

# A NEARLY OPTIMAL RESOURCE ALLOCATION ALGORITHM FOR THE DOWNLINK OF OFDMA 2-D NETWORKS WITH MULTICELL INTERFERENCE

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## ABSTRACT

In this paper, we address the problem of power control and sub-carrier assignment for the downlink of a sectorized two-dimensional (2-D) OFDMA cellular network assuming statistical Channel State Information (CSI) and *fractional frequency reuse*. The latter reuse scheme has been recommended for several cellular systems such as WiMAX. In this context, we provide a resource allocation algorithm with low computational complexity that can be implemented in a distributed fashion without the intervention of any central controller. The performance of this allocation scheme is analyzed assuming fast fading Rayleigh channels and Gaussian distributed multicell interference. Interestingly, it is shown that the proposed algorithm is asymptotically equivalent to an optimal resource allocation *i.e.*, its total transmit power is equal in the limit of large numbers of users to the transmit power associated with a global solution to the joint resource allocation problem.

## 1. INTRODUCTION

The problem of OFDMA resource allocation has lately gained considerable research interest since the adoption of this multiple access scheme in a number of current and future wireless standards such as IEEE 802.16e specifications for wireless metropolitan area networks (WiMax) and Third Generation Partnership Project - Long Term Evolution (3GPP-LTE). In principle, performing resource allocation for OFDMA cellular systems requires to solve the problem of transmit power control and subcarrier assignment jointly in all the cells composing the system. Unfortunately, this optimization problem is difficult to solve in most of the practical situations. As a consequence, most of the related works in the literature focus on the single cell case (*e.g.*, [1, 2, 3]). As for the fewer works that address the more involved multicell allocation problem, we cite [4, 5] in the case of perfect CSI at the transmitters side, and [6] in the case of imperfect CSI. In the latter work, *all* the available subcarriers are likely to be used by different base stations and are thus subject to multicell interference. In such a configuration, interference may reach excessive levels, especially for users located at cells borders.

Similarly to [6], we assume in this paper that the the CSI at the base station is limited to some channel statistics. However, contrary to this work, we consider that a certain subset of subcarriers is shared orthogonally between the adjacent base stations (and is thus “protected” from multicell interference) while the remaining subcarriers are “non protected” since they are reused by different base stations. This so-called *fractional frequency reuse* is recommended in a number of standards *e.g.*, in [7] for IEEE 802.16 (WiMax). In this context, we investigate the problem of power control and subcarrier assignment for the downlink of OFDMA systems allowing to satisfy

all users’ rate requirements while spending the least power possible at the transmitters’ side. In our previous work [8, 9], the solution to this problem is characterized in the special case of one-dimensional (1-D) cellular networks where all users and base stations are located on a line. In [10], we identify the *asymptotic* behaviour of the solution to the above resource allocation problem in the more realistic 2-D setting as the number of users in each cell tends to infinity. In the present work, our aim is to propose suboptimal resource allocation strategies for OFDMA 2-D networks and to study their performance with respect to the the above optimization problem. Our proposed allocation algorithm is presented in Section 4. This algorithm assumes that users of each cell are divided prior to resource allocation into two groups separated by a fixed curve. The first group is composed of closer users to the base station who are constrained to modulate non protected subcarriers from the subset of reused subcarriers. The second group comprises the farthest users who are constrained to modulate protected subcarriers. The relevant choice of the curves that separate the two groups of users in each cell is addressed in Section 5. Theorem 1 states that these separating curves can be chosen in such a way that the limit of the transmit power of the proposed *sub-optimal* algorithm as the number of users grows to infinity is equal to the limit of the transmit power of an *optimal* solution to the resource allocation problem. Finally, our results are sustained by the simulations carried out in Section 6.

## 2. SYSTEM MODEL

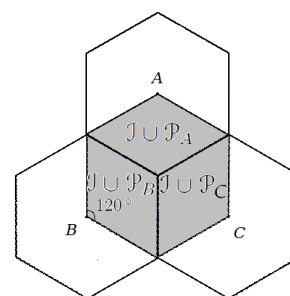


Fig. 1. 3-cells system model and the frequency reuse scheme

We consider the downlink of a sectorized OFDMA cellular system composed of hexagonal cells. Each cell in the system is divided into three  $120^\circ$  sectors. Typically, the major part of interference undergone by a user is generated by the nearest interfering base station. This is due to the decay profile with respect to distance of the

path loss function. It is thus reasonable to assume that interference generated by farther base stations can be neglected. We focus therefore on three interfering sectors of three adjacent cells, say cells  $A$ ,  $B$  and  $C$ , as illustrated by Figure 1. The three cells have the same surface. We denote by  $K^A$ ,  $K^B$  and  $K^C$  the number of users in the considered sector of cell  $A$ ,  $B$  and  $C$  respectively. We denote by  $K = K^A + K^B + K^C$  the total number of users. The total number of available subcarriers is denoted by  $N$ . The signal received by user  $k$  in cell  $c$  ( $c \in \{A, B, C\}$ ) at subcarrier  $n \in \{0, 1, \dots, N-1\}$  during the  $m$ th OFDM block is given by

$$y_k(n, m) = H_k^c(n, m)s_k(n, m) + w_k(n, m), \quad (1)$$

where  $s_k(n, m)$  represents the data symbol destined to user  $k$ , and where  $w_k(n, m)$  is a random process that encompasses both the thermal noise and the possible multicell interference. Random variable  $H_k^c(n, m)$  stands for the frequency-domain channel coefficient associated with user  $k$  at the  $n$ th subcarrier and the  $m$ th OFDM block. The realizations of this random variable are assumed to be only known at the receiver side and unknown at the base station. Random variables  $\{H_k^c(n, m)\}_{n,m}$  are identically distributed w.r.t  $n$  and  $m$  and are Rayleigh distributed with variance  $\rho_k = \mathbb{E}[|H_k^c(m, n)|^2]$  which is assumed constant w.r.t  $n$  and  $m$ . This holds for example in the case of decorrelated Gaussian distributed time-domain channel coefficients. Furthermore, for each  $n \in \{0 \dots N-1\}$ , random process  $\{H_k^c(n, m)\}_m$  is assumed to be ergodic. Finally, variance  $\rho_k$  is assumed to be known at the transmitter side and **vanishes with the distance between base station  $c$  and user  $k$  following a given path loss model**. As already stated, we assume that fractional frequency reuse is applied. According to this scheme (which is illustrated in Figure 1), a certain subset of subcarriers  $\mathcal{I} \subset \{0, 1, \dots, N-1\}$  is reused in the three cells. If user  $k$  modulates such a subcarrier  $n \in \mathcal{I}$ , the process  $w_k(n, m)$  represents both thermal noise of variance  $\sigma^2$  and interference. Its variance is denoted in this case by  $\sigma_k^2$  and is assumed to be constant w.r.t both  $n$  and  $m$  as will be made clear later on. The reuse factor  $\alpha$  is the ratio between the number of reused subcarriers and the total number of subcarriers:

$$\alpha = \frac{\text{card}(\mathcal{I})}{N}.$$

The remaining  $(1 - \alpha)N$  subcarriers are shared by the three sectors in an orthogonal way, such that each base station  $c$  ( $c = A, B, C$ ) has at its disposal a subset  $\mathcal{P}_c$  of cardinality  $\frac{1-\alpha}{3}N$ . If user  $k$  modulates a subcarrier  $n \in \mathcal{P}_c$ , then process  $w_k(n, m)$  will contain only thermal noise with variance  $\sigma^2$ . Finally,  $\mathcal{I} \cup \mathcal{P}_A \cup \mathcal{P}_B \cup \mathcal{P}_C = \{0, 1, \dots, N-1\}$ . Denote by  $\mathcal{N}_k \subset \{0 \dots N-1\}$  the subset of subcarriers assigned to user  $k$ . We assume that this subset may contain subcarriers from both the ‘‘interference’’ subset  $\mathcal{I}$  and the ‘‘protected’’ subset  $\mathcal{P}_c$ . We denote by  $\gamma_{k,1}^c N$  (resp.  $\gamma_{k,2}^c N$ ) the number of subcarriers modulated by user  $k$  in  $\mathcal{I}$  (resp.  $\mathcal{P}_c$ ). In other words,

$$\gamma_{k,1}^c = \text{card}(\mathcal{I} \cap \mathcal{N}_k)/N \quad \gamma_{k,2}^c = \text{card}(\mathcal{P}_c \cap \mathcal{N}_k)/N.$$

Note that by definition of  $\gamma_{k,1}^c$  and  $\gamma_{k,2}^c$ , these so-called sharing factors should be selected such that

$$\sum_{k=1}^{K^c} \gamma_{k,1}^c \leq \alpha, \quad \sum_{k=1}^{K^c} \gamma_{k,2}^c \leq \frac{1-\alpha}{3}.$$

Define  $P_{k,1}^c$  (resp.  $P_{k,2}^c$ ) as the power transmitted on the subcarriers assigned to user  $k$  in  $\mathcal{I}$  (resp. in  $\mathcal{P}_c$ ) i.e.,  $P_{k,1}^c = E[|s_k(n, m)|^2]$  if  $n \in \mathcal{I}$ ,  $P_{k,2}^c = E[|s_k(n, m)|^2]$  if  $n \in \mathcal{P}_c$ . Denote by  $W_{k,1}^c = \gamma_{k,1}^c P_{k,1}^c$  (resp.  $W_{k,2}^c = \gamma_{k,2}^c P_{k,2}^c$ ) the average power transmitted to

user  $k$  in  $\mathcal{I}$  (resp.  $\mathcal{P}_c$ ). Parameters  $\{\gamma_{k,i}^c, P_{k,i}^c\}_{i=1,2}$  will be designated in the sequel as the *resource allocation parameters*. We now describe the adopted model for the multicell interference. Consider one of the subcarriers  $n$  assigned to user  $k$  of cell  $A$  in the non protected subset  $\mathcal{I}$ . We assume that the variance  $\sigma_k^2$  of the additive noise process  $w_k(n, m)$  depends only on the position of user  $k$  and the average powers  $Q_1^B = \sum_{k=1}^{K^B} W_{k,1}^B$  and  $Q_1^C = \sum_{k=1}^{K^C} W_{k,1}^C$  transmitted respectively by base stations  $B$  and  $C$  in  $\mathcal{I}$ . This assumption is valid for instance in OFDMA systems that adopt random subcarrier assignment. Putting all pieces together:

$$\mathbb{E}[|w_k(n, m)|^2] = \begin{cases} \sigma^2 & \text{if } n \in \mathcal{P}_c \\ \sigma_k^2 = \sigma^2 + \sum_{\tilde{c}=B,C} \mathbb{E}[|H_k^{\tilde{c}}(n, m)|^2] Q_1^{\tilde{c}} & \text{if } n \in \mathcal{I} \end{cases} \quad (2)$$

where  $H_k^{\tilde{c}}(n, m)$  ( $\tilde{c} = B, C$ ) represents the channel between base station  $\tilde{c}$  and user  $k$  of cell  $A$  at subcarrier  $n$  and OFDM block  $m$ . Of course, the average channel gain  $\mathbb{E}[|H_k^{\tilde{c}}(n, m)|^2]$  depends on the position of user  $k$  via the path loss model. Thus, if two users  $k$  and  $l$  of cell  $A$  are located on the same line perpendicular to the axis  $BC$  such that  $k$  is closer to base station  $A$ , then  $\sigma_k^2 \leq \sigma_l^2$ .

### 3. JOINT RESOURCE ALLOCATION PROBLEM

Assume that each user  $k$  has a rate requirement of  $R_k$  nats/s/Hz. Consider the problem of determination of the resource allocation parameters for the three interfering sectors. This parameters must be selected such that the target rate of each user is satisfied and such that the power spent by the three base stations is minimized. Due to the ergodicity of the process  $\{H_k(n, m)\}_m$  for each subcarrier  $n$ , the rate  $R_k$  can be satisfied provided that it is smaller than the ergodic capacity  $C_k$  associated with user  $k$ . Unfortunately, the exact expression of  $C_k$  is difficult to obtain due to the fact that the noise-plus-interference  $\{w_k(n, m)\}_{n,m}$  is not a Gaussian process in general. Nonetheless, if we endow the input symbols  $s_k(n, m)$  with Gaussian distribution, the mutual information between  $s_k(n, m)$  and the received signal  $y_k(n, m)$  in (1) is at its minimum when  $\{w_k(n, m)\}$  is Gaussian distributed. Therefore, we adopt in the sequel the approximation of the multicell interference by a Gaussian process as it provides a lower bound on the mutual information. We now focus on cell  $A$ . We denote by  $g_{k,1}(Q_1^B, Q_1^C)$  and  $g_{k,2}$  the channel Gain-to-Interference-plus-Noise Ratio and Gain-to-Noise Ratio on the subcarriers of the subset  $\mathcal{I}$  (resp.  $\mathcal{P}_A$ ) associated with user  $k$  i.e.,

$$g_{k,1}(Q_1^B, Q_1^C) = \frac{\rho_k}{\sigma_k^2}, \quad g_{k,2} = \frac{\rho_k}{\sigma^2}.$$

Capacity  $C_k = C_k(\gamma_{k,1}^A, \gamma_{k,2}^A, W_{k,1}^A, W_{k,2}^A, Q_1^B, Q_1^C)$  is given by

$$C_k = \gamma_{k,1}^A \mathbb{E} \left[ \log \left( 1 + g_{k,1}(Q_1^B, Q_1^C) \frac{W_{k,1}^A}{\gamma_{k,1}^A} Z \right) \right] + \gamma_{k,2}^A \mathbb{E} \left[ \log \left( 1 + g_{k,2} \frac{W_{k,2}^A}{\gamma_{k,2}^A} Z \right) \right]. \quad (3)$$

Here,  $Z$  stands for an exponentially distributed random variable. The multicell resource allocation problem can now be defined as follows.

**Problem 1** Minimize the power spent by the three base stations  $Q =$

$$\sum_{c=A,B,C} \sum_{k=1}^{K^c} (W_{k,1}^c + W_{k,2}^c) \text{ w.r.t } \{\gamma_{k,1}^c, \gamma_{k,2}^c, W_{k,1}^c, W_{k,2}^c\}_{c=A,B,C, k=1 \dots K^c}$$

under the following constraints:

$$\begin{aligned} \mathbf{C1} : \forall k, R_k \leq C_k \quad \mathbf{C3} : \sum_{k=1}^{K^c} \gamma_{k,2}^c &= \frac{1-\alpha}{3} \\ \mathbf{C2} : \sum_{k=1}^{K^c} \gamma_{k,1}^c &= \alpha \quad \mathbf{C4} : \forall k, \gamma_{k,i}^c, W_{k,i}^c \geq 0 \quad (i = 1, 2). \end{aligned}$$

Since the ergodic capacity  $C_k$  is not a convex function of the optimization variables, Problem 1 cannot be solved using convex optimization tools. It is therefore of interest to propose practical allocation algorithms that provides suboptimal solutions to this problem.

#### 4. PROPOSED RESOURCE ALLOCATION ALGORITHM

In [10], we showed that under some assumptions that will be made clear in Section 5, any global solution to Problem 1 has the following *asymptotic* property: The power allocated to users of each cell  $c$  who modulates subcarriers from *both* the protected subset  $\mathcal{P}_c$  and the non protected subset  $\mathcal{I}$  becomes negligible as the number  $K$  of users increases. Given this result, one can suggest the suboptimal (w.r.t Problem 1) resource allocation algorithm given below. For a given user  $k$  in cell  $c$ , we denote by  $(x_k, y_k)$  his/her position in the Cartesian coordinate system associated with this cell as illustrated in Figure 2. Let  $d_{\text{subopt}}^c(\cdot)$  be a continuous function defined on  $[-D, D]$  which takes its values in  $[0, D]$  (where  $D$  stands for the radius of the cell as shown in Figure 2). In our proposed algorithm, we use such a function to define a curve that separates the users of cell  $c$  into two subsets. The first subset  $\mathcal{K}_I^c$  contains users of cell  $c$  who are constrained to modulate only in the non protected subset  $\mathcal{I}$  and is defined as follows:

$$\mathcal{K}_I^c = \{k \in \{1 \dots K^c\} \mid y_k \leq d_{\text{subopt}}^c(x_k)\}.$$

The second subset  $\mathcal{K}_P^c$  is composed of the rest of users who are constrained to modulate only in the protected band  $\mathcal{P}_c$ :

$$\mathcal{K}_P^c = \{k \in \{1 \dots K^c\} \mid y_k > d_{\text{subopt}}^c(x_k)\}.$$

Note that curves  $d_{\text{subopt}}^A(\cdot)$ ,  $d_{\text{subopt}}^B(\cdot)$  and  $d_{\text{subopt}}^C(\cdot)$  are fixed in advance prior to resource allocation. Relevant selection of these separating curves is postponed to Section 5.

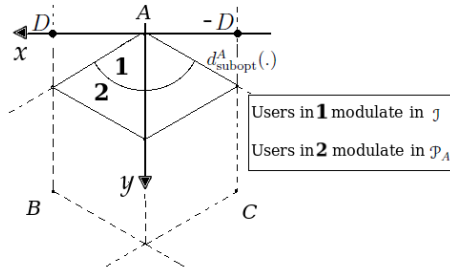


Fig. 2. Fixed separating curve in cell A

##### 4.1. Resource Allocation for Protected Users $\{\mathcal{K}_P^c\}_{c=A,B,C}$

Since users  $\mathcal{K}_P^c$  in each cell  $c$  are constrained to modulate only the subcarriers of subset  $\mathcal{P}_c$ , they are not subject to multicell interference. Resource allocation for such users can thus be done independently in each cell by solving a simple single cell optimization problem. Let us focus on cell A for example. By definition of the subset

$k \in \mathcal{K}_P^A$ , the resource allocation parameters in the non protected subset  $\mathcal{I}$  are arbitrarily set to zero i.e.,  $\gamma_{k,1}^A = P_{k,1}^A = 0$ . Denote by  $C_k(\gamma_{k,2}^A, P_{k,2}^A)$  the ergodic capacity associated with user  $k$  in the protected subset  $\mathcal{P}_A$  i.e.,

$$C_k(\gamma_{k,2}^A, P_{k,2}^A) = \gamma_{k,2}^A \mathbb{E} \left[ \log \left( 1 + g_{k,2} P_{k,2}^A Z \right) \right].$$

Resource allocation parameters  $\gamma_{k,2}^A, P_{k,2}^A$  can be obtained by solving the following classical single cell problem.

**[Single cell problem in band  $\mathcal{P}_c$ ] Minimize the transmit power  $\sum_{k \in \mathcal{K}_P^A} \gamma_{k,2}^A P_{k,2}^A$  of base station A under rate constraint  $R_k \leq C_k(\gamma_{k,2}^A, P_{k,2}^A)$  for each  $k \in \mathcal{K}_P^A$  such that  $\sum_k \gamma_{k,2}^A = \frac{1-\alpha}{3}$ .**

The above problem is convex in variables  $\{\gamma_{k,2}^A, W_{k,2}^A\}_{k \in \mathcal{K}_P^A}$ , where  $W_{k,2}^A = \gamma_{k,2}^A P_{k,2}^A$ . Its solution can be obtained (see [8, 9]) by solving the associated KKT conditions. It is given by:

$$P_{k,2}^A = g_{k,2}^{-1} f^{-1}(g_{k,2} \beta_2^A) \quad (4)$$

$$\gamma_{k,2}^A = \frac{R_k}{C(g_{k,2} \beta_2^A)}, \quad (5)$$

where  $f$  and  $C$  are increasing functions defined on  $\mathbb{R}_+$  by  $f(x) = \mathbb{E}[\log(1+xZ)] / \mathbb{E}[\frac{Z}{1+xZ}] - x$  and  $C(x) = \mathbb{E}[\log(1+f^{-1}(x)Z)]$ . Parameter  $\beta_2^A$  is obtained by writing that constraint  $\sum_k \gamma_{k,2}^A = \frac{1-\alpha}{3}$  holds. In other words,  $\beta_2^A$  is the unique solution to:

$$\sum_{k \in \mathcal{K}_P^A} \frac{R_k}{C(g_{k,2} \beta_2^A)} = \frac{1-\alpha}{3}. \quad (6)$$

Resource allocation parameters for users of cells B and C can be similarly obtained. The following procedure performs the above resource allocation for protected users.

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##### Algorithm 1 Resource allocation for protected users

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for all  $c = A, B, C$  do
   $\beta_2^c \leftarrow$  Solve (6)
  for all  $k \in \mathcal{K}_P^c$  do
     $P_{k,2}^c \leftarrow$  (4)
     $\gamma_{k,2}^c \leftarrow$  (5)
  end for
end for
return  $\{\gamma_{k,2}^c, P_{k,2}^c\}_{c=A,B, k \in \mathcal{K}_P^c}$ 

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##### 4.2. Resource Allocation for Interfering Users $\{\mathcal{K}_I^c\}_{c=A,B,C}$

For users  $\mathcal{K}_I^c$  in each cell  $c$ , resource allocation parameters in the protected subset  $\mathcal{P}_c$  are arbitrarily set to zero i.e.,  $\gamma_{k,2}^c = P_{k,2}^c = 0$ . The remaining resource allocation parameters  $\{\gamma_{k,1}^c, P_{k,1}^c\}_{c=A,B,C, k \in \mathcal{K}_I^c}$  are the solution to the following multicell allocation problem. Recall the definition of  $Q_1^c = \sum_{k \in \mathcal{K}_I^c} \gamma_{k,1}^c P_{k,1}^c$  as the average power transmitted by base station  $c$  ( $c = A, B, C$ ) in the interference subset  $\mathcal{I}$ . For each cell  $c \in \{A, B, C\}$ , denote by  $\bar{c}$  and  $\bar{\bar{c}}$  the other two cells. For example,  $\bar{A} = B$  and  $\bar{\bar{A}} = C$ . Define  $C_k(\gamma_{k,1}^c, P_{k,1}^c, Q_1^{\bar{c}}, Q_1^{\bar{\bar{c}}})$  as the ergodic capacity associated with user  $k$  in subset  $\mathcal{I}$ :

$$C_k(\gamma_{k,1}^c, P_{k,1}^c, Q_1^{\bar{c}}, Q_1^{\bar{\bar{c}}}) = \gamma_{k,1}^c \mathbb{E} \left[ \log \left( 1 + g_{k,1} (Q_1^{\bar{c}}, Q_1^{\bar{\bar{c}}}) P_{k,1}^c Z \right) \right].$$

Resource allocation parameters  $\gamma_{k,1}^c, P_{k,1}^c$  for users in  $\{\mathcal{K}_I^c\}_{c=A,B,C}$  can be obtained by solving the following optimization problem.

**Problem 2 [Multicell problem in band  $\mathcal{I}$ ]** Minimize the total transmit power  $\sum_{c=A,B,C} \sum_{k \in \mathcal{K}_I^c} \gamma_{k,1}^c P_{k,1}^c$  w.r.t.  $\{\gamma_{k,1}^c, P_{k,1}^c\}_{k=1 \dots K^c}$   $c=A,B,C$  under the following constraints:

$$\mathbf{C1} : \forall c, \forall k \in \mathcal{K}_I^c, R_k \leq C_k(\gamma_{k,1}^c, P_{k,1}^c, Q_1^{\bar{c}}, Q_1^{\bar{c}})$$

$$\mathbf{C2} : \forall c, \sum_{k \in \mathcal{K}_I^c} \gamma_{k,1}^c = \alpha \quad \mathbf{C3} : \forall c, \forall k \in \mathcal{K}_I^c, \gamma_{k,1}^c, P_{k,1}^c \geq 0.$$

Note that Problem 2 may not be always feasible. Indeed, since the protected subcarriers are forbidden for users  $\mathcal{K}_I^c$ , the multicell interference may in some cases reach excessive levels and prevent some users from satisfying their rate requirements. Fortunately, we will see that if the predefined separating curves  $d_{\text{subopt}}^A(\cdot)$ ,  $d_{\text{subopt}}^B(\cdot)$  and  $d_{\text{subopt}}^C(\cdot)$  are relevantly chosen, then the latter problem is feasible, at least for sufficiently large numbers of users. Moreover, one can use an approach similar to [8, 9] to show that any global solution to the above problem satisfies the following property. There exist six positive numbers  $\{\beta_1^c, Q_1^c\}_{c=A,B,C}$  such that:

$$P_{k,1}^c = g_{k,1}^{-1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) f^{-1}(g_{k,1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) \beta_1^c) \quad (7)$$

$$\gamma_{k,1}^c = \frac{R_k}{C(g_{k,1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) \beta_1^c)}, \quad (8)$$

where for each  $c = A, B, C$  and for a fixed value of  $Q_1^{\bar{c}}$  and  $Q_1^{\bar{c}}$ ,  $(\beta_1^c, Q_1^c)$  is the unique solution to the following system of equations:

$$\sum_{k \in \mathcal{K}_I^c} \frac{R_k}{C(g_{k,1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) \beta_1^c)} = \alpha, \quad (9)$$

$$Q_1^c = \sum_{k \in \mathcal{K}_I^c} R_k \frac{g_{k,1}^{-1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) f^{-1}(g_{k,1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) \beta_1^c)}{C(g_{k,1}(Q_1^{\bar{c}}, Q_1^{\bar{c}}) \beta_1^c)}. \quad (10)$$

Note that equation (9) is equivalent to the constraint  $\mathbf{C2}$ :  $\sum_k \gamma_{k,1}^c = \alpha$ , while equation (10) is nothing else than the definition of the average power  $Q_1^c = \sum_{k \in \mathcal{K}_I^c} \gamma_{k,1}^c P_{k,1}^c$  transmitted by base station  $c$  in subset  $\mathcal{I}$ . Using arguments already developed in our previous work [9], one can prove that when Problem 2 is feasible, then the system of six equations (9)-(10) for  $c = A, B, C$  admits a unique solution  $\beta_1^A, Q_1^A, \beta_1^B, Q_1^B, \beta_1^C, Q_1^C$  and that this solution can be obtained by Algorithm 2. Of course, the feasibility of Problem 2

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**Algorithm 2** Ping-pong algorithm for three interfering cells

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**Initialization:**  $Q_1^A \leftarrow 0, Q_1^B \leftarrow 0, Q_1^C \leftarrow 0$

**repeat**

$(\beta_1^A, Q_1^A) \leftarrow$  Solve (9)-(10) for  $c = A$

$(\beta_1^B, Q_1^B) \leftarrow$  Solve (9)-(10) for  $c = B$

$(\beta_1^C, Q_1^C) \leftarrow$  Solve (9)-(10) for  $c = C$

**until** convergence

**for all**  $c = A, B, C$  **do**

$\{\gamma_{k,1}^c, P_{k,1}^c\}_{k \in \mathcal{K}_I^c} \leftarrow$  (7)-(8)

**end for**

**return**  $\{\gamma_{k,1}^c, P_{k,1}^c\}_{c=A,B,C, k \in \mathcal{K}_I^c}$

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depends on the choice of the separating curves  $\{d_{\text{subopt}}^c(\cdot)\}_{c=A,B,C}$ . Section 5 addresses the relevant selection of these curves such that Algorithm 2 converges for sufficiently large numbers of users. Finally, note that Algorithm 2 can be implemented in a distributed manner since it only requires exchanging the successive values of  $Q_1^A, Q_1^B$  and  $Q_1^C$  between the three base stations.

### 4.3. Summary: Distributed Resource Allocation Algorithm

The proposed distributed resource allocation scheme for both protected and interfering users can finally be summarized by Algorithm 3.

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**Algorithm 3** Proposed resource allocation algorithm

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**for all**  $c = A, B, C$  **do**

$\mathcal{K}_P^c \leftarrow \{k \in \{1 \dots K^c\} \mid y_k > d_{\text{subopt}}^c(x_k)\}$

$\mathcal{K}_I^c \leftarrow \{k \in \{1 \dots K^c\} \mid y_k \leq d_{\text{subopt}}^c(x_k)\}$

**end for**

$\{\gamma_{k,2}^c, P_{k,2}^c\}_{c=A,B,C, k \in \mathcal{K}_P^c} \leftarrow$  Algorithm 1

$\{\gamma_{k,1}^c, P_{k,1}^c\}_{c=A,B,C, k \in \mathcal{K}_I^c} \leftarrow$  Algorithm 2

**return**  $\{\gamma_{k,1}^c, P_{k,1}^c, \gamma_{k,2}^c, P_{k,2}^c\}_{c=A,B,C, k=1 \dots K^c}$

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## 5. DETERMINATION OF CURVES $\{d_{\text{subopt}}^c(\cdot)\}_{c=A,B,C}$ AND ASYMPTOTIC OPTIMALITY OF ALGORITHM 3

Our aim now is to relevantly select the separating curves  $d_{\text{subopt}}^A(x)$ ,  $d_{\text{subopt}}^B(x)$  and  $d_{\text{subopt}}^C(x)$ . For that sake, we study the asymptotic behaviour of the total transmit power associated with the proposed allocation algorithm (Algorithm 3) as the number  $K = K^A + K^B + K^C$  of users tends to infinity in the following way. Denote by  $B$  the total bandwidth of the system and define  $r_k$  as the target data rate of user  $k$  in nats/s *i.e.*,  $r_k = BR_k$  where  $R_k$  is the data rate requirement of user  $k$  in nats/s/Hz. Assume that  $K^c/K \rightarrow 1/3$  *i.e.*, the number of users in each cell is asymptotically equivalent. Since the sum  $\sum_k r_k$  of rate requirements tends to infinity as well, we let the bandwidth  $B$  grow to infinity and we assume that  $K/B \rightarrow t$  where  $t$  is a positive real number. Define for each cell  $c = A, B, C$  the following measure  $\nu^{c,(K)}$  on the Borel sets of  $\mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$  as

$$\nu^{c,(K)}(I, J, L) = \frac{1}{K^c} \sum_{k=1}^{K^c} \delta_{(r_k, x_k, y_k)}(I, J, L),$$

where  $I, J, L$  are intervals of  $\mathbb{R}_+, \mathbb{R}, \mathbb{R}_+$  respectively and where  $\delta_{(r_k, x_k, y_k)}$  is the Dirac measure at  $(r_k, x_k, y_k)$ . Note that  $\nu^{c,(K)}(I, J, L)$  can be interpreted as the number of users of cell  $c$  whose rate requirements in nats/s are inside  $I$ , whose x-coordinates are inside  $J$  and whose y-coordinate are inside  $L$ , normalized by  $K^c$ . We furthermore make the following assumption.

**Assumption 1** As  $K \rightarrow \infty$ , measure  $\nu^{c,(K)}$  converges weakly to a measure  $\nu^c$ . Moreover,  $\nu^c$  is the measure product of a limit rate distribution  $\zeta^c$  times a limit location distribution  $\lambda^c$ . Finally,  $\lambda^c$  is absolutely continuous with respect to the Lebesgue measure on  $\mathbb{R}^2$ .

As the number  $K$  of users tends to infinity as above, we showed in [10] that in any global solution to Problem 1, the total power allocated to users modulating subcarriers from *both* the protected subset  $\mathcal{P}_c$  and the non protected subset  $\mathcal{I}$  tends to zero. In other words, a given user in each cell  $c$  either modulates protected subcarriers or non protected subcarriers, but not both. We also characterized a curve  $x \mapsto d^c(x)$  in each cell  $c$  that geographically separates the two groups of protected and non protected users. Here, we propose to fix the separating curves  $\{d_{\text{subopt}}^c(\cdot)\}_c$  associated with the proposed suboptimal allocation algorithm (Algorithm 3) to be equal to the asymptotic curves  $\{d^c(\cdot)\}_c$ . In this case, denote by  $Q_{\text{subopt}}^{(K)} = \sum_{c=A,B,C} \sum_{k=1}^{K^c} \sum_{i=1,2} \gamma_{k,i}^c P_{k,i}^c$  the total transmit power of the



resource allocation  $\{\gamma_{k,i}^c, P_{k,i}^c\}_{i=1,2, c,k}$  resulting from applying Algorithm 3. Define  $Q_T^{(K)} = \sum_{c=A,B,C} \sum_{k=1}^{K^c} \sum_{i=1,2} \gamma_{k,i}^{c,*} P_{k,i}^{c,*}$  as the total transmit power of an optimal solution  $\{\gamma_{k,i}^{c,*}, P_{k,i}^{c,*}\}_{i=1,2, c,k}$  to Problem 1. The following theorem states that Algorithm 3 is asymptotically optimal.

**Theorem 1** Assume that the separating curves  $\{d_{\text{subopt}}^c(\cdot)\}_{c=A,B,C}$  are set such that  $d_{\text{subopt}}^c(x) = d^c(x)$  for all  $x \in [-D, D]$ , where  $\{d^c(\cdot)\}_{c=A,B,C}$  are the asymptotic separating curves given by Theorem 3 in [10]. If Assumption 1 is valid, then the following holds:

$$\lim_{K \rightarrow \infty} Q_{\text{subopt}}^{(K)} = \lim_{K \rightarrow \infty} Q_T^{(K)}.$$

The proof of Theorem 1 will be given in the extended version of this paper.

## 6. SIMULATIONS

In our simulations, we considered a free space propagation model with a carrier frequency  $f_0 = 2.4\text{GHz}$ . Path loss in dB of user  $k$  in cell  $c$  ( $c = A, B, C$ ) is thus given by  $\rho_k(\text{dB}) = 20 \log_{10}(|ck|) + 100.04$ , where  $|ck|$  stands for the distance between user  $k$  and base station  $c$ . The positions of users in each sector are assumed to be uniformly distributed random variables. We also assume that all users have the same target rate, and that  $K^A = K^B = K^C$ . Denote by  $r_t = \sum_{k=1}^{K^c} r_k$  the sum rate per sector measured in bits/s. Let us study the performance of the proposed allocation algorithm (Algorithm 3) in the case where the separating curves  $\{d_{\text{subopt}}^c(\cdot)\}_{c=A,B,C}$  are selected as in Section 5 (see Figure 3). To that end, we compute

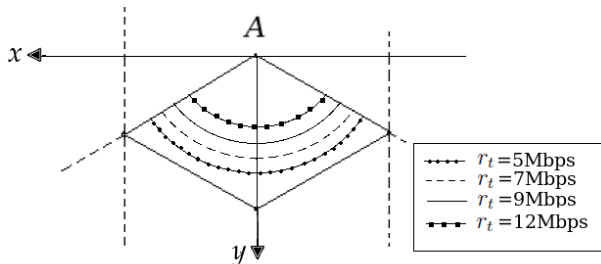


Fig. 3. Asymptotically optimal separation curve  $d^A(\cdot)$ .

the transmit powers  $Q_{\text{subopt}}^{(K)}$  spent when Algorithm 3 is applied for a large number of realizations of the random positions of users. We next evaluate the associated mean value  $\mathbb{E}[Q_{\text{subopt}}^{(K)}]$  (expectation is taken w.r.t the random positions of users) for different values of the total number  $K$  of users and we compare it with the asymptotic optimal transmit power  $Q_T = \lim_{K \rightarrow \infty} Q_T^{(K)}$  as given by Theorem 3 of [10]. The results of this comparison are illustrated in Figure 4. Note that the difference between  $Q_{\text{subopt}}^{(K)}$  and  $Q_T$  decreases with the number of users. This difference can be considered negligible even for a moderate number of users equal to 50 per sector. This sustains that the proposed allocation algorithm is asymptotically optimal.

In Figure 5, we compare our proposed resource allocation algorithm to the distributed allocation scheme introduced in [11]. The latter sets the value of the reuse factor  $\alpha$  to one *i.e.*, all the available subcarriers can be reused in all the cells. The comparison is done for different values of the sum rate  $r_t$ . Note that considerable gains can be achieved by applying our proposed allocation scheme.

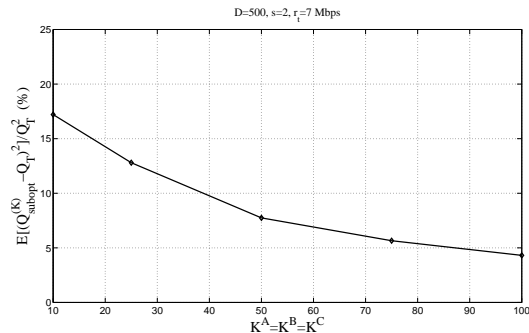


Fig. 4.  $\mathbb{E} \left( Q_{\text{subopt}}^{(K)} - Q_T \right)^2 / Q_T^2$  vs. number of users per sector

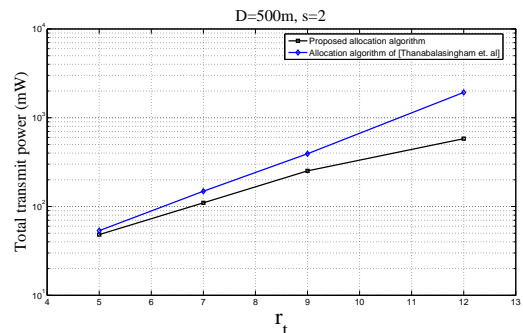


Fig. 5. Comparison between the proposed allocation algorithm and the scheme of [11] for  $K^A = K^B = K^C = 25$

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