Complexity Analysis of Block Equalization Approach for PolMux QAM Coherent Systems

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Abstract: The computational load of block CMA equalizers is addressed. Compared to the adaptive CMA, we show block approaches increase the convergence speed by ~ 10 but only the complexity by ~ 4 in 112Gbit/s PolMux 16QAM systems.

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1. Introduction

Coherent detection combined with digital signal processing (DSP) is a strong candidate for 100Gbit/s optical transmission and beyond. DSP has especially offered the ability to compensate for residual chromatic dispersion (CD) and polarization mode dispersion (PMD) through a Multiple-Input Multiple-Output (MIMO) equalizer whose coefficients are calculated using the Constant Modulus Algorithm (CMA) [1]. The CMA is generally implemented using the stochastic gradient algorithm on a sample-by-sample basis.

In order to enhance the performance in terms of the convergence speed, we recently proposed the blockwise approach for the CMA [2]. In this paper, we describe first the Fixed step-size Block CMA (BF-CMA), pseudo-Newton Block CMA (BN-CMA) and the Optimal step-size Block CMA (BO-CMA). Second, the computational load of these block algorithms is compared with that of the well-known sample-by-sample adaptive CMA equalization.

2. Block equalization approach

Let $y_{p,a}(t)$ be the continuous-time received signal on the polarization p. The signal $y_{p,a}(t)$ is a noisy filtered version of the symbol sequence by the residual CD and the PMD. By sampling at twice the baud rate $1/T_s$, we obtain the bivariate discrete-time received signal $\mathbf{y}_p(n) = [y_{p,a}(nT_s), y_{p,a}(nT_s + T_s/2)]^{\mathrm{T}}$ where the superscript $(.)^{\mathrm{T}}$ stands for the transposition. Let $\mathbf{w}_{p,q}(\ell)$ be the ℓ-th component (of size 1×2) for the fractionally-spaced equalizer between polarizations p and q. As the equalizer is assumed to be of length L, its scalar output associated with the polarization p writes

$$z_p(n) = \sum_{\ell=0}^{L-1} \left(\overline{\mathbf{w}_{p,1}(\ell)}\mathbf{y}_1(n-\ell) + \overline{\mathbf{w}_{p,2}(\ell)}\mathbf{y}_2(n-\ell) \right) = \mathbf{w}_p^{\mathrm{H}} \mathbf{y}^{(L)}(n) \tag{1}$$

with $\mathbf{w}_p = [\mathbf{w}_{p,1}(0), \cdots, \mathbf{w}_{p,1}(L-1), \mathbf{w}_{p,2}(0), \cdots, \mathbf{w}_{p,2}(L-1)]^{\mathrm{T}}$, and $\mathbf{y}^{(L)}(n) = [\mathbf{y}_1(n)^{\mathrm{T}}, \mathbf{y}_1(n-1)^{\mathrm{T}}, \cdots, \mathbf{y}_1(n-L+1)^{\mathrm{T}}, \mathbf{y}_2(n)^{\mathrm{T}}, \mathbf{y}_2(n-1)^{\mathrm{T}}, \cdots, \mathbf{y}_2(n-L+1)^{\mathrm{T}}]^{\mathrm{T}}$. The overline $\overline{(.)}$ and the superscript $(.)^{\mathrm{H}}$ stand for complex conjugaison and conjugate transposition respectively.

The CMA criterion is used to calculate the coefficients $\mathbf{w}_p$ for $p = 1,2$. We consider a block of duration NT_s, *i.e.*, we have N available vectors $\mathbf{y}^{(L)}(n)$. By denoting the estimated equalizer at the i-th iteration of the (block) gradient-like algorithm by $\mathbf{w}_p^i$, the equalizer coefficients can be updated as follows:

- **BF-CMA:** $\mathbf{w}_p^{i+1} = \mathbf{w}_p^i - \mu\Delta^i$ where $\Delta^i = \frac{1}{N}\sum_{n=0}^{N-1}(|z_p^i(n)|^2 - R)\overline{z_p^i(n)}\mathbf{y}^{(L)}(n)$ and $z_p^i(n) = (\mathbf{w}_p^i)^{\mathrm{H}}\mathbf{y}^{(L)}(n)$.
- **BN-CMA:** $\mathbf{w}_p^{i+1} = \mathbf{w}_p^i - \mu\mathbf{G}^i\Delta^i$ with $\mathbf{G}^i = \lambda^{-1}\mathbf{G}^{i-1} - \frac{\lambda^{-2}\mathbf{G}^{i-1}\mathbf{m}\mathbf{m}^{\mathrm{H}}\mathbf{G}^{i-1}}{(2(1-\lambda)\frac{1}{N}\sum_{n=0}^{N-1}|z_p^i(n)|^2)^{-1} + \lambda^{-1}\mathbf{m}^{\mathrm{H}}\mathbf{G}^{i-1}\mathbf{m}}$ where $\mathbf{m} = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{y}_L(n)$, $0 \leq \lambda \leq 1$, $\lambda + \mu = 1$ and $G^0 = \delta\mathbf{Id}$ with the identity matrix $\mathbf{Id}$ and $\delta > 0$.
- **BO-CMA:** $\mathbf{w}_p^{i+1} = \mathbf{w}_p^i - \mu^i\Delta^i$, where μ^i is the real root of a third-order degree polynomial provided in [2].

3. Convergence performance

A 112Gbit/s transmission is achieved by multiplexing both polarizations with 16-QAM modulated signals which corresponds to 14Gbaud transmission per polarization. We used a square root raised cosine filter with a roll-off factor equal to 1 at the transmitter. At the receiver side, a matched filter and an anti-aliasing filter (fifth-order Bessel filter with bandwidth equal to 80% the baud rate) are applied, and the signal is sampled at twice the symbol rate. No laser phase noise and no frequency offset were considered. The block size to estimate the equalizers is set to $N = 1000$ in order to ensure convergence [2]. The length of the equalizers is $L = 3$, *i.e.*, $2L$ coefficients are needed to filter each polarization due the the oversampling. The OSNR (in 0.1nm) is 20dB. Finally, we have CD=1000ps/nm, polarization rotation=$\pi/4$, and DGD=50ps.

In Fig. 1, we plot the BER for the block algorithms with respect to the number of iterations. Extensive numerical investigations have shown that we have to fix $\mu = 0.02$ (BF-CMA and BN-CMA) and $\delta = 0.9$ (BN-CMA). As for the BO-CMA, we have chosen the same optimal step-size for both polarizations at each iteration. The BO-CMA is clearly the fastest one since only 25 iterations are required to obtain a BER $\sim 10^{-3}$ in comparison to 35 iterations for the BN-CMA and 40 iterations for the BF-CMA. In Fig. 2, we plot the BER for two standard equalizers based on sample-by-sample equalization with respect to the number of iterations (which is here equivalent to the number of observations). We have considered adaptive CMA with fixed step-size (AF-CMA) and adaptive CMA with pseudo-Newton update based step-size (AN-CMA). Clearly, we observe that the adaptive equalizers converge much slower than the block ones.

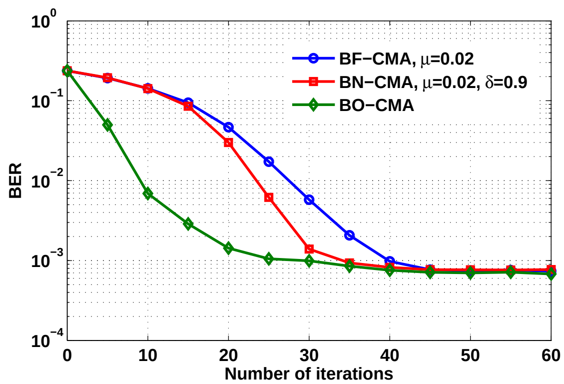

Fig. 1. BER vs. #iterations for block approaches

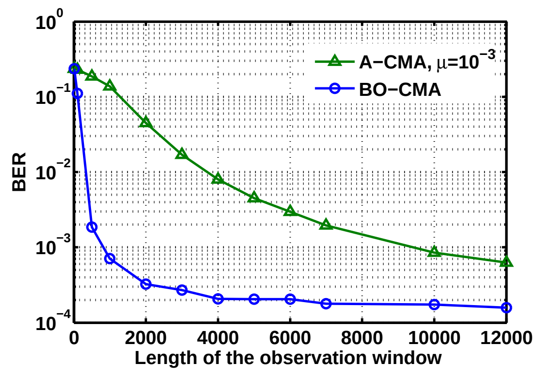

Fig. 2. BER vs. #iterations for adaptive approaches

4. Complexity analysis

Table. 1 inspects the computational load for aforementioned algorithms by measuring the number of flops (complex multiplications) required to reach convergence (BER below the FEC limit $\sim 10^{-3}$). The number of iterations required for these different algorithms are chosen according to Figs. 1 and 2. The AF-CMA was found the least complex at the expense of the largest observation window to reach convergence. The AN-CMA leads to a significant increase of complexity ($\times 16$) without a substantial gain in convergence speed. In contrast, all the blockwise approaches enable us to reduce dramatically the convergence speed at the expense of a slightly more expensive computational cost since this cost has to be multiplied by 4 for BF-CMA, 3.5 for BN-CMA, and 4.5 for the BO-CMA compared to AF-CMA. Notice that the optimal step-size evaluation is done once per iteration since the same is used for both polarizations.

	Sample-by-sample		**Block** ($N = 1000$)		
	AF-CMA [1]	AN-CMA [3]	BF-CMA	BN-CMA	BO-CMA
Update equation (per iter. and polar.)	$2(4L+1)$	$80L^2+8L+5$	$2N(4L+1)$	$2N(4L+1)+80L^2+8L+5$	$2N(4L+1)$
Polynomial evaluation (per iter.)	-	-	-	-	$4N(3L+1)+4L$
Number of iterations	10000	6000	40	35	25
Total flops ($\times 10^3$)	**520**	**8988**	**2080**	**1872**	**2300**

Tab. 1. Computational load for sample-by-sample vs. block CMA

5. Conclusion

The complexity of several block CMA equalizers are analyzed and compared with the standard adaptive CMA equalizer. In our realistic simulation set-up, the block CMA approaches leads to speed up the convergence by a factor ~ 10 but to increase the computational load limited to ~ 4.

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References

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