Maximization of the Sum of Energy-Efficiency For Type-I HARQ Under The Rician Channel

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***Abstract*—In this paper, we address the resource allocation (RA) problem for type-I hybrid automatic repeat request in a multiuser context, with the objective of maximizing the sum of the users' energy efficiency. The resources to allocate consist in per-link transmit energy and proportion of the bandwidth. We assume the use of practical modulation and coding schemes, and we impose a minimum goodput constraint per-link. The RA is performed assuming that only statistical channel state information is available, and considering the Rician channel model. We provide numerical examples highlighting the importance of explicitly taking into account the Rician channel model during the RA.**

***Index Terms*—Energy efficiency, HARQ.**

I. INTRODUCTION

The hybrid automatic repeat request (HARQ) mechanism combines the use of forward error correction (FEC) and automatic repeat request (ARQ), increasing the data rate of wireless communications under time-varying channel. In this paper, our objective is to perform the resource allocation (RA) for type-I HARQ based systems in a multiuser context assuming the use of practical modulation and coding schemes (MCS). Especially, we focus on maximizing the energy efficiency (EE), defined as the ratio between the number of information bits that can be transmitted without error and the energy consumed to transmit them, which has gained much interest from the scientific community [1]. More precisely, we aim to maximize the sum of the users' EE (SEE). Also, we assume that only statistical channel state information (CSI) is available to perform the RA, which is realistic assumption for assisted device-to-device (D2D) communications [2] or ad hoc networks [3] in which a node called resource manager (RM) collects the CSI of the links to perform the RA.

In this context, the statistical distribution of the channel is of importance and, in this paper, we focus on the Rician channel model [4]. This model is of interest since it represents well the realistic statistical behavior of the channel in the presence of a line of sight (LoS) between the transmitter and the receiver, see for instance [5] in the context of millimeter wave communications, or [6] for indoor communications. Moreover, the Rician channel is versatile since it encompasses both the Rayleigh and the additive white Gaussian noise (AWGN) channels as special cases [4].

The RA problem with the objective of maximizing EE-related metrics with no HARQ and using practical MCS is addressed in the single user context in [7], and in the multiuser context in [8]. Especially, in [8], the authors solve the problem of the maximization of the EE of the network assuming that perfect CSI is available at the base station.

When HARQ is used in a multiuser context using orthogonal frequency division multiple access (OFDMA), [9]–[12] address the maximization of EE-related metrics. Among these works, in [9], [10] the authors consider the use of capacity achieving codes while in [11], [12] the authors consider the use of practical MCS. More precisely, in [11], the authors derive a suboptimal procedure to maximize the harmonic mean of the user's EE using type-I HARQ, the Rayleigh channel, statistical CSI and considering the use of relay. In [12], the maximum SEE (MSEE) problem is solved for type-II HARQ considering the Rayleigh channel and statistical CSI.

Hence, the works addressing the RA problem in a multiuser context with EE-related metrics and using HARQ, statistical CSI and practical MCS, e.g. [11], [12], only consider the Rayleigh channel. The main contribution of this paper is to provide an algorithm to solve the MSEE problem for type-I HARQ under the Rician channel when assuming statistical CSI and practical MCS. Notice that our derivations can handle the Rician channel without additional complexity as compared with the conventional Rayleigh one. Our numerical results emphasize that substantial gains can be achieved considering the Rician channel i.e. by considering the existence of a LoS instead of only considering the channel variance, as done in the conventional Rayleigh channel.

The rest of the paper is organised as follows. In Section II, we present the system model and we formulate the RA problem. Section III is devoted to the optimal solution of the MSEE problem. In Section IV, we investigate the results of the proposed algorithm through numerical simulations. Finally, in Section V, we draw concluding remarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Channel model and HARQ mechanism

Let us consider a network with a total bandwidth B divided in N_c subcarriers, which are shared between L active links using the OFDMA as the multi-access technology. Notice that our derivations extend straightforwardly to any multiple access multicarrier scheme and to single-carrier frequency division multiplexing as long as there is no interference. We suppose that the RM centralizes the statistical CSI of the links to perform the RA. We assume for each link a multipath channel, which is constant within one OFDMA symbol and varies independently from symbol to symbol. Hence, similarly to [13], the received signal on link ℓ on the nth subcarrier at OFDMA symbol i writes as

$$Y_\ell(i,n) = H_\ell(i,n)X_\ell(i,n) + Z_\ell(i,n), \tag{1}$$

where, $H_\ell(i,n) \sim \mathcal{CN}(a_\ell, \zeta_\ell^2)$ where $\mathcal{N}(a_\ell, \zeta_\ell^2)$ stands for the complex Gaussian distribution with mean a_ℓ and variance ζ_ℓ^2, $X_\ell(i,n)$ is the transmitted symbol on the nth subcarrier of the ith OFDMA symbol and $Z_\ell(i,n) \sim \mathcal{CN}(0, N_0 B/N_c)$, with N_0 the noise level in the power spectral density. We can define the average gain-to-noise ratio (GNR) G_ℓ and Rician K-factor K_ℓ of the ℓth link as

$$G_\ell := \frac{\mathbb{E}[|H_\ell(i,n)|^2]}{N_0} = \frac{\Omega_\ell}{N_0}, \tag{2}$$

$$K_\ell := \frac{|a_\ell|^2}{\zeta_\ell^2}, \tag{3}$$

with $\Omega_\ell := |a_\ell|^2 + \zeta_\ell^2$. Notice that $K_\ell = 0$ corresponds to the Rayleigh channel while $K_\ell \to +\infty$ corresponds to the AWGN channel. It is assumed that the RM only knows the average GNR and the Rician K-factor of each link to perform the RA. Moreover, we suppose the use of both time and frequency interleavers such that each modulated symbol experiences independent channel realization i.e. the channel can be seen as fast fading.

We assume that, at the medium access (MAC) layer, each link uses a type-I HARQ scheme. The information bits are grouped into packets of $\mathcal{L}_\ell$ bits, which are encoded by a FEC with rate R_ℓ to obtain the MAC packets. A MAC packet is sent on the channel at most $\mathcal{M}$ times. At the received side, after decoding the mth received packet, the information bits are checked using a cyclic redundancy check (CRC) which is assumed to be error free. An acknowledgement (ACK) is sent if the information bits are correctly decoded, while otherwise a negative ACK (NACK) is sent. Since type-I HARQ is considered, the MAC packets received in error are discarded.

B. Energy consumption model

On link ℓ, a quadrature amplitude modulation (QAM) with m_ℓ bits per symbol is used. Let n_ℓ be the number of subcarriers allocated to the ℓth link, and let us define $\gamma_\ell := n_\ell/N_c$ as the proportion of bandwidth allocated to this link. Because only statistical CSI is available, the same power is used on all the subcarriers, and we define $P_\ell := \mathbb{E}[|X_\ell(j,n)|^2]$ as the transmit power allocated to the ℓth link.

To send and receive a packet, the power expenditure is composed of the transmit power and the circuitry consumption, leading to the following expression for the power consumption of the ℓth link:

$$P_{T,\ell} := N_c \gamma_k P_\ell \kappa_\ell^{-1} + P_{ctx,\ell} + P_{crx,\ell}, \tag{4}$$

with $\kappa_\ell \leq 1$ the efficiency of the power amplifier and $P_{ctx,\ell}$ (resp. $P_{crx,k}$) the circuitry power consumption of the transmitter (resp. the receiver).

C. Energy efficiency

The energy efficiency of the ℓth link is the ratio between its goodput η_ℓ, i.e. the number of informations bits that can be transmitted without error per second, and its power consumption which writes

$$\mathcal{E}_\ell := \frac{\eta_\ell[\text{bits/s}]}{P_{T,\ell}[\text{W}]}. \tag{5}$$

For type-I HARQ and when the channel has no correlation between the HARQ rounds, the goodput is given by [14]:

$$\eta_\ell = B\alpha_\ell\gamma_\ell(1 - q_\ell(G_\ell E_\ell)), \tag{6}$$

where $q_\ell(G_\ell E_\ell)$ is the packet error rate (PER) on the ℓth link, $E_\ell := N_c P_k/B$ and $\alpha_\ell := m_\ell R_\ell$. Notice that because type-I HARQ is considered on block fading channel, (6) is independent of $\mathcal{M}$. By plugging (6) and (4) into (5), we can express the EE of the ℓth link as:

$$\mathcal{E}_\ell(E_\ell, \gamma_\ell) = \frac{1 - q_\ell(G_\ell E_\ell)}{A_\ell E_\ell + C_\ell \gamma_\ell^{-1}}, \tag{7}$$

with $A_\ell := \kappa_\ell^{-1}/\alpha_\ell$ and $C_\ell := (P_{ctx,\ell} + P_{crx,\ell})/(B\alpha_\ell)$.

D. Assumptions on the packet error rate

We now make several assumptions on the PER $q_\ell(x)$.

Assumption 1. $q_\ell(x)$ *is strictly decreasing and strictly convex.*

Assumption 2. $\lim_{x\to\infty} q_\ell(x) = 0$ *and* $\lim_{x\to\infty} q'_\ell(x) = 0$.

One can check that these assumptions hold for the PER approximation provided in [13] for the Rician fast fading channel.

E. Considered constraints

In this paper, we consider a quality of service (QoS) constraint by imposing a target per-link minimum goodput constraint $\eta_\ell^{(t)}$, which writes as:

$$B\alpha_\ell\gamma_\ell(1 - q_\ell(G_\ell E_\ell)) \geq \eta_\ell^{(t)}. \tag{8}$$

In order to be able to apply convex optimization tools in the next section, we now rewrite constraint (8) in the following convex manner:

$$\eta_\ell^{(0)}\gamma_\ell^{-1} + q_\ell(G_\ell E_\ell) - 1 \leq 0. \tag{9}$$

with $\eta_\ell^{(0)} := \eta_\ell^{(t)}/(B\alpha_\ell)$. Also, from the definition of the bandwidth parameter, the following inequality has to hold:

$$\sum_{\ell=1}^{L} \gamma_\ell \leq 1. \tag{10}$$

F. Problems formulation

Our objective is to solve the MSEE problem with per-link minimum goodput constraint, which writes as

Problem 1. *The MSEE problem writes*

$$\max_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} \mathcal{E}_\ell(E_\ell, \gamma_\ell), \tag{11}$$

$$\text{s.t.} \quad \eta_\ell^{(0)}\gamma_\ell^{-1} + q_\ell(G_\ell E_\ell) - 1 \leq 0, \quad \forall \ell, \tag{12}$$

$$\sum_{\ell=1}^{L} \gamma_\ell \leq 1, \tag{13}$$

where $\mathbf{E} := [E_1, \ldots, E_L]$, $\boldsymbol{\gamma} := [\gamma_1, \ldots, \gamma_L]$ are the resources that we want to allocate to the links.

III. SEE MAXIMIZATION

We can prove the Problem 1 is the maximization of a sum of ratios with concave numerators and convex denominators over a convex set. In [15], an efficient approach to solve this type of problem is proposed. This iterative approach is successfully used in the RA context for instance in [16]. At the ith iteration, it requires to solve the following problem, which is the minimization of a convex function over a convex set:

Problem 2.

$$\min_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} u_\ell^{(i)} q_\ell(G_\ell E_\ell) + u_\ell^{(i)}\beta_\ell^{(i)}(A_\ell E_\ell + C_\ell \gamma_\ell^{-1}), \tag{14}$$

$$\text{s.t.} \quad (12), (13) \tag{15}$$

with, for all ℓ, $u_\ell^{(i)} \geq 0$ and $\beta_\ell^{(i)} \geq 0$ depend on the optimal solution of the $(i-1)$th iteration, as detailed later. The Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient to find the global optimal solution of Problem 2 [17]. The rest of this section is devoted to the solution of these conditions. Defining $\boldsymbol{\delta} := [\delta_1, \ldots, \delta_L]$ and λ as the Lagrangian multipliers associated with constraints (12) and (13), respectively, the KKT conditions of Problem 2 write for all ℓ as:

$$G_\ell q'_\ell(G_\ell E_\ell)(u_\ell^{(i)} + \delta_\ell) + u_\ell^{(i)}\beta_\ell^{(i)} A_\ell = 0, \tag{16}$$

$$-u_\ell^{(i)}\beta_\ell^{(i)} C_\ell \gamma_\ell^{-2} + \lambda - \delta_\ell \eta_\ell^{(0)} \gamma_\ell^{-2} = 0. \tag{17}$$

Also, the following complementary slackness conditions hold at the optimum:

$$\delta_\ell(\eta_\ell^{(0)}\gamma_\ell^{-1} - 1 + q_\ell(G_\ell E_\ell)) = 0, \tag{18}$$

$$\lambda(\sum_{\ell=1}^{L} \gamma_\ell - 1) = 0. \tag{19}$$

To solve the optimality conditions (16)-(19), our approach is in two steps: first, we assume the value of λ as known and we exhibit the optimal solution as a function of this multiplier. Second, we search for the optimal value of this multiplier.

1) Solution for fixed λ: There are two possible cases: either $\delta_\ell = 0$, or $\delta_\ell > 0$. Let us define I_λ (resp. $\bar{I}_\lambda$) as the set of links with $\delta_\ell = 0$ (resp. $\delta_\ell > 0$). In the following, we find the optimal values of E_ℓ and γ_ℓ for the links in these sets, and we give a characterization enabling to check whether a link belongs to I_λ or $\bar{I}_\lambda$.

Case 1: $\delta_\ell = 0$. We obtain the optimal value of E_ℓ, denoted by $E^*_{\ell,1}$, from (16) and the optimal value of γ_ℓ, denoted by $\gamma^*_{\ell,1}(\lambda)$, from (17) as:

$$E^*_{\ell,1} = q'^{-1}_\ell(-\beta_\ell^{(i)} A_\ell G_\ell^{-1}) G_\ell^{-1}, \tag{20}$$

$$\gamma^*_{\ell,1}(\lambda) = (u_\ell^{(i)}\beta_\ell^{(i)} C_\ell \lambda^{-1})^{1/2}, \tag{21}$$

where q'^{-1}_ℓ is the inverse of q'_ℓ with respect to the composition (i.e. $q'^{-1}_\ell(q'_\ell(x)) = 1$), which exists and is strictly increasing due to the strict convexity of q_ℓ induced by Assumption 1.

Also, we can prove that a link is in I_λ iff the following inequality holds

$$\lambda \leq \lambda_\ell^T, \tag{22}$$

with $\lambda_\ell^T := u_\ell \beta_\ell C_\ell (\eta_\ell^{(0)})^{-2}(1 - q_\ell(G_\ell E_\ell^T))^2$, with $E_\ell^T := G_\ell^{-1} q'^{-1}_\ell(-\beta_\ell A_\ell G_\ell^{-1})$. The direct part of the previous statement is obtained because a link in I_λ has to satisfy the goodput constraint, and hence plugging (20) and (21) into (12) yields λ_ℓ^T. The converse is obtained by contradiction: assuming a link ℓ in $\bar{I}_\lambda$ and assuming that (22) holds, we prove that the condition (18) cannot hold for ℓ with $\delta_\ell > 0$. Hence, the links belonging to I_λ satisfy inequality (22) with the corresponding optimal parameters given by (20) and (21).

Case 2: $\delta_\ell > 0$. To find the optimal value of E_ℓ (resp. γ_ℓ) denoted by $E^*_{\ell,2}(\lambda)$ (resp. $\gamma^*_{\ell,2}(\lambda)$) for the links in $\bar{I}_\lambda$, first, (18) yields:

$$\gamma^*_{\ell,2}(\lambda) = \frac{\eta_\ell^{(0)}}{1 - q_\ell(G_\ell E^*_{\ell,2}(\lambda))}. \tag{23}$$

Then, using (23), we solve the KKT conditions (16) and (17). From (17), we obtain $\delta_\ell = (\lambda \gamma^{*2}_{\ell,2}(\lambda) - u_\ell^{(i)}\beta_\ell^{(i)} C_\ell)/\eta_\ell^{(0)}$ which we plug into (16), yielding, using (23):

$$h_{\ell,\lambda}(G_\ell E^*_{\ell,2}(\lambda)) = 0, \tag{24}$$

with $h_{\ell,\lambda}(x) := G_\ell q'_\ell(x)(u_\ell - u_\ell^{(i)}\beta_\ell^{(i)} C_\ell/\eta_\ell^{(0)} + \lambda\eta_\ell^{(0)}/(1 - q_\ell(x))^2) + u_\ell^{(i)}\beta_\ell^{(i)} A_\ell$. If a solution of (24) exists and is unique, then it is the optimal solution for given λ. We can prove that we are only interested in the solution of (24) in $[E_\ell^T, +\infty)$. We hence prove the unicity of such a solution in the next lemma.

Lemma 1. *For all the links in $\bar{I}_\lambda$, $h_{\ell,\lambda}(G_\ell x)$ has a unique root in $[E_\ell^T, +\infty)$.*

Proof: Due to space limitations, we only give a sketch of the proof. The proof is four steps: first, we check that $h_{\ell,\lambda}(G_\ell E_\ell^T) \leq 0$. Second, by computing the first order derivative of $h_{\ell,\lambda}$, it can be established that this function is strictly increasing until some $x_{\ell,\lambda}$, with possibly $x_{\ell,\lambda} = +\infty$. Third, we check that $h_{\ell,\lambda}(x_{\ell,\lambda}) > 0$. Finally, we end the proof by checking that, $\forall x > x_{\ell,\lambda}$, $h_{\ell,\lambda}(x) > 0$. ■

From Lemma 1 we deduce that we can find the unique value of $E^*_{\ell,2}(\lambda)$ solving (24) using the bisection method. Also, $\gamma^*_{\ell,2}(\lambda)$ can be found using (23). Hence, we found the optimal values of E_ℓ and γ_ℓ for all $\lambda > 0$.

2) Search for the optimal value of λ: To find λ^*, the optimal value of λ, first, from (17), we deduce that $\lambda^* > 0$. Second, we use the complementary slackness condition (19). To this end, let us define the following function representing the sum of the bandwidth parameter for given value Λ of the Lagrangian multiplier associated with constraint (13):

$$\Gamma : \Lambda \mapsto \sum_{\ell \in I_\Lambda} \gamma^*_{\ell,1}(\Lambda) + \sum_{\ell \in \bar{I}_\Lambda} \gamma^*_{\ell,2}(\Lambda) \tag{25}$$

Due to (19), λ^* is such that $\Gamma(\lambda^*) = 1$. A property of Γ is given in the next lemma.

Lemma 2. *The function $\Gamma(\Lambda)$ defined in (25) is continuous and non increasing on $\mathbb{R}^{+*}$ and $\exists \lambda^*$ such that $\Gamma(\lambda^*) = 1$.*

Proof: The proof is given in the Appendix. ■

Hence, λ^* can be found by a linesearch method. Finally, the optimal solution of Problem 1 can be found by alternating the following two steps until convergence to its global optimal solution [16].

1) Find the optimal solution of Problem 2 by solving $\Gamma(\lambda^*) = 1$, and then by computing the optimal values of E_ℓ and γ_ℓ using (20)-(21) for the links in I_{λ^*}, and by solving $h_{\ell,\lambda^*}(G_\ell E_\ell) = 0$ and using (23) for the links in $\bar{I}_{\lambda^*}$.
2) For all ℓ, update $u^{(i)}_\ell$ and $\beta^{(i)}_\ell$ based on the modified Newton method detailed in [16, Eqs. (33)-(34)].

IV. Numerical Examples

We use the convolutional code with generator polynomial $[171, 133]_8$, and we use the quadrature phase shift keying (QPSK) modulation, i.e. $\forall \ell$, $m_\ell = 2$. The number of links is $L = 5$ and the link distances D_ℓ are uniformly drawn in $[50 \text{ m}, 1 \text{ km}]$. We set $B = 5$ MHz, $N_0 = -170$ dBm/Hz and $\mathcal{L}_\ell = 128$. The carrier frequency is $f_c = 2400$ MHz and we put $\zeta^2_\ell = (4\pi f_c/c)^{-2} D_\ell^{-3}$ where c is the celerity of light in vacuum. We assume that the required goodput per-link is equal for all links. We put $\forall \ell$, $P_{ctx,\ell} = P_{crx,\ell} = 0.05$ W and $\kappa_\ell = 0.5$. All the results are averaged over 50 networks realizations.

First, we consider two scenarios: in the first one, all the links are Rayleigh distributed (i.e. $K_\ell = 0$ for all ℓ), and in the second one all the links are Rician distributed with $K_\ell = 10$. In Fig. 1, we plot the SEE of the MSEE and the SEE obtained from the minimum power (MPO) criterion, obtained from [13], versus the minimum required goodput. As expected: the MSEE gives much higher SEE than the MPO. Also, we can see that the SEE obtained on the Rician channel is higher than the one on the Rayleigh channel.

Second, we define $\mathbf{E}^*_R$ and $\boldsymbol{\gamma}^*_R$ as the optimal values of $\mathbf{E}$ and $\boldsymbol{\gamma}$ when the Rician channel is considered, respectively. We also define $\mathbf{E}^*_C$ and $\boldsymbol{\gamma}^*_C$ as the optimal values of $\mathbf{E}$ and $\boldsymbol{\gamma}$ when considering the Rayleigh channel, respectively. In

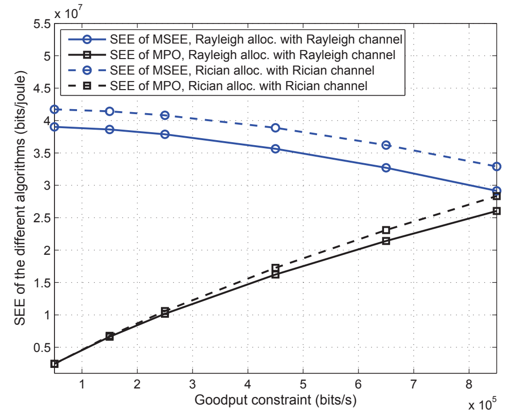

Fig. 1: SEE obtained with the proposed algorithms versus the per-link minimum goodput constraint.

Fig. 2, we plot the SEE gains between the Rician and the Rayleighs allocations under the Rician channel by computing $100 \times (\sum_{\ell=1}^{L} \mathcal{E}_\ell(\mathbf{E}^*_R, \boldsymbol{\gamma}^*_R) / (\sum_{\ell=1}^{L} \mathcal{E}_\ell(\mathbf{E}^*_C, \boldsymbol{\gamma}^*_C)) - 1)$, versus the minimum goodput constraint. We observe that the SEE gain increases with the minimum goodput constraint, and that for $\eta^{(t)}_\ell = 8.5 \times 10^5$ bits/s, we can achieve a SEE gain of about 11%, which is interesting since considering the Rician channel does not induce additional complexity as compared with considering the conventional Rayleigh channel. We also compute the average EE gains per-link for this value of $\eta^{(t)}_\ell$, and this gain is about 19%, meaning that, for a fixed number of information bits to transmit, about 16% energy can be saved in average when considering the Rician channel instead of the Rayleigh one, which is equivalent to say that the energy consumed when considering the Rayleigh channel is about 19% higher than when considering the Rician one.

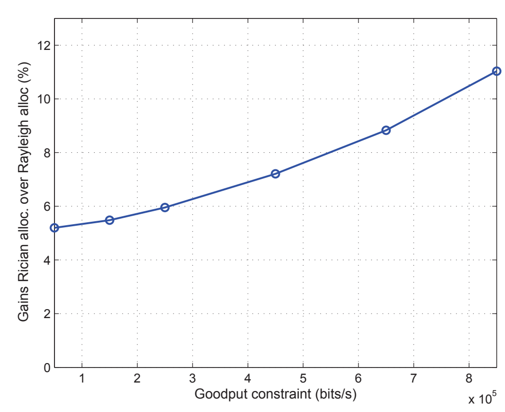

Fig. 2: SEE gains between the Rician and the Rayleighs allocations under the Rician channel, versus the per-link minimum goodput constraint.

V. Conclusion

In this paper, we addressed the problem of joint bandwidth and power allocation for type-I HARQ when practical MCS are used along with statistical CSI. We optimally solved the MSEE problem when the Rician channel is assumed. Through simulations, we exhibited the interest of taking into account the distribution of the channel and especially of taking into account the existence of a LoS between the transmitter and the receiver instead of only using the channel variance, given by the pathloss.

APPENDIX

Here, we prove Lemma 2. Let us define k'_m a one-to-one mapping from $\{1, \ldots, L\}$ in itself such that $\lambda^T_{k'_1} \leq \cdots \leq \lambda^T_{k'_L}$ where $\lambda^T_{k'_i}$ is defined in (22). Also, let us rewrite Γ in a more convenient form

$$\Gamma(\Lambda) = \sum_{\ell \in I_\Lambda} \sqrt{\frac{u_\ell C_\ell \beta_\ell}{\Lambda}} + \sum_{\ell \in \bar{I}_\Lambda} \gamma^*_{\ell,2}(\Lambda) \tag{26}$$

where, for all the links in $\bar{I}_\Lambda$, $\gamma^*_{\ell,2}$ is given in (23) and $E^*_{\ell,2}(\Lambda)$ is the unique root of $h_{\ell,\Lambda}(G_\ell E^*_{\ell,2}(\Lambda))$. We remind that a link ℓ is in I_λ iff $\Lambda < \lambda^T_\ell$. To prove Lemma 2, we first prove that Γ is continuous and non increasing on every open set $(\lambda^T_{k'_i}, \lambda^T_{k'_{i+1}})$, second we prove that Γ is continuous in every $\lambda^T_{k'_i}$ and finally we prove that for sufficiently large Λ, $\Gamma(\Lambda) \leq 1$. We can see that the first term in the right hand side (RHS) of (26) is continuous and strictly decreasing on $(\lambda^T_{k'_i}, \lambda^T_{k'_{i+1}})$. Let us now focus on the second term of the RHS. To ensure the desired property, we prove in the following that $E^*_{\ell,2}(\Lambda)$ defined as the unique root of (24), which is continuous in Λ, is also decreasing in Λ, and we use (23). To study the variation of $E^*_{\ell,2}(\Lambda)$ as a function of Λ, let us define without loss of generality $\lambda^T_\ell < \lambda_1 < \lambda_2$, and let us evaluate h_{ℓ,λ_2} in $E^*_{\ell,2}(\lambda_1)$:

$$h_{\ell,\lambda_2}(E^*_{\ell,2}(\lambda_1)) = q'_\ell(G_\ell E^*_{\ell,2}(\lambda_1)) \frac{(\lambda_2 - \lambda_1)\eta^{(0)}_\ell}{(1 - q_\ell(G_\ell E^*_{\ell,2}(\lambda_1))^2} < 0 \tag{27}$$

From (27) and because of the result from Lemma 1, we can deduce that $E^*_{\ell,2}(\lambda_2)$, the unique root of $h_{\ell,\lambda_2}(G_\ell E^*_{\ell,2}(\lambda_2))$, is such that $E^*_{\ell,2}(\lambda_2) \geq E^*_{\ell,2}(\lambda_1)$. Hence, $E^*_{\ell,2}(\Lambda)$ is a non decreasing function of Λ, and as a consequence, due to (23), $\gamma^*_{\ell,2}(\Lambda)$ is a non increasing function of Λ. We have hence proved that Γ is continuous and decreasing on every open set $(\lambda^T_{k'_i}, \lambda^T_{k'_{i+1}})$.

Second, we check that Γ is continuous in $\lambda^T_{k'_i}$. To do so, one can show that $\lim_{\Lambda \nearrow \lambda^T_{k'_i}} \gamma^*_{\ell,1}(\Lambda) = \lim_{\Lambda \searrow \lambda^T_{k'_i}} \gamma^*_{\ell,2}(\Lambda)$. Putting all pieces together, it results that Γ is continuous and non increasing on $\mathbb{R}^{+*}$.

Finally, by letting Λ be sufficiently large, one can show that $E^*_{\ell,2}(\Lambda)$ goes to the infinity, which means that $\gamma^*_{\ell,2}(\Lambda)$ goes to $\eta^{(0)}_\ell$, and we have $\sum_{\ell=1}^L \eta^{(0)}_\ell \leq 1$ (otherwise the problem would be infeasible), hence it exists λ^* such that $\Gamma(\lambda^*) = 1$, which concludes the proof.

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