A POWER ALLOCATION ALGORITHM FOR OFDM GAUSSIAN INTERFERENCE CHANNELS

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ABSTRACT

Recently a new approximate expression of the capacity region for a flat-fading Gaussian interference channel has been proposed. This expression handles interference in a optimal manner. Based on this expression, we propose to develop a new power allocation algorithm improving the achievable rate region for non-flat fading Gaussian interference channel using a OFDM modulation. In the ADSL environment undergoing crosstalk, we numerically show that our power allocation scheme provides substantial gain compared to the uniform power allocation as well as the optimal power allocation based on metric treating interference as noise.

1. INTRODUCTION

In wireline (such as ADSL when cross-talk occurs) or wireless (such as ad hoc network) systems, multi-user inferference can strongly limit the global performance of the above-mentioned systems. Therefore, for several years now, designing new codes and/or new power allocation algorithms that properly handle the interference is a challenging task. Even more challenging is the characterization of the capacity region associated with the so-called interference channel since it is still an open problem. The capacity region has only been fixed for very simple schemes. In the noiseless case, the capacity region of the two-user interference channel is described in [1]. It is especially shown that the precoding scheme proposed in [2] is optimal. As for the noisy context (corresponding to adding a white Gaussian noise), the capacity is only known for the strong interference case, that is to say, when interference-to-noise ratios (INR) are much stronger than signal-to-noise ratios (SNR), and is done in [2, 3]. For all other INR ranges, the capacity region remains unknown. Nevertheless, a very simple Han-Kobayashi type scheme has been recently proposed in [4], which has been proven to perform within one bit of the capacity. The authors also derive new tighter bounds of the capacity region that depend on the so-called interference level through the so-called generalized degree of freedom function. The interference level and the generalized degree of freedom are two new notions introduced in [4].

To deal with interference issue, there are finally three approaches:

- the first one handles interference as a potentially useful signal as done in [4]. Nevertheless although no practical scheme with low computational load exists yet, the approach still provides a theoretical point of view;
- the second one removes the interference by orthogonalizing the transmission links. This can be done through TDMA, FDMA and so on;
- the third one treats the interference as noise. Usually a power allocation has to be done in order to mitigate the negative impact of the interference.

When interference occurs, it is thus interesting to find a relevant power allocation scheme (subject to some realistic constraints) in order to increase the capacity region. In the last decade, many power allocation algorithms have been proposed in the OFDM context. One can mention Iterative WaterFilling (IWF) [5] which corresponds to an extension of the well known waterfilling algorithm [6]. Albeit better than an uniform power allocation, IWF is suboptimal, especially in highly asymmetric scenarios. This is due to its distributed structure and consequently, the selfish optimum it achieves. Another interesting spectrum management technique is the so-called Optimal Spectrum Balancing (OSB) proposed in [7]. Using a centralized control and OFDM modulation, OSB is proven to converge to a global optimal solution, thus finding the best achievable rate region if the number of subcarriers is high enough. However, the cost in computational load is very high which prevents to use it as soon as the number of users becomes higher than four. Based on the same idea, near-optimal lower-complexity algorithms have been proposed in the literature: Iterative Spectrum Balancing (ISB) in [8], Successive Convex Approximation for Low-complExity (SCALE) in [9] and Autonomous Spectrum Balancing (ASB) in [10].

When interference is removed through an orthogonal access scheme, the power and bandwidth allocation issue maximizing the capacity region or more simply the sum capacity usually boils down to a convex optimization problem which can be solved easily by standard tools [11].

When interference is handled as in [4], no power allocation scheme has been proposed yet. In this paper, we propose to fill up the gap. Therefore we develop a new power allocation algorithm based on the capacity region expressions introduced in [4]. To do that, we assume i) two active users disturbed by a Gaussian Interference Channel, ii) a perfect Channel State Information at the Transmitter and iii) an OFDM modulated transmit signal. This algorithm may be useful as soon as a practical code could exploit the generalized degrees of freedom of the channel. In simulation, we observe that the best power allocation scheme enables us to outperfom the uniform power allocation scheme significantly.

The rest of the paper is organized as follows: in Section 2, we introduce the power allocation problem that we would like to solve. To do that, we especially need to introduce recent results on the capacity region and the so-called generalized degrees of freedom of a Gaussian interference channel. In Section 3, we remark that the problem is convex if the number of subcarriers is large enough which enables us to use standard convex optimization tools to numerically solve our problem. Section 4 is devoted to performance illustrations of this new approach in an ADSL environment. Concluding remarks are drawn in Section 5.

2. PROBLEM STATMENT

We will consider a two-user baseband Gaussian interference channel composed by K subcarriers associated with an OFDM modulation as shown in Fig. 1 and represented by the following equations.

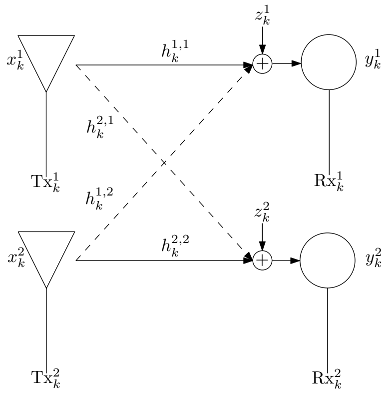

Fig. 1. The k-th subcarrier of a two-user Gaussian interference channel.

$$\begin{aligned} y_k^1 &= h_k^{1,1} x_k^1 + h_k^{1,2} x_k^2 + z_k^1 \\ y_k^2 &= h_k^{2,2} x_k^2 + h_k^{2,1} x_k^1 + z_k^2, \end{aligned} \tag{1}$$

where, for each subcarrier k, $\{x_k^n\}, \{y_k^n\}$ are the complex-valued transmitted and received signals and $\{h_k^{m,n}\}$ are the channel gains, for $n, m = 1, 2$. Let be $P_k^n \triangleq \mathbb{E}[|x_k^n|^2]$. Processes z_k^n are independent and identically distributed (i.i.d.) complex-valued circularly-symmetric Gaussian noises with zero mean and variance N_0 per complex dimension. The transmitted signal for user n is also subject to a total power constraint $P_{\max}^n$, such that, $\sum_{k=1}^K P_k^n \leq P_{\max}^n$.

We assume that the channel gains are known at both the transmitter and receiver's sides. This assumption is realistic in an ADSL environment for which the channel is clearly static. In such an environment, the cross-terms for each subcarrier $h_k^{1,2}$ and $h_k^{2,1}$ represent either the so-called FEXT or NEXT. Then, as noticed in [4], each subcarrier k can be characterized by the so-called *interference levels* α_k^1 and α_k^2 defined as follows

$$\alpha_k^n \triangleq \frac{\log \mathrm{INR}_k^n}{\log \mathrm{SNR}_k^n}.$$

for $n = 1, 2$, with

$$\mathrm{INR}_k^n = \frac{|h_k^{n,n'}|^2 P_k^{n'}}{N_0} \quad \text{and} \quad \mathrm{SNR}_k^n = \frac{|h_k^{n,n}|^2 P_k^n}{N_0}.$$

In [4], it is proven that, at high SNR, the capacity of user n at the subcarrier k only depends on SNR_k^n and α_k^n (thus can be denoted by $C_k^n(\mathrm{SNR}_k^n, \alpha_k^n)$) and is accurately approximated as follows

$$C_k^n(\mathrm{SNR}_k^n, \alpha_k^n) \approx d(\alpha_k^n) \log(1 + \mathrm{SNR}_k^n), \tag{2}$$

when $\mathrm{SNR}_k^n \to \infty$, $\mathrm{INR}_k^n \to \infty$. The term $d(\alpha_k^n)$ defines the so-called *generalized degree of freedom* (g.d.f.) and $\log(1 + \mathrm{SNR}_k^n)$ is the n-th user channel capacity in sub-channel k in interference-less case. A closed-form expression for $d(\alpha)$ is given in [4] and is reminded below

$$d(\alpha) \leq \begin{cases} 1 - \alpha, & 0 \leq \alpha < \frac{1}{2} \\ \alpha, & \frac{1}{2} \leq \alpha < \frac{2}{3} \\ 1 - \frac{\alpha}{2}, & \frac{2}{3} \leq \alpha < 1 \\ \frac{\alpha}{2}, & 1 \leq \alpha < 2 \\ 1, & \alpha \geq 2 \end{cases} \tag{3}$$

with equality when $\mathrm{SNR}_k^n \to \infty$. To be more precise, in [4], it is proven that the rate defined in Eqs.(2)-(3) is achievable by means of a Han-Kobayashi type scheme and is close to the true capacity within one bit for every value of α. We are aware that previous equation is only available, on the one hand, at high SNR and high INR context, i.e., when the noise is strongly lower that the signal and the interference, and, on the other hand, at interference-free context (the generalized degree of freedom is then obviously equal to 1). Consequently, in the simulation part, we have to check that the obtained power allocation enables us to work with such SNR and INR assumptions.

As a summary, each user's global capacity can be written as

$$C^n(\mathbf{SNR}^n, \boldsymbol{\alpha}^n) = \sum_{k=1}^K C_k^n(\mathrm{SNR}_k^n, \alpha_k^n), \tag{4}$$

for $n = 1, 2$ and where $\mathbf{SNR}^n = [\mathrm{SNR}_1^n \ldots \mathrm{SNR}_K^n]$, $\boldsymbol{\alpha}^n = [\alpha_1^n \ldots \alpha_K^n]$ and $C_k^n(\mathrm{SNR}_k^n, \alpha_k^n)$ is defined in Eq. (2).

Our goal is now to find a relevant power allocation scheme using this new way of treating interference in order to maximize the transmission rate of one user keeping the transmission rate of the other user greater than a minimum target rate. Before going further, we need to adapt Eqs. (2)-(4) in order to take into account the power spectral densities of each user and also the loss in capacity due to a practical code. We assume that the subcarrier spacing is equal to Δ_f and the symbol rate is f_s. Let D_k^n be the number of bits that each user sends per subcarrier k use. According to Eq. (2), D_k^n can write as

$$D_k^n \triangleq d(\alpha_k^n) \log\left(1 + \frac{1}{\Gamma} \frac{|h_k^{n,n}|^2 P_k^n}{(\sigma_k^n)^2}\right), \tag{5}$$

where $(\sigma_k^n)^2$ are the transmit and noise power spectral densities, respectively. Moreover, Γ is the SNR gap to capacity which takes into account the practical losses. Finally the data rate of the n-th user (in bits/s) takes the following form

$$R^n = f_s \sum_{k=1}^K D_k^n \tag{6}$$

and each user is of course subject to a total power constraint

$$\Delta_f \sum_{k=1}^K P_k^n \leq P_{\max}^n.$$

Therefore our power allocation issue boils down to the following optimization problem

$$\begin{aligned} & \max_{\mathbf{P}^1, \mathbf{P}^2} R^2 \\ \text{s.t.} \quad & R^1 \geq R_{\text{target}}^1 \\ & \Delta_f \textstyle\sum_{k=1}^K P_k^1 \leq P_{\max}^1 \\ & \Delta_f \textstyle\sum_{k=1}^K P_k^2 \leq P_{\max}^2 \end{aligned} \tag{7}$$

where $\mathbf{P}^n = [P_1^n \dots P_K^n]$ and R^1_{target} is a minimum target data rate for user 1.

3. NEW POWER ALLOCATION ALGORITHM

Due to the nature of the objective and constraint functions, the optimization problem defined by Eq. (7) is clearly highly non-convex and thus, conventional convex optimization techniques do not apply here. The most direct, but computationally intractable approach to solve this optimization problem would be to perform an exhaustive search across all possible power spectral densities combinations according to a certain power granularity Δ_s which is fixed by practical limitations of the transceivers. Of course, this way for solving the optimization problem is exponentially complex in the number of subcarriers as the problem is coupled across all frequencies.

In [7], it is proven that the optimization problem described in Eq. (7) is equivalent to the following optimization problem

$$\begin{array}{ll} & \max_{\mathbf{P}^1,\mathbf{P}^2} wR^1 + (1-w)R^2 \\ \text{s.t.} & \Delta_f \sum_{k=1}^K P_k^1 \leq P^1_{\max} \\ & \Delta_f \sum_{k=1}^K P_k^2 \leq P^2_{\max} \end{array} \tag{8}$$

for any w as soon as an OFDM modulation is employed when the number of subcarriers is large enough. In contrast, the equivalence between both problems is regardless of the closed-form expressions of R^1 and R^2 with respect to the users' power spectral densities.

Notice that in [7], R^n is still provived by (6) but with

$$D_k^n = \log\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 P_k^n}{|h_k^{n,n'}|^2 P_k^{n'} + (\sigma_k^n)^2}\right) \tag{9}$$

for $(n,n') \in \{1,2\}^2$, instead of the expression given in Eq. (5). Actually in [7], interference is treated as extra noise. Consequently we have to solve a similar problem to [7] in which only the objective function has been modified.

In [7], it is also proven that the duality gap between the primal problem defined by Eq. (8) and the so-called dual problem tends to zero when OFDM modulation is used with a sufficiently large number of subcarriers. Consequently the primal problem can be fixed by solving the dual problem which is always convex independently of the property of the primal objective function. This enables us to carry out standard convex optimization tools such as the so-called KKT conditions for fixing our optimization problem [11]. Simultaneously, in [12], it is also obtained that the optimization problem given by Eq. (8), whatever the link between the data rate R^n and the power spectral densites $\mathbf{P}^n$, offers a duality gap to its dual problem going to zero when an OFDM modulation is used with a sufficiently large number of subcarriers. As a consequence, our optimization problem for which the objective data rate function is based on an approximation of the true capacity (given by Eq. (5)) can be solved by using standard convex optimization tools, namely, by solving the dual problem, as soon as the number of OFDM subcarriers is large enough.

We remind that the dual problem is as follows: let $L(\mathbf{P}^1, \mathbf{P}^2, \lambda_1, \lambda_2)$ be the Lagrangian function defined by

$$\begin{aligned} L(\mathbf{P}^1,\mathbf{P}^2,\lambda_1,\lambda_2) &= wR^1 + (1-w)R^2 \\ &+ \lambda_1\left(P^1_{\max} - \Delta_f \sum_{k=1}^K P_k^1\right) \\ &+ \lambda_2\left(P^2_{\max} - \Delta_f \sum_{k=1}^K P_k^2\right) \end{aligned}$$

where λ_n is the Lagrangian multiplier associated with the n-th user power constraint. Then we introduce the so-called Lagrangian dual function $g(\lambda_1,\lambda_2)$ defined as

$$g(\lambda_1,\lambda_2) = \max_{\mathbf{P}^1,\mathbf{P}^2} L(\mathbf{P}^1,\mathbf{P}^2,\lambda_1,\lambda_2). \tag{10}$$

The dual problem is fixed by minimizing $g(\lambda_1,\lambda_2)$ with respect to λ_1 and λ_2. This last minimization can be easily computed since the dual function g is always convex with respect to λ_1 and λ_2.

Notice that the maximization to be done in Eq. (10) seems to have a high computational cost. Nevertheless this 2K-dimensional maximization step can be strongly simplified by remarking that

$$L(\mathbf{P}^1,\mathbf{P}^2,\lambda_1,\lambda_2) = \sum_{k=1}^K L_k(P_k^1,P_k^2,\lambda_1,\lambda_2)$$

with

$$\begin{aligned} L_k(P_k^1,P_k^2,\lambda_1,\lambda_2) &= wf_sD_k^1 + (1-w)f_sD_k^2 \\ &+ \lambda_1(P^1_{\max}/K - \Delta_f P_k^1) \\ &+ \lambda_2(P^2_{\max}/K - \Delta_f P_k^2). \end{aligned}$$

and so can be decomposed in K 2-dimensional maximization step and leads to a complexity linear in the number of subcarriers. Finally minimizing the dual function can be done by using the so-called sub-gradient method or the so-called bisection one [7, 11].

The algorithm associated with the optimization problem in [7] is called the *Optimal Spectrum Balancing* (OSB) since it optimizes the achievable rate region but only when the interference is viewed as noise, that is to say when the interference is not managed. Therefore, in the sequel, the algorithm proposed in [7] is called *OSB - No Interference Management* (OSB-NIM). In contrast, we propose optimal power allocation algorithm with an approximation of the true capacity region which enables us to manage efficiently the interference, and thus the algorithm proposed in this paper is named *OSB - Optimal Interference Management* (OSB-OIM).

4. NUMERICAL RESULTS

In this section, we wish to examine the performance of OSB when applied to the generalized degrees of freedom bitloading function shown in Eq. (5). In order to compare the results with well established schemes, we use the classical scenario of downstream ADSL, following the same approach as in [7]. Indeed, since this channel can be seen as a parallel Gaussian interference channel, this configuration perfectly suits our purpose. Figure 2 shows the practical context of our problem: two simplified Central office-based (CO) and Remote Terminal-based (RT) ADSL deployments, interfering with each other.

The system's parameters are as follows: a line diameter of 0.5 mm (24-AWG) is used. The capacity gap is set to $\Gamma = 12.9$

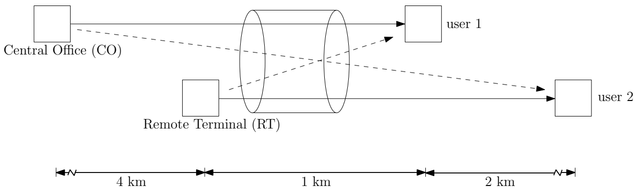

Fig. 2. Interfering CO-based and RT-based ADSL links.

dB, the p.s.d. granularity to $\Delta_s = 0.5$ dBm/Hz, the tone spacing to $\Delta_f = 4.3125$ kHz and the OFDM symbol rate to $f_s = 4$ kHz. Each modem is entitled to a maximum transmit power of 20.4 dBm [13]. Also, the background noise p.s.d. is assumed to be -140 dBm/Hz and no spectral mask is applied to the compared algorithms. On the other hand, as can be seen in Figure 2, the CO line is 5 km long and the RT line is 3 km long, situated at 4 km from the CO. The channel's attenuation pattern can then be empirically calculated [14]. Figure 3 shows the resulting channel and crosstalk transfer functions for the simulated scenario.

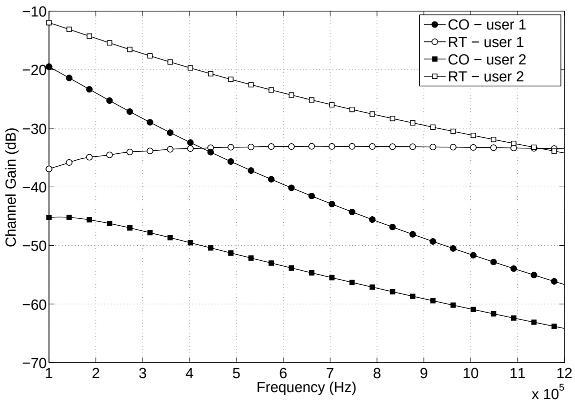

Fig. 3. Typical direct and crosstalk loop coupling.

In Figure 4, we plot the different rate regions obtained by using the conventional OSB-NIM (as in [7]) and the proposed OSB-OIM (as in Eq. (5)). IWF and uniform power allocations were also included in the simulations for comparison.

We can observe the substantial gains obtained by the generalized degrees of freedom bitloading. For example, when a 4-Mbps service is required from the CO link, the OSB-OIM configuration can ensure up to 10-Mbps on the RT link, whereas the OSB-NIM scheme can only simultaneously send 8-Mbps on the RT link. IWF does even worst by sending a maximum of 3-Mpbs over the RT link. It is worth noting, however, that both OSB-OIM and OSB-NIM systems need a centralized control to operate, where an optimal tradeoff between the users' requirements and the channel's parameters can be found. IWF, on the other hand, is a distributed algorithm, where the power control solution is greedy and which tends to overestimate the channels capabilities. Finally non-uniform power allocations (based on OSB-NIM or OSB-OIM) enable us to have substiantial gains compared to the uniform power allocations.

The p.s.d. corresponding to the same 4-Mbps service on the CO link are shown in Figures 5 and 6.

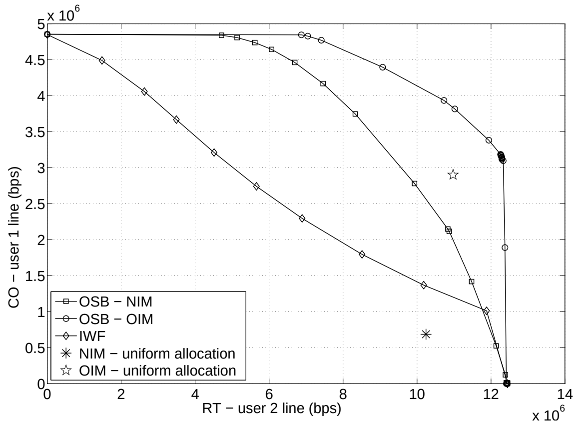

Fig. 4. Rate regions in downstream ADSL.

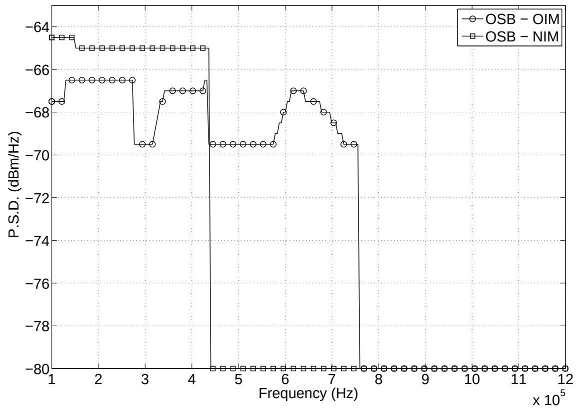

Fig. 5. PSD's on the CO line in downstream ADSL (CO line at 4Mbps).

Interestingly, we can observe that, when interference is considered as noise, OSB-NIM leads to orthogonal p.s.d.'s for the CO and RT lines and thus to the so-called OFDMA in the specific simulation set-up. As expected for this configuration, this abrupt cut in the PSD distributions occurs at approximately 450 kHz, when the SNR of the CO line becomes too low to support any transmission. Indeed, this can be confirmed by observing Fig. 3, where we can see that, at this frequency, the interference becomes stronger than direct channel gain. Morever, thanks to Figs. 5 and 6, we validate the assumptions done on the value of the SNR and INR. Indeed, when the INR is equal to zero, the generalized degree of freedom based approach is valid. When the INR does not vanish, the INR and the SNR have to be high. Both figures satisfy this constraint since the interference and signal levels (if not zero) are around -70 dBm/Hz whereas the noise is about -140 dBm/Hz.

On the other hand, we also show that, when the interference is optimally managed, frequencies are shared among both users and OFDMA is then not advocated anymore.

Finally, to fully validate our approach, we show an example of the converge rate of the OSB-OIM algorithm in Fig. 7, compared

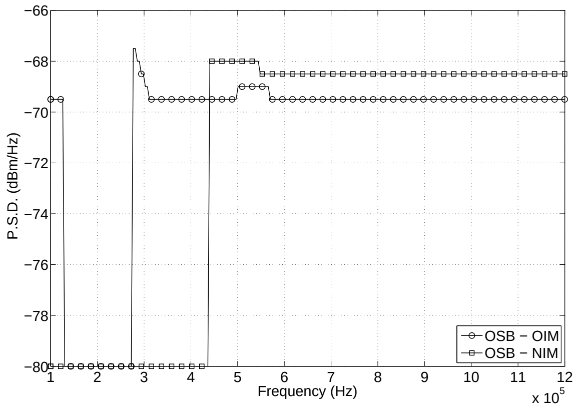

Fig. 6. PSD's on the RT line in downstream ADSL (CO line at 4Mbps).

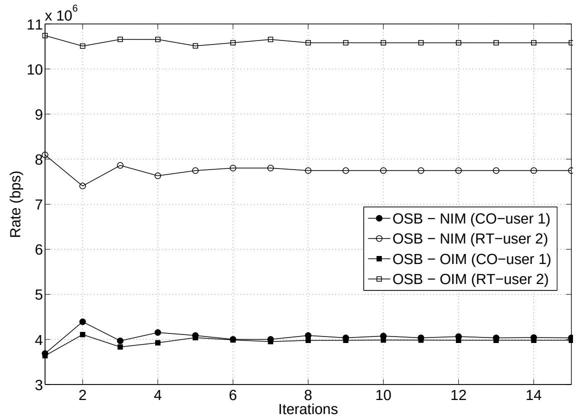

Fig. 7. Convergence of the OSB-OIM algorithm (CO line at 4Mbps).

to the classic OSB-NIM algorithm. We observe that, in this example, even with a much more complicated objective function, the OSB-OIM algorithm converges at approximately the same speed as the OSB-NIM, giving satisfactory results in less than 8 iterations.

5. CONCLUSION

In this paper, we propose a power allocation algorithm for the OFDM based Gaussian interference channel where the generalized degrees of freedom offered by the channel are optimally exploited. Numerical results show that the achievable data rate region obtained by our new power allocation algorithm is significantly improved compared to the uniform power allocation and to the optimal power allocation algorithm when interference is viewed as an extra noise.

An important and challenging issue for future research is the development of new power allocation algorithms for OFDM Gaussian interference channel exploiting the generalized degrees of freedom when more than two users are active.

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