Outage Probability derivations for PDL-disturbed Coherent Optical Communication

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Abstract: We derive in closed-form the outage probability for different statistical models of PDL and evaluate their accuracy. From the resulting expressions, we quantify its impact on optical transmissions as an SNR penalty.

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1. Introduction

Recent works about the design of coherent receivers for optical communications have mainly focused on the mitigation of linear impairments such as chromatic dispersion (CD) and polarization mode dispersion (PMD). In contrast, polarization-dependent loss (PDL) impairment, which could not be mitigated by signal processing techniques, has received much less attention. As with PMD, PDL varies randomly over time [1], and thus can be modeled as a random variable. Therefore different models have been proposed for the related probability density function (pdf) [1–4].

In the frequency domain, the received signal $\mathbf{Y}(\omega)$ on both polarizations can be written as follows: $\mathbf{Y}(\omega) = \mathbf{H}(\omega)\mathbf{X}(\omega)+\mathbf{N}(\omega)$ where $\mathbf{X}(\omega)$ is the transmitted signal and $\mathbf{N}(\omega)$ is the additive white Gaussian noise (AWGN) with variance N_0 per real dimension [5]. The transfer matrix $\mathbf{H}(\omega)$ corresponds to the concatenation of the linear impairments (CD, PMD, and PDL) [4].

From an information-theoretic point-of-view, as PDL is a random phenomenon varying slowly compared to the codeword duration, the appropriate tool for analyzing PDL is the so-called *outage probability*. Let $C(\mathbf{H})$ and r be the capacity for one channel realization $\mathbf{H}(\omega)$ and the required data rate respectively. The outage probability, denoted by P_{out}, is defined as the probability that $C(\mathbf{H})$ is less than r, *i.e.*, $P_{\text{out}} = \text{Prob}_{\mathbf{H}}\{C(\mathbf{H}) < r\}$.

The purpose of the paper is twofold: we express P_{out} in closed-form, then we numerically evaluate the loss in performance due to the presence of PDL.

2. Closed-form expressions for the outage probability

Assuming Gaussian codeword, the capacity for one channel realization takes the following form: $C(\mathbf{H}) = (1/2\pi)\int_{-\pi B}^{\pi B}\log_2\left(\det\left(\mathbf{I}_2+\rho\mathbf{H}(\omega)\mathbf{H}(\omega)^{\text{H}}\right)\right)\mathrm{d}\omega$ where B is the bandwidth, the superscript $(.)^{\text{H}}$ stands for the hermitian operator, $\rho = E_s/N_0$ with E_s the symbol energy. We remind that $\mathbf{H}(\omega)$ is a concatenation of a PDL matrix $\mathbf{H}_{\text{PDL}}(\omega)$, and two unitary matrices corresponding to the CD and PMD impairments. As a consequence, $C(\mathbf{H})$ only depends on $\mathbf{H}_{\text{PDL}}(\omega)$ and thus simplifies as follows: $C(\mathbf{H}) = (1/2\pi)\int_{-\pi B}^{\pi B}\log_2\left(\det\left(\mathbf{I}_2+\rho\mathbf{H}_{\text{PDL}}(\omega)\mathbf{H}_{\text{PDL}}(\omega)^{\text{H}}\right)\right)\mathrm{d}\omega$.

The matrix $\mathbf{H}_{\text{PDL}}(\omega)$ is usually independent of ω and is modeled by

$$\mathbf{H}_{\text{PDL}} = \mathbf{R}_\alpha\mathbf{D}_\gamma\mathbf{R}_\beta,\ \text{with } \mathbf{D}_\gamma = \begin{bmatrix}\sqrt{1-\gamma} & 0\\ 0 & \sqrt{1+\gamma}\end{bmatrix} \text{ and } \Gamma = 10\log_{10}\left(\frac{1+\gamma}{1-\gamma}\right) \tag{1}$$

where $\mathbf{R}_\alpha$ and $\mathbf{R}_\beta$ are two random rotation matrices, and Γ (inherently in dB) is the so-called PDL coefficient.

By replacing $\mathbf{I}_2$ with $\mathbf{R}_\alpha\mathbf{R}_\alpha^{\text{H}}$ in the expression of the capacity, and by using the property of commutation of the determinant, we obtain that $C(\mathbf{H}) = B\log_2\left(\det\left(\mathbf{I}_2+\rho\mathbf{D}_\gamma^2\right)\right)$. Consequently, the outage probability is equal to

$$P_{\text{out}} = \text{Prob}\{\gamma^2 > f(R,\rho)\} \text{ with } f(R,\rho) = 1-(2^R-1-2\rho)/\rho^2, \text{ and } R = r/B \tag{2}$$

We remark that we only need the pdf of γ for the derivations of P_{out}. A few models for this pdf already exist: γ constant [4], γ Gaussian [2], Γ Gaussian [1] and Γ Maxwellian [3]. We add another model by truncating the "γ Gaussian" model in order to force $|\gamma|$ to be less than 1. After algebraic manipulations on cumulative density functions, we obtain the different closed-form expressions for P_{out} shown in the following Table where $Q(x)$ stands for the Gaussian tail function, $T(R,\rho) = \frac{20}{\log(10)}\text{artanh}(\sqrt{f(R,\rho)})$. The coefficients used are defined as in the literature: μ_g and σ_g in [2], K is a normalization coefficient, μ_Γ is the mean PDL in dB [1], and σ_m has been defined in [3].

γ constant	$P_{\text{out}} = 1$ if $\rho < \frac{\sqrt{1+(1-\gamma^2)(2^R-1)}-1}{(1-\gamma^2)}$, 0 elsewhere
γ Gaussian: $\mathcal{N}(\mu_g, \sigma_g^2)$	$P_{\text{out}} = Q\left(\frac{\sqrt{f(R,\rho)}+\mu_g}{\sigma_g}\right) + Q\left(\frac{\sqrt{f(R,\rho)}-\mu_g}{\sigma_g}\right)$
γ truncated Gaussian: $\text{pdf}_\gamma(x) = \begin{cases} \frac{K}{\sqrt{2\pi\sigma_g^2}} e^{-\frac{(x-\mu_g)^2}{2\sigma_g^2}} & \text{if } \lvert x\rvert \leq 1, \\ 0 \text{ elsewhere} \end{cases}$	$P_{\text{out}} = \begin{cases} K\left[Q\left(\frac{\sqrt{f(R,\rho)}+\mu_g}{\sigma_g}\right) + Q\left(\frac{\sqrt{f(R,\rho)}-\mu_g}{\sigma_g}\right) - Q\left(\frac{1+\mu_g}{\sigma_g}\right) - Q\left(\frac{1-\mu_g}{\sigma_g}\right)\right] & \text{if } \rho < \frac{1}{2}\left(2^R-1\right) \\ 0 & \text{elsewhere} \end{cases}$
Γ Gaussian: $\mathcal{N}(0, \mu_\Gamma)$	$P_{\text{out}} = 2Q\left(\frac{T(R,\rho)}{\sqrt{\mu_\Gamma}}\right)$ if $\rho < \frac{1}{2}\left(2^R-1\right)$, 0 elsewhere
Γ Maxwellian	$P_{\text{out}} = 2Q\left(\frac{T(R,\rho)}{\sigma_m}\right) + \sqrt{\frac{2}{\pi\sigma_m^2}}T(R,\rho)e^{\frac{-T(R,\rho)^2}{2\sigma_m^2}}$ if $\rho < \frac{1}{2}\left(2^R-1\right)$, 0 elsewhere

3. Numerical evaluations

In Fig. 1, numerical evaluations of P_{out} according to the proposed closed-form expressions and an evaluation of P_{out} under a more phenomenological PDL model are plotted versus ρ when $R = 4$ bit/s/Hz and $\mathbb{E}[\Gamma] \simeq 3$ dB. Since PDL is generated by elements (such as connectors, splices, etc.) spread along the transmission channel, the PDL matrix $\tilde{\mathbf{H}}_{\text{PDL}}$ may be a concatenation of elementary PDL matrices, such that, $\tilde{\mathbf{H}}_{\text{PDL}} = \sqrt{2}\underline{\mathbf{H}}_{\text{PDL}}/\|\underline{\mathbf{H}}_{\text{PDL}}\|_F$ where $\|\bullet\|_F$ is the Frobenius norm, and $\underline{\mathbf{H}}_{\text{PDL}} = \prod_{\ell=1}^{N}(\mathbf{R}_{\alpha_\ell}\mathbf{D}_{\gamma_\ell}\mathbf{B}_{\phi_\ell})$ with $\mathbf{B}_\phi$ is a birefringence diagonal matrix whose diagonal is $[e^{i\phi}, e^{-i\phi}]$. We assume that α_ℓ and ϕ_ℓ are uniformly chosen in $[0, 2\pi]$, and γ_ℓ is truncated Gaussian distributed. Even though $\tilde{\mathbf{H}}_{\text{PDL}}$ does not have the same form as $\mathbf{H}_{\text{PDL}}$, we compute an equivalent γ for each $\tilde{\mathbf{H}}_{\text{PDL}}$ by identifying $(1+\gamma)/(1-\gamma)$ with the square condition number of $\tilde{\mathbf{H}}_{\text{PDL}}$. As the outage probability can not be expressed in closed-form under this phenomenological PDL model, we evaluate it via Monte-Carlo simulations by setting $N = 100$. We observe that the most relevant model (for any outage probability value) is the Maxwellian one.

In Fig. 2, we plot the SNR penalty versus the mean PDL for different values of P_{out} assuming Maxwellian model and $R = 4$ bit/s/Hz. The SNR penalty is defined as the ratio between the SNR needed to yield given P_{out} and R when PDL occurs and the SNR needed to offer this rate in a PDL-free communication. Given a standard mean PDL of 3 dB, the SNR penalty thus is between 1 and 2 dB according to the chosen P_{out}. Moreover, one can prove that the SNR penalty is upper-bounded by $(2^R-1)/(2(\sqrt{2^R}-1))$. This bound is an interesting insight for designing new systems.

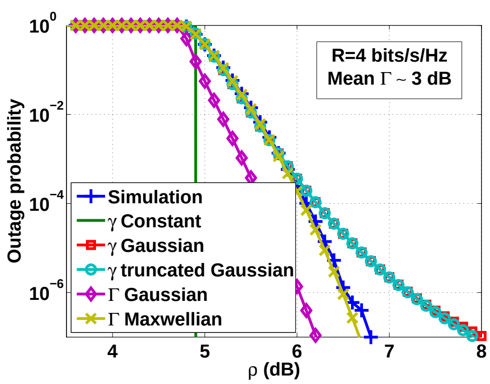

Fig. 1. P_{out} versus ρ for various PDL models

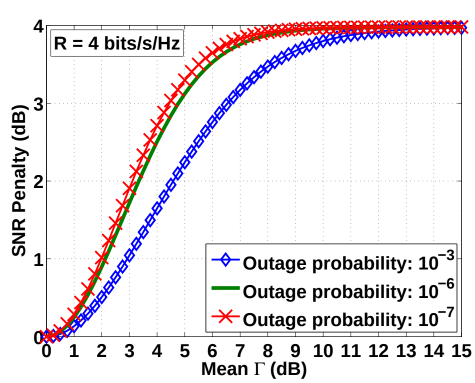

Fig. 2. SNR Penalty for Maxwellian model

4. Conclusion

Closed-form expressions for outage probability have been obtained for various PDL models. Numerical evaluations have shown an agreement between this models (especially the Maxwellian one) and a phenomenological model for useful outage probabilities ($\sim 10^{-6}$). SNR penalty has been evaluated and offers practical insight for system design.

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