\
\gamma_a &:= \text{cum}(Y_r, Y_r, Y_r, Y_i) = E[Y_r^3 Y_i] - 3E[Y_r^2]E[Y_r Y_i] & \\
&= E[Y_r^3 Y_i] \;, & (6)\\
\gamma_b &:= \text{cum}(Y_r, Y_i, Y_i, Y_i) = E[Y_r Y_i^3] - 3E[Y_i^2]E[Y_r Y_i] & \\
&= E[Y_r Y_i^3] \;, \quad (E[Y_r Y_i] = 0). & (7)
\end{aligned}$$

Cartwright's estimator is defined by:

$$\tan(4\theta) = 4\left(\frac{\gamma_a - \gamma_b}{\gamma}\right) \Rightarrow \theta = \frac{1}{4}\text{atan}\left[4\left(\frac{\gamma_a - \gamma_b}{\gamma}\right)\right]. \quad (8)$$

To verify that Cartwright's estimator is the fourth-power estimator in (2), we equate the in-phase and quadrature components of:

$$\begin{aligned}
EY^4(n) &= e^{j4\theta}EX^4(n) = \cos(4\theta)EX^4(n) + j\sin(4\theta)EX^4(n)\\
EY^4(n) &= E(Y_r(n) + jY_i(n))^4 = E[Y_r^4(n) + Y_i^4(n) - 6Y_r^2(n)\\
&\quad \times Y_i^2(n)] + 4jE[Y_r^3(n)Y_i(n) - Y_r(n)Y_i^3(n)]\\
&= \gamma + 4j(\gamma_a - \gamma_b). \qquad (9)
\end{aligned}$$

It follows that $\gamma = \cos(4\theta)EX^4(n)$ and $4(\gamma_a - \gamma_b) = \sin(4\theta)EX^4(n)$, which implies the equivalence between estimators (2) and (8). Cartwright's (fourth-power) estimator requires only that $EX^4(n) \neq 0$ and the independence between $X(n)$ and additive circularly and normally distributed noise $N(n)$, and it can be applied to both square and cross-QAM constellations, as opposed to the estimator proposed in [4], which can be applied only to square-QAM constellations.

It is interesting to remark that three other phase estimators, derived using completely different arguments, are equivalent to the fourth-power estimator. An alternative robust phase estimator with guaranteed convergence has been proposed in [2] for square-QAM constellations. Herein, the carrier acquisition problem is reduced to the blind source separation problem of the linear mixture of the in-phase and quadrature-phase components of the received signal, and a cumulant-based source separation criterion is proposed to estimate the unknown phase-offset [2]. In [1], [11, pp. 271-277], a low SNR approximation of the likelihood function, assuming PSK input constellations, is shown to have the same form as the estimator [2]. Furthermore, it is justified that this estimator can be used even for general QAM constellations [11, pp. 271-277]. By relying on Godard's quartic criterion [8], Foschini has shown an alternative derivation of this phase estimator in [5]. Next, we describe briefly the estimator proposed in [2], which relies on the observation that the in-phase and quadrature components of a square-QAM constellation are independent.

Let ϕ denote an estimate of the unknown phase offset θ, define the "rotated" output $\tilde{Y}(n) := \exp(-j\phi)\; Y(n)$, and assume that $X(n)$ belongs to a square-QAM constellation. In the absence of noise and if $\phi = \theta$, then the in-phase and quadrature components of $\tilde{Y}(n) = X(n)$ are independent. Thus, the joint cumulants of the in-phase ($\tilde{Y}_r(n)$) and quadrature ($\tilde{Y}_i(n)$) components of $\tilde{Y}(n)$ are equal to zero

$$\begin{aligned}
\tilde{\gamma}_a &:= \text{cum}(\tilde{Y}_r(n), \tilde{Y}_r(n), \tilde{Y}_r(n), \tilde{Y}_i(n)) = 0 \;,\\
\tilde{\gamma}_b &:= \text{cum}(\tilde{Y}_r(n), \tilde{Y}_i(n), \tilde{Y}_i(n), \tilde{Y}_i(n)) = 0 \;, \qquad (10)
\end{aligned}$$

and[2] $\tilde{\gamma}_a - \tilde{\gamma}_b = 0$. It is interesting to remark that (10) continues to hold true even in the presence of additive circularly and normally distributed noise $N(n)$, because the cumulants of the in-phase and quadrature components of $N(n)$ cancel out. By taking into account (9), it follows that $\tilde{\gamma}_a - \tilde{\gamma}_b = (E\tilde{Y}^4(n) - E\tilde{Y}^{*4}(n))/8j$. Thus, θ can be estimated from:

$$\begin{aligned}
\theta_a &:= \arg\min_\phi (E\tilde{Y}^4(n) - E\tilde{Y}^{*4}(n))\\
&= \arg\min_\phi (e^{-j4\phi}EY^4(n) - e^{j4\phi}EY^{*4}(n)). \qquad (11)
\end{aligned}$$

If we consider the polar representation $EY^4(n) = \lambda^4\exp(j4\theta)$, from (11) we obtain that $\theta_a = \arg\min_\phi \lambda^4(\exp(-j4(\phi-\theta)) - \exp(j4(\phi-\theta)))$, which implies that $\theta_a = \theta$ modulo a $\pi/4$-phase ambiguity. Hence, estimator (11) is the same as the fourth-power estimator (2). By taking advantage of the sign of $\tilde{\gamma} := (E\tilde{Y}^4(n) + E\tilde{Y}^{*4}(n))/2$ (see (5), (9)), the $\pi/4$-phase ambiguity inherent in (11) can be reduced to a $\pi/2$-phase ambiguity (since if $\theta_a - \theta = \pi/4$ modulo $\pi/2$, then $\tilde{\gamma} = -EX^4(n) \neq EX^4(n)$).

In practice, many communication systems utilizing QAM constellations employ also coding, which implies that the SNR available at the synchronizer will be reduced by an amount proportional to the coding gain. In order to evaluate correctly the performance of these phase estimators at all SNR levels, next we provide an exact expression for the large sample variance of the power-law estimator, which is valid for any SNR level and it is not restricted to the high SNR regime as is the case with the approximate asymptotic expression presented in [12]. The next section will show that

[1] The notation *w.p. 1* denotes convergence with probability one.

[2] The reader can easily check that $\tilde{\gamma}_a = -\tilde{\gamma}_b$, [4].

the expression of [12] is not valid for low and medium SNRs ($\leq$ 20 dB).

Theorem 1. *Assuming that the i.i.d. symbol stream $X(n)$ is coming from a finite dimensional QAM-constellation and that the additive noise $N(n)$ is circularly and normally distributed and independent of $X(n)$, then the estimate (3) is asymptotically normally distributed with zero mean and the asymptotic variance:*

$$\lim_{N\to\infty} N(\hat{\theta}-\theta)^2 = \frac{\mu_{Y,44} - EX^8(n)}{32(EX^4(n))^2}, \quad (12)$$

with[3] $\mu_{Y,40} := EY^4(n) = e^{j4\theta}EX^4(n)$, *and*

$$\mu_{Y,44} := E|X(n)|^8 + 16E|X(n)|^6E|N(n)|^2 + 36E|X(n)|^4 \times E|N(n)|^4 + 16E|X(n)|^2E|N(n)|^6 + E|N(n)|^8. \quad (13)$$

Proof. Please see [13]. □

The asymptotic variance (12) does not depend on the unknown phase θ, but only on the input symbol constellation and the SNR. This confirms the conclusion drawn in [3] stating that the standard deviation of (8) appears to be constant with respect to the true value of θ. We evaluate next the asymptotic performance of a phase estimator based on an alternative set of statistics that was proposed in [4].

B. HOS-Based Phase Estimator of [6]

The phase estimator [4] extracts the unknown phase information $\theta \in (-\pi/4, \pi/4)$ using the relations:

$$\cot(2\theta) = \frac{\gamma_a - \gamma_b}{2\gamma} \quad \text{if} \quad \left|\frac{\gamma}{\gamma_x}\right| \geq 0.125 \Leftrightarrow$$
$$\theta \in \left(-\frac{\pi}{4}, -\frac{\pi}{8}\right] \cup \left[\frac{\pi}{8}, \frac{\pi}{4}\right), \quad (14)$$
$$\tan(2\theta) = \frac{2(\gamma_a - \gamma_b)}{\gamma_x - 4\gamma} \quad \text{if} \quad \left|\frac{\gamma}{\gamma_x}\right| < 0.125 \Leftrightarrow$$
$$\theta \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right), \quad (15)$$

with $\gamma_x := E[|X|^4] - 2\{E|X|^2\}^2$ and

$$\gamma := \text{cum}\{Y_r(n), Y_r(n), Y_i(n), Y_i(n)\} = E[Y_r^2(n)Y_i^2(n)] - E[Y_r^2(n)]E[Y_i^2(n)] = 0.25\sin^2(2\theta)\gamma_x. \quad (16)$$

Let $\hat{\gamma}_a$, $\hat{\gamma}_b$, and $\hat{\gamma}$ denote sample estimates for γ_a, γ_b, and γ, respectively, and define by $\hat{\theta}_1$ and $\hat{\theta}_2$ the sample estimates corresponding to (14) and (15), respectively. The next theorem, whose proof is deferred due to space limitations to [13], establishes the asymptotic performance of $\hat{\theta}_1$ and $\hat{\theta}_2$.

Theorem 2. *Assuming that the i.i.d. symbol stream $X(n)$ is coming from a finite dimensional QAM-constellation and that the additive noise $N(n)$ is circularly and normally distributed and independent of $X(n)$, then the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ are asymptotically normally distributed with zero mean and asymptotic variances:*

$$\lim_{N\to\infty} N(\hat{\theta}_1-\theta)^2 = \frac{\varrho_{11} + \cot^2(2\theta)\varrho_{22} - 2\cot(2\theta)\varrho_{12}}{\gamma_x^2},$$
$$if \quad \theta \in \left(-\frac{\pi}{4}, -\frac{\pi}{8}\right] \cup \left[\frac{\pi}{8}, \frac{\pi}{4}\right), \quad (17)$$
$$\lim_{N\to\infty} N(\hat{\theta}_2-\theta)^2 = \frac{\varrho_{11} + 4\tan^2(2\theta)\varrho_{22} + 4\tan(2\theta)\varrho_{12}}{\gamma_x^2},$$
$$if \quad \theta \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right), \quad (18)$$

where:

$$\varrho_{11} := \lim_{N\to\infty} NE[(\hat{\gamma}_a - \hat{\gamma}_b) - (\gamma_a - \gamma_b)]^2 = -\frac{|EX^4(n)|^2}{32} + \frac{\cos(8\theta)[(EX^4(n))^2 - EX^8(n)] + \mu_{Y,44}}{32}, \quad (19)$$

$$\varrho_{12} := \lim_{N\to\infty} NE\{(\hat{\gamma} - \gamma)[(\hat{\gamma}_a - \hat{\gamma}_b) - (\gamma_a - \gamma_b)]\} = \frac{-\sin(8\theta)[EX^8(n) - 2(EX^4(n))^2] + 2\text{Im}\{\mu_{Y,62}\}}{64} - \frac{4\sin(4\theta)EX^4(n)[\mu_{Y,22} - 3\mu_{Y,11}^2]}{64} - \frac{8(E|X(n)|^2 + E|N(n)|^2)\text{Im}\{\mu_{Y,51}\}}{64}, \quad (20)$$

$$\varrho_{22} := \lim_{N\to\infty} NE(\hat{\gamma} - \gamma)^2 = \frac{\cos(8\theta)EX^8(n) + 3\mu_{Y,44}}{128} - \frac{4\text{Re}\{\mu_{Y,62}\} + 48\mu_{Y,11}^4 + 6[\cos(4\theta)EX^4(n) - \mu_{Y,22}]^2}{128} - \frac{32\mu_{Y,11}^2[\cos(4\theta)EX^4(n) - 2E|Y(n)|^4]}{128} + \frac{16[\text{Re}\{\mu_{Y,51}\} - \mu_{Y,33}]\mu_{Y,11}}{128}, \quad (21)$$

$\mu_{Y,44}$ *is given by (13), and*

$$\mu_{Y,62} := e^{j4\theta}[EX^6(n)X^{*2}(n) + 12EX^5(n)X^*(n)E|N(n)|^2 + 15EX^4(n)E|N(n)|^4], \quad (22)$$
$$\mu_{Y,51} := e^{j4\theta}[EX^5(n)X^*(n) + 5EX^4(n)E|N(n)|^2], \quad (23)$$
$$\mu_{Y,33} := E|X(n)|^6 + 9E|X(n)|^4E|N(n)|^2 + 9E|X(n)|^2E|N(n)|^4 + E|N(n)|^6, \quad (24)$$
$$\mu_{Y,22} := E|X(n)|^4 + 4E|X(n)|^2E|N(n)|^2 + E|N(n)|^4, \quad (25)$$
$$\mu_{Y,11} := E|X(n)|^2 + E|N(n)|^2. \quad (26)$$

Opposed to the power-law estimator, the asymptotic performance of the Chen etal. estimator [4] depends on the phase offset θ. As the simulation results will show (see Figure 5), the asymptotic performance of this estimator deteriorates significantly whenever the a-priori intervals (14), (15) are missed, and for any SNR it exhibits a larger variance than the power-law estimator.

IV. Performance Comparisons

In this section, computer simulations are performed to assess the relative merits of the proposed phase estimators by comparing the theoretical (asymptotic) limits and the experimental standard deviations of the investigated estimators. Two additional estimators have been analyzed: the fractionally-sampled (FS) power-law estimator and the reduced-constellation power estimator. The FS-power estimator recovers the unknown phase offset θ by exploiting

[3]The notation $\mu_{Y,kl} := EY^k(n)Y^{*l}(n)$ stands for the $(k+l)th$-moment of $Y(n)$.

all the samples obtained by fractionally-sampling (oversampling) the received continuous-time waveform in the estimator (3). A raised-cosine pulse shape with roll-off factor 0.3 and an oversampling factor $P = 3$ are assumed throughout the simulations. The reduced-constellation power estimator relies also on (3), but only the received samples that are larger in magnitude than a given threshold are processed [10, p. 1382], [6, p. 1482]. Thus, only the points closest to the four corners of the constellation are processed. The asymptotic performance of these two additional estimators can be established using the result of Theorem 1, but due to space limitations their expressions will not be presented.

In Figures 1-a and b, we have plotted the experimental and theoretical standard deviations of all these estimators versus SNR, assuming a square 256-QAM constellation, $\theta = 15^0 (= \pi/12)$, $N = 512$ samples, $MC = 300$ Monte-Carlo runs, and additive normally distributed noise. The threshold in the reduced-constellation power estimator has been set up so that only the received samples corresponding to the 12 points of the input 256-QAM constellation with the largest radii are processed. The solid line denotes the stochastic Cramér-Rao bound (CRB$= 1/(N \cdot \text{SNR})$) corresponding to the phase estimate. Figure 1 shows that the power-law estimator performs better than the Chen etal. estimator [4] at all SNR levels, but worse than the reduced-constellation power estimator at high SNRs (SNR$\geq$ 20 dB). The FS-based power estimator appears to have the worst performance. The reduced performance of the FS-power estimator is due to the increased "self-noise" generated by the residual intersymbol interference effects. For this reason, we have not pursued further the analysis of FS-based power-law estimators.

In Figure 2, we have plotted separately the theoretical and experimental standard deviations of the power-law, the reduced-constellation power-law, and the Chen etal. (15) estimators, assuming $MC = 300$ Monte-Carlo simulation runs, $N = 512$ samples, $\theta = \pi/12$, and a 256-QAM input constellation. The experimental values are well predicted by the asymptotic limits for all three estimators, but the CRB seems to be a loose bound. In Figure 3, the experimental and theoretical standard deviations of the power-law and the Chen etal. estimators are plotted versus the number of samples (N), assuming SNR$=$ 10 dB, $MC = 300$ Monte-Carlo runs, $\theta = \pi/12$. It turns out that both estimators achieve the asymptotic bound even when a reduced number of samples $N = 250 \div 500$ are used.

In Figure 4-a, the asymptotic performance of the Chen etal. estimator (14) is analyzed, assuming $\theta = \pi/5$, $MC = 300$, and $N = 512$. Figures 4-b and 5 show that the performance of the Chen etal. estimator depends on the unknown phase θ and has a larger standard deviation than the power-law estimator for any phase offset θ (Figure 5) and for any SNR-level (Figure 4-b). In Figure 5, the theoretical standard deviations (17) and (18) are plotted on the interval $(-\pi/4, \pi/4)$ assuming perfect a-priori knowledge of the intervals (14), (15) where θ lies. However, in the presence of a wrong a-priori knowledge on θ ($|\theta| \geq \pi/4$) the performance of estimator [4] deteriorates significantly.

In Figures 6 and 7, we have analyzed the performance of the power-law and the reduced-constellation power-law estimators in the case of a cross 128-QAM constellation, assuming $\theta = \pi/12$, $MC = 300$, $N = 4000$ samples. For such constellations, the Chen etal. estimator cannot be used since the in-phase and quadrature components of the input symbol stream are not independent. In Figures 6 and 7-a, the experimental and asymptotic standard deviations of the power-law and the reduced-constellation power-law estimators are plotted for different SNR levels. Figures 7-a,b show that the asymptotic limit predicts well the experimental results for all SNR-levels and number of samples $N \geq 1000$. It appears also that for cross-QAM constellations, the power-law estimator exhibits very slow convergence rate and good estimates of the phase-offset can be obtained only by using a large number of samples ($N > 5,000$). Finally, Figure 8 reveals that the approximate asymptotic limit derived in [12] does not predict well the exact asymptotic limit of the power-law estimator for small and medium SNRs (SNR$\leq$ 20dB).

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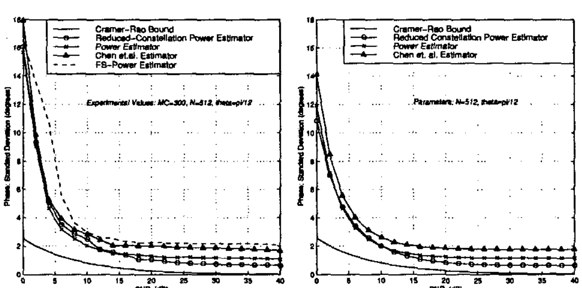

Fig. 1. Standard Deviation vs. SNR a) Experimental Values b) Asymptotic Values (256 square-QAM)

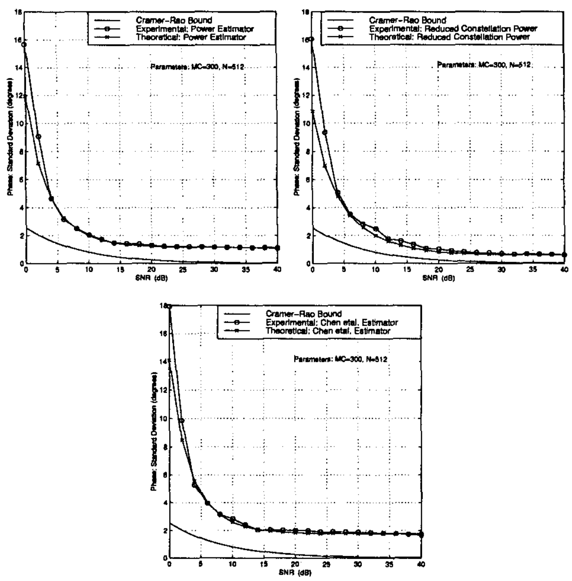

Fig. 2. Standard Deviation vs. SNR: Experimental/Theoretical Values a) Power Estimator b) Reduced-Constellation Power Estimator c) Chen etal. Estimator (256 square-QAM)

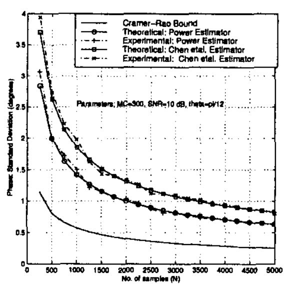

Fig. 3. Standard Deviation vs. No. of Samples: Power Estimator vs. Chen etal. Estimator (256 square-QAM)

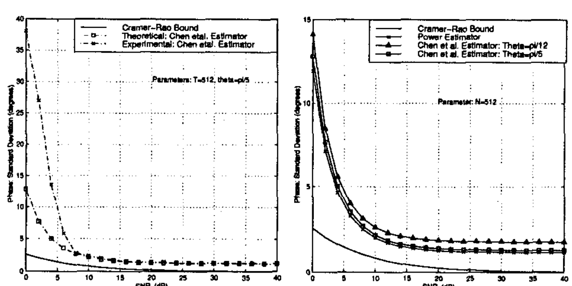

Fig. 4. Standard Deviation vs. SNR a) Chen etal. Estimator ($\theta = \pi/5$) b) Asymptotic Limits (256 square-QAM)

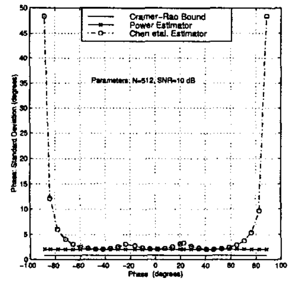

Fig. 5. Standard Deviation vs. Phase offset: Asymptotic Limit (256 square-QAM)

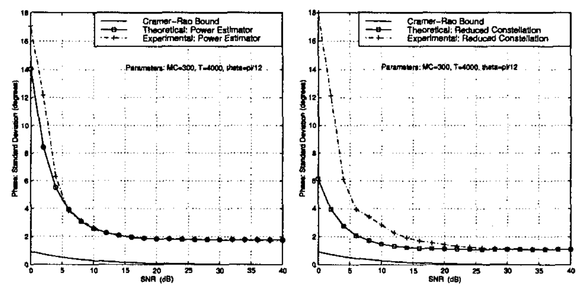

Fig. 6. Standard Deviation vs. SNR a) Power Estimator b) Reduced-Constellation Power Estimator (128 cross-QAM)

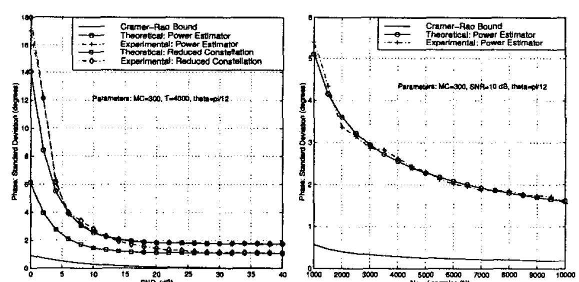

Fig. 7. Standard Deviation vs. SNR/Data: a) Reduced-Constellation Power-Law and Power-Law Estimators b) Power Estimator (128 cross-QAM)

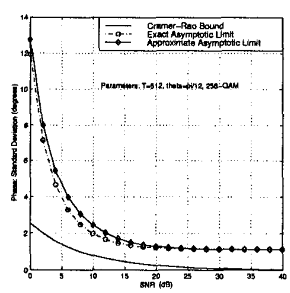

Fig. 8. Standard Deviation vs. SNR: Exact and Approximate Asymptotic Limits (256 square-QAM)