A FOURTH-ORDER ALGORITHM FOR BLIND CHARACTERIZATION OF OFDM SIGNALS

Abdelaziz Bouzegzi[(1)], *Pierre Jallon*[(1)], *and Philippe Ciblat*[(2)]

[(1)] CEA-LETI - MINATEC, Grenoble, France
[(2)] ENST, Paris, France

Email : abdelaziz.bouzegzi@cea.fr, pierre.jallon@cea.fr, philippe.ciblat@enst.fr

ABSTRACT

In the framework of cognitive radio, electro-magnetic environment sensing is a crucial task. In order to distinguish various systems relying on OFDM modulations from each others (such as WiMAX, WiFi, DVB-T), we need to be able to estimate precisely the inter-carrier spacing used in the transmitted signal. When the ratio between cyclic prefix and OFDM symbol duration is small or when the multipath propagation channel is almost as large as the cyclic prefix, standard approaches based on detection of cyclic prefix via an autocorrelation fall down. Therefore we propose a new algorithm to estimate the parameters of an OFDM modulated signal (especially the inter-carrier spacing) relying on the fourth order statistics of the received signal. We theoretically prove its robustness to multipath channels, time offset and frequency offset. Then its performance is analysed through numerical simulations and compared to standard approach which confirms the accuracy of the new algorithm.

1. INTRODUCTION

The concept of cognitive radio introduced by [1] consists in developing flexible terminal which adapts its transmission parameters to its environment. A cognitive terminal has therefore sensing capabilities to characterize its spectral environment and to recognize the standard and/or the modulation parameters of others cognitive terminal/access points.

This paper focuses on such a recognition issue and we assume that the cognitive terminal expects to recognize systems based on OFDM modulation. This assumption is not restrictive since OFDM modulation is currently the most popular modulation scheme (e.g. WiFi, WiMAX, DAB, DVB-T, 3GPP/LTE). Note that an interesting property for standard recognition is that the abovementioned systems differ from one another in the value of the inter-carrier spacing (equal to 15.625kHz, 10.94kHz, 312.5kHz, 1kHz 1.116kHz, 15kHz for Fixed WiMAX, Mobile WiMAX, WiFi, DAB, DVBT, 3GPP/LTE respectively). It is hence sufficient to estimate the inter-carrier spacing of an OFDM modulated signal to identify the used standard. Nevertheless, to have a complete estimator of the modulation parameters of an OFDM signal, the blind estimation issue of the guard interval duration has also to be addressed. Obviously, these results also apply to military contexts.

The estimation of the OFDM signal design parameters has already given rise to several contributions [2, 3, 4, 5, 6]. All these methods are developed in the context of cyclic-prefixed OFDM (CP-OFDM) and are based on the second-order statistics. Indeed, they exploit the periodicity induced by the presence of the cyclic-prefix in the following way : an estimation of the useful time of the OFDM symbol (which is equal to the inverse of the inter-carrier spacing) is first performed by evaluating the autocorrelation function of the receive signal at any lag. Since cyclic prefix occurs, a peak can be exhibited at the lag equal to the useful time of the OFDM symbol. This peak actually appears periodically with the period equal to the the whole OFDM symbol duration. This then enables to estimate the whole OFDM symbol time duration and additionally the cyclic prefix duration.

This autocorrelation based method (introduced in [2, 3, 4, 5, 6]) of course does not work well or even fails when i) the cyclic prefix length is small compared to the whole OFDM symbol length, or ii) the multipath propagation channel length is almost equal to the cyclic prefix length with strong inter-symbol interference, or iii) other kinds of OFDM such as zero-padded OFDM (ZP-OFDM) are considered. Obviously, under one out of these assumptions, the periodicity of the received signal provided by the cyclic prefix is significantly reduced and even often destroyed.

In this paper, we adopt another approach to estimate OFDM signal design parameters. We introduce a flexible OFDM receiver and a cost function (based on the kurtosis of the receiver output symbols) exhibiting a global extrema when the receiver's parameters match the modulation parameters of the received signal. Though more complex, this method has several advantages over the autocorrelation based method: it is robust to the length of the cyclic prefix, to the multipath propagation channel and to the kind of OFDM. It can also be used for time synchronization and for frequency offset estimation and compensation.

This paper is organized as follows: in Section 2 we describe the architecture of the OFDM flexible receiver. In Section 3 we introduce the cost function (based on the kurtosis) and we prove that its optimization leads to an accurate estimation of the OFDM parameters such as the inter-carrier spacing, the length of the prefix. In Section 4, we show that our method is robust to timing offset and carrier frequency offset. Section 5 is devoted to numerical illustrations and comparison with autocorrelation based method.

2. OFDM RECEIVER WITH FLEXIBLE PARAMETERS

We assume the cyclic-prefix OFDM modulated signal. The transmit OFDM signal $s_a(t)$ is considered to get a inter-carrier spacing equal to $1/NT_c$ where N is the number of subcarriers and where $1/T_c$ is the information symbol rate in absence of guard interval.

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The duration of the useful part of the OFDM symbol is then NT_c. As usual, in order to vanish the inter-symbol interference, we add a cyclic prefix of length DT_c. Then the whole OFDM symbol duration is $T_s = (N + D)T_c$. Moreover the transmit continuous-time signal $s_a(t)$ takes the following form

$$s_a(t) = \frac{1}{\sqrt{N}} \sum_{k\in\mathbb{Z}} \sum_{n=0}^{N-1} a_{k,n} e^{-2i\pi \frac{n(t-DT_c-kT_s)}{NT_c}} g_a(t - kT_s) \quad (1)$$

where $a_{k,n}$ are the transmitted data symbols assumed to be independent and identically distributed (i.i.d) with zero-mean and variance E_s, and where $g_a(t)$ the shaping function equals to 1 if $0 \leq t < T_s$ and 0 otherwise. Finally we consider an observation window of duration T.

The transmit signal passes through a multipath channel composed by L paths. The amplitude and the delay of the l^{th} path are denoted by λ_l and τ_l respectively. Then, in noiseless case, the continuous-time received signal takes the following form

$$y_a(t) = \left(\sum_{l=1}^{L} \lambda_l s_a(t - \tau_l) \right) e^{2i\pi\Delta f t} \quad (2)$$

where Δf is the frequency offset due to the oscillator drift or the Doppler effect.

The discrete-time receive signal, obtained by sampling $y_a(t)$ at rate $1/T_e$, thus writes:

$$y(n) = \left(\sum_{l=1}^{L} \lambda_l s_a(nT_e - \tau_l) \right) e^{2i\pi\delta f n}$$

where $\delta f = \Delta f T_e$ is the normalized carrier frequency offset, and T_e satisfies the Shannon condition in regard to the bandwidth of received signal. Moreover we assume that the OFDM prefix size has correctly been chosen, i.e. $\max_l \tau_l - \min_l \tau_l < DT_c$. Due to the Shannon sampling theorem, one can notice that we are able to obtain the value of $y_a(t)$ for any t with the sole knowledge of the samples $y(n)$ by means of interpolators.

For sake of simplicity, we first assume that $\min_l \tau_l = 0$ (no time offset) and $\delta f = 0$ (no carrier frequency offset). Extended results to the general case (i.e., in presence of time offset and carrier frequency offset) will be provided in Section 4.

We remind that the OFDM receiver does not have the knowledge of T_c, N, D, channel parameters, and transmit data. In order to be carried out, the OFDM receiver firstly requires the knowledge of NT_c and DT_c. Once NT_c and DT_c estimated, usual blind channel estimation techniques can be considered in order to retrieve the transmit data. In this paper, we only focus on the estimation issue of NT_c and DT_c. Their estimated values are denoted by $\widehat{NT_c}$ and $\widehat{DT_c}$ respectively. Thanks to the value of $\widehat{NT_c}$ and $\widehat{DT_c}$, the OFDM receiver will operate as follows:

1. Consider the following sequence for all k and for all $p \in \{0, \hat{P} - 1\}$

   $$r_{k,p} = y_a(pT_e + \widehat{DT_c} + k(\widehat{NT_c} + \widehat{DT_c})) \quad (3)$$

   where $\hat{P}$ is the integer given by $\lfloor \widehat{NT_c}/T_e \rfloor$ and where $\lfloor . \rfloor$ stands for the integer part. Note that if $\widehat{NT_c} = NT_c$ and $\widehat{DT_c} = DT_c$, then $r_{k,p}$ corresponds to the p^{th} element of the k^{th} OFDM block once the cyclic prefix has been removed.

2. Recover the transmitted symbols by applying the normalized Fourier transform:

   $$\forall n \in \{0, \cdots, N' - 1\}, \hat{a}_{k,n} = \frac{1}{\sqrt{\hat{P}}} \sum_{p=0}^{\hat{P}-1} r_{k,p} e^{2i\pi p \frac{nT_e}{\widehat{NT_c}}} \quad (4)$$

   As explained below, the value of N' has a negligible influence on the estimation performance as soon as $N' \leq N$. In practice, N' is chosen to be equal to 64 which is the usual minimal value for N whatever the encountered system.

If the estimation step works well ($\widehat{NT_c} = NT_c$ and $\widehat{DT_c} = DT_c$), the decoded symbol $\hat{a}_{k,n}$ (at block k and at subcarrier n) is expected to be proportional to the transmitted symbol $a_{k,n}$, i.e.,

$$\hat{a}_{k,n} = \mu_n a_{k,n} \quad (5)$$

where μ_n is an unknown constant depending on the channel.

3. PARAMETERS ESTIMATION ALGORITHM

The underlying idea of our alogrithm is that Eq. (5) may hold for every value of k and v if and **only if** $\widehat{NT_c} = NT_c$ and $\widehat{DT_c} = DT_c$. If $\widehat{NT_c} \neq NT_c$ or $\widehat{DT_c} \neq DT_c$, an extra term associated with inter-carrier and/or inter-symbol interference should appear in Eq. (5). As done in blind equalization [7, 8], the presence or the absence of interference can be evaluated via the the so-called kurtosis, namely the normalized fourth order cumulant of decoded symbols $\hat{a}_{k,n}$ defined as

$$\kappa(\hat{a}_{k,n}) = \frac{\text{cum}(\hat{a}_{k,n}, \hat{a}^*_{k,n}, \hat{a}_{k,n}, \hat{a}^*_{k,n})}{\left(\mathbb{E}[|\hat{a}_{k,n}|^2]\right)^2} \quad (6)$$

where the superscript $(.)^*$ stands for the complex conjugate.

Our objective is hence to prove that the kurtosis of each decoded symbols defined as a function of $\widehat{NT_c}$ and $\widehat{DT_c}$ reaches its global minimum value if and **only if** $\widehat{NT_c} = NT_c$ and $\widehat{DT_c} = DT_c$. Thanks to this theoretical result, we will be able to build a practical estimation algorithm of NT_c and DT_c based on the minimization of the kurtosis.

The main results of the paper lay in the following theorems. Before going further, notice that the transmitted symbols $a_{k,n}$ are i.i.d. symbols, thus their kurtosis $\kappa(a_{k,v})$ are identical (independent of k and n) and are simply denoted by $\kappa(a)$ in the sequel. We remind that the kurtosis is negative for standard linear modulations (PAM, PSK, QAM).

Theorem 1 *Consider the decoded symbols at subcarrier ν and OFDM symbol k. We have the following result*

$$\textit{Given } (k, \nu),\ \kappa(\hat{a}_{k,\nu}) \geq \kappa(a)$$

and the equality is achieved if and only if

- $\forall p \in \{0, \hat{P} - 1\}$, *the samples $r_{k,p}$ from which are extracted $\hat{a}_{k,\nu}$ belong to the same transmitted OFDM symbol, and*
- $\widehat{NT_c} = NT_c$.

If the equality holds, we also have that $\hat{a}_{k,\nu} = \mu_v a_{k',\nu}$ with μ_v a constant depending on the channel response at subcarrier ν. Note that $\hat{a}_{k,\nu}$ corresponds to the symbol transmitted at the carrier ν, but not necessarily at the k^{th} OFDM symbol.

The proof of Theorem 1 is drawn in Appendix. In practice, the result of theorem 1 can not be used since it concerns only one decoded symbol : its kurtosis can not be estimated. The practical result of this paper is based on the following theorem:

Theorem 2 *For any decoded symbols, i.e. for any subcarrier ν and any OFDM symbol k, the following inequality holds*

$$\forall(k,\nu)\,,\,\kappa(\hat{a}_{k,\nu}) \geq \kappa(a).$$

Equality holds if and only if $\widehat{NT_c} = NT_c$ and $\widehat{DT_c} = DT_c$.

In the sequel, a sketch of proof of Theorem 2 is given: as proved in Theorem 1, for each decoded symbol $\hat{a}_{k,\nu}$, its kurtosis reaches its global minimum value if and only if $\widehat{NT_c} = NT_c$ and $r_{k,p}$ for $p \in \{0, \hat{P}-1\}$ belong to the same transmit OFDM symbol. To prove Theorem 2, it only remains to show $\widehat{DT_c} = DT_c$. If $\widehat{DT_c} \neq DT_c$, it is always possible to find a k_* such as the set of points $r_{k_*,p}$ for $p \in \{0, \hat{P}-1\}$ belong to two OFDM symbols. Then, thanks to Theorem 1, we have $\kappa\left(\hat{a}_{k_*,\nu}\right) > \kappa\left(a\right)$ and equality if and only if $\widehat{DT_c} = DT_c$.

The question now is : how estimating the kurtosis ? Theorem 2 concerns the kurtosis of the decoded sequence of symbols $\hat{a}_{k,n}$. This kurtosis can be estimated in a very classical way. For instance, if PSK modulation (with strictly more than 2 state) or QAM modulation are considered, we get $\mathrm{cum}(\hat{a}_{k,n}, \hat{a}^*_{k,n}, \hat{a}_{k,n}, \hat{a}^*_{k,n}) = \mathbb{E}[|\hat{a}_{k,\nu}|^4] - 2(\mathbb{E}[|a_{k,\nu}|^2])^2$. Then $\kappa(\hat{a}_{k,\nu})$ is estimated by $\hat{\kappa}$ defined as follows

$$\hat{\kappa} = \frac{\sum_{k=0}^{M'-1}\sum_{\nu=0}^{N'-1}|\hat{a}_{k,\nu}|^4 - 2\left(\sum_{k=0}^{M'-1}\sum_{\nu=0}^{N'-1}|\hat{a}_{k,\nu}|^2\right)^2}{\left(\sum_{k=0}^{M'-1}\sum_{\nu=0}^{N'-1}|\hat{a}_{k,\nu}|^2\right)^2} \tag{7}$$

with $M' = \lfloor T/(\widehat{NT_c} + \widehat{DT_c})\rfloor$.

Thanks to theorem 2, one can ensure that the minimization of the kurtosis leads the identifiability of NT_c and DT_c. In practice (i.e., in noisy context and when only a finite number of observations is available), we are only able to provide an estimate of NT_c and DT_c by minimizing the estimate of the kurtosis.

4. ROBUSTNESS TO VARIOUS IMPAIRMENTS

4.1. Impact of time offset

We consider that $\min_l \tau_l > 0$. Due to this time offset, the blind OFDM receiver should ensure that its first sample $r_{0,0}$ depends of the transmitted signal. For doing that, a preliminar step has to be added to the receiver. This step consists in dropping the first $\lfloor\hat{\tau}_1/T_e\rfloor$ samples (if the delays of paths are listed in ascending order). Theorem 2 in which parameter τ_1 has also to be estimated can easily be extended.

4.2. Impact of carrier frequency offset

If the received signal undergoes a carrier frequency offset ($\delta f \neq 0$), another extra step has to be added to the receiver to estimate and compensate it. The received samples given by (3) should then be modified into

$$r_{k,p} = y_a(pT_e + \widehat{DT_c} + k(\widehat{NT_c} + \widehat{DT_c}))e^{-2i\pi\widehat{\delta f}t/T_e} \tag{8}$$

where $\widehat{\delta f}$ is an estimate of δf. We can prove, with similar techniques, that the proposed cost function based on Eq. (8) instead of Eq. (3) is minimal for the true value of NT_c, DT_c, and δf.

5. SIMULATIONS RESULTS

Numerical simulations have been computed to analyse the performance of the proposed algorithm. No frequency offset has been considered, and we set $\min_l \tau_l = 0$. Except otherwise stated, the number of carriers (N) is fixed to 256, the ratio between the cyclic prefix length over the useful OFDM symbol time (CP $:= D/N$) is equal to $1/4$. The duration of the useful part of OFDM symbol (NT_c) is 32μs. The number of available OFDM symbols is 5. The sampling rate T_e is chosen as $T_e = (4/5)T_c$.

The multipath propagation channel is built as follows : each channel realization is composed by 10 paths for which each path delay is uniformally distributed between $[0, \tau_{\max}]$ (unless otherwise stated, $\tau_{\max} = \mathrm{CP}/2$) and each path magnitude is Gaussian distributed with same variance. Each curve is averaged over 1000 Monte-Carlo runs.

Let $M = \lfloor T/T_e \rfloor$. Given a SNR, the noise variance is defined as follows

$$\sigma^2 = \frac{T_c}{T_e}\frac{1}{M}\sum_{m=0}^{M-1}\left|\sum_{l=1}^{L}\lambda_l s_a(mT_e - \tau_l)\right|^2 10^{-\mathrm{SNR}/10}$$

As we treat an estimation problem, the performance measure may be the Mean Square Error on NT_c and DT_c. Nevertheless, our practical problem related to radio cognitive is to identify the right system (WiMAX, WiFi, DAB, DVB-T or 3GPP/LTE, etc) by comparing the $\widehat{NT_c}$ to its theoretical value for each considered system. As seen in introduction, the smallest gap between two inter-carrier spacing values is little larger than 1%. Therefore, for our practical system identification issue, we only need an estimation of $1/NT_c$ up to 1%. Consequently rather than considering MSE as performance measure, we consider that the inter-carrier spacing estimation is correct if $1/\widehat{NT_c}$ is close to $1/NT_c$ up to 1%. In practice, we have calculated our OFDM flexible receiver for each $\widehat{NT_c}$ belonging to the grid of step 0.3μs starting at 5μs and ending at 50μs. This leads to a gap equal to 1% between two adjacent tested inter-carrier spacing compared to the true value 32μs. For each considered value of $\widehat{NT_c}$, $\widehat{DT_c}$ takes values in the set $\widehat{NT_c} \times \{1/2, 1/4, 1/8, 1/16, 1/32\}$. Finally the performance has been evaluated as the number of correct detection, i.e. the number of realizations for which the $\widehat{NT_c}$ and $\widehat{DT_c}$ are equal to NT_c and DT_c up to 0.3μs. Due to lack of space, we do not plot the performance on DT_c for certain figures. Actually we have observed that detections on NT_c or DT_c always yield similar performance.

In Figure 1, we display the correct detection rate for the proposed algorithm (denoted by HoS for High Order Statistics) and the autocorrelation based method (denoted by SoS for Second Order Statistics) versus SNR when various cyclic prefix OFDM lengths have been employed (CP $= \{1/4, 1/8, /16, 1/32\}$). We remark that if CP is equal to $1/4$, both algorithms offer the same performance. In contrast, as soon as CP is strictly less than $1/4$, our algorithm still works well whereas the autocorrelation based method fails down. Notice that standard CP varies from $1/32$ (DVB-T in France) to $1/4$ (WiFi). Consequently, our algorithm is more appropriate for the cognitive radio than the autocorrelation method.

In Figure 2, we plot the correct detection rate versus SNR for various channel lengths ($\tau_{\max} = \{\mathrm{CP}, \mathrm{CP}/2, \mathrm{CP}/4, 0\}$) with CP $= 1/4$. We oberve that our algorithm outperforms the autocorrelation based algorithm as soon as the channel length is more

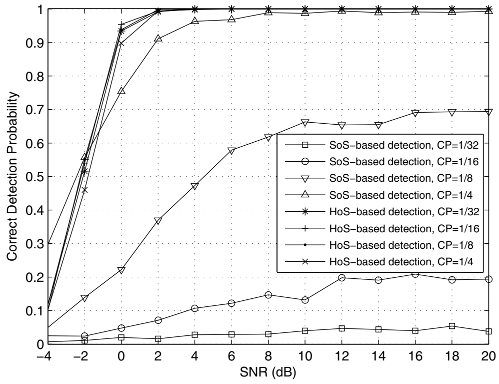

Fig. 1. Detection rate versus SNR for various CP

than $\mathrm{CP}/2$. Our algorithm is more robust to realistic propagation environment when the channel lies on at least half the cyclic prefix.

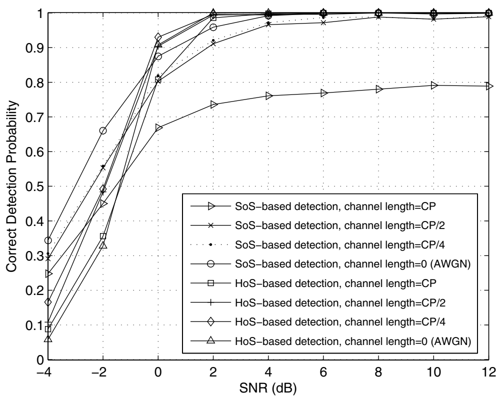

Fig. 2. Detection rate versus SNR for various $\tau_{\max}$

In Figure 3, we analyse the robustness of both algorithms to the number of available OFDM symbols for $\mathrm{SNR} = 10\mathrm{dB}$. Although high-order statistics are more difficult to estimate than second-order statistics, we remark that the proposed algorithm is able to detect accurately the OFDM parameters with less OFDM symbols than the autocorrelation based method. Actually this can be easily explained as follows : due to Eq. (7), kurtosis can be well evaluated since $M'N'$ samples are available to estimate it.

In Figure 4, we inspect the robustness of both algorithms to carrier offset. We see that the detection rate of our method is still quite good despite the presence of carrier offset.

In Figure 5, we plot the detection rate of both algorithms versus SNR when each subcarrier of each OFDM symbol may be modulated by different modulations (QPSK, 8-PSK, 16-QAM, 64-QAM). These four modulations are equilikely and have the same variance. Due to this non i.i.d. assumption on $a_{k,n}$, the assumptions on which Theorem 1 rely are not satisfied anymore. Nevertheless, we show that, in practice, our algorithm still works well and even better than the autocorrelation based algorithm. As

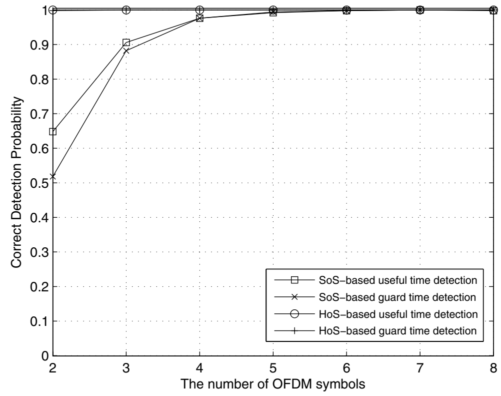

Fig. 3. Detection rate versus number of OFDM symbols

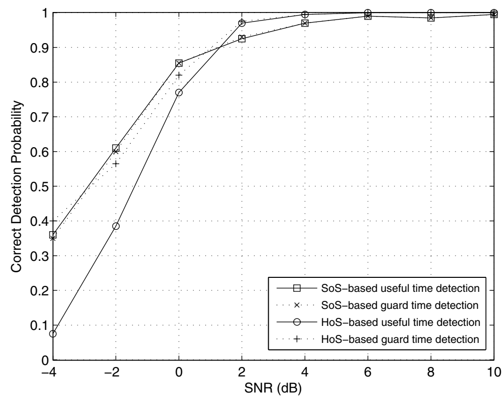

Fig. 4. Detection rate versus SNR in presence of carrier offset

the cognitive radio context applies to wireless communications, it is standard to satisfy i.i.d. assumption (except for few null subcarriers) because neither spectral power allocation (inside the same OFDM block) nor adaptive modulation (for adjacent OFDM blocks) is carried out due to the mis-knowledge of the channel at the transmitter.

6. CONCLUSION

In this paper, we proposed a new flexible OFDM receiver able to detect the OFDM parameters. The receiver exploited the fourth order statistics of the receive signal. Its performance has been evaluated by means of numerical simulations. We showed that the new method outperforms the autocorrelation based method in some useful contexts.

A. APPENDIX - PROOF OF THEOREM 1

Without loss of generality, we only focus on the first estimated OFDM block. According to Eqs. (2) and (3), the first estimated

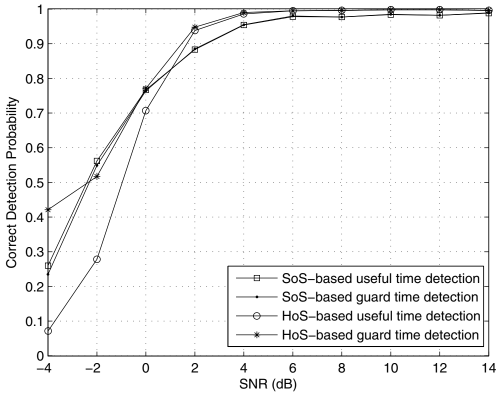

Fig. 5. Detection rate versus SNR in adaptive modulation context

OFDM block is composed by

$$r_{0,p} = \sum_{l=1}^{L} \lambda_l s_a(pT_e + \widehat{DT}_c - \tau_l), \quad \forall p \in \{0, \hat{P}-1\}$$

Thanks to Eqs. (4) and (1), the decoded symbols $\hat{a}_{0,v}$ writes then, $\forall v \in \{0, N'\}$

$$\hat{a}_{0,v} = \frac{1}{\sqrt{\hat{P}N}} \sum_{l=1}^{L} \hat{a}_{0,v}^{(l)} \tag{9}$$

with

$$\begin{aligned} \hat{a}_{0,v}^{(l)} &= \sum_{k\in\mathbb{Z}} \sum_{n=0}^{N-1} \tilde{a}_{k,n}^{(l)} \sum_{p=0}^{\hat{P}-1} e^{-2i\pi pT_e(\frac{n}{NT_c} - \frac{v}{\widehat{NT_c}})} \\ &\times \quad \lambda_l g_a(pT_e + \widehat{DT}_c - \tau_l - kT_s) \end{aligned} \tag{10}$$

and

$$\tilde{a}_{k,n}^{(l)} = a_{k,n} e^{-2i\pi \frac{n}{NT_c}(\widehat{DT}_c - DT_c - \tau_l - kT_s)}. \tag{11}$$

To prove Theorem 1, we will first show that, for each path l, we get

$$\kappa(\hat{a}_{0,v}^{(l)}) \geq \kappa\left(\tilde{a}_{k,v}^{(l)}\right), \quad \forall k \tag{12}$$

and the equality holds for one particular $k = k_0$ if and only if the conditions of Theorem 1 are satisfied, and in particular if the decoded symbol $\hat{a}_{0,v}^{(l)}$ is proportional to one $\tilde{a}_{k_0,\nu}^{(l)}$. Due to Eq. (11), it is equivalent to be proportional to $\tilde{a}_{k_0,\nu}^{(l)}$ and to the transmit symbol $a_{k_0,v}$. Note that k_0 does not depend on l.

As the summation over p is finite in Eq. (10) and as the function $g_a(t)$ has a finite support, the summation over k in Eq. (10) is also finite. Let Ω_l be the following set

$$\Omega_l = \{k \mid \exists p \in \{0, \hat{P}-1\} \text{s.t.} g_a(pT_e + \widehat{DT}_c - \tau_l - kT_s) = 1\}$$

Let us consider that $\operatorname{card}(\Omega_l) > 1$. Under such an assumption, it is clear that the decoded symbol $\hat{a}_{0,v}^{(l)}$ depends at least from 1 transmit symbols of each transmit OFDM symbol. So $\hat{a}_{0,v}^{(l)}$ is a linear combination of several symbols which implies that the inequality (12) is a strict inequality. Consequently, in order to obtain equality in Eq. (12), we need $\operatorname{card}(\Omega_l) = 1$. Let us now consider that $\operatorname{card}(\Omega_l) = 1$. Let k_0 be the unique element of Ω_l. Under this assumption, we have that $r_{0,p}$ for any $p \in \{0, \hat{P}-1\}$ belongs to the same k_0^{th} transmit OFDM symbol. Then $\hat{a}_{0,v}^{(l)}$ simplifies as follows

$$\hat{a}_{0,v}^{(l)} \quad = \quad \lambda_l \sum_{n=0}^{N-1} \tilde{a}_{k_0,n}^{(l)} e^{i\theta_n} \frac{\sin\left(\pi \frac{\hat{P}T_e}{NT_c}\left(n - v\frac{NT_c}{\widehat{NT_c}}\right)\right)}{\sin\left(\pi \frac{T_e}{NT_c}\left(n - v\frac{NT_c}{\widehat{NT_c}}\right)\right)}$$

where θ_n still depends on n.

Once again, as $\hat{a}_{0,v}^{(l)}$ is a linear combination of $\tilde{a}_{k_0,n}^{(l)}$, Eq. (12) holds. Equality occurs when the weights of the linear combination vanish except one. These weights are zero if and only if it exists n_0 such that

$$\begin{cases} \dfrac{\sin\left(\pi \frac{\hat{P}T_e}{NT_c}\left(n - v\frac{NT_c}{\widehat{NT_c}}\right)\right)}{\sin\left(\pi \frac{T_e}{NT_c}\left(n - v\frac{NT_c}{\widehat{NT_c}}\right)\right)} \neq 0 & \text{if } n = n_0 \\ \dfrac{\sin\left(\pi \frac{\hat{P}T_e}{NT_c}\left(n - v\frac{NT_c}{\widehat{NT_c}}\right)\right)}{\sin\left(\pi \frac{T_e}{NT_c}\left(n - v\frac{NT_c}{\widehat{NT_c}}\right)\right)} = 0 & \text{otherwise} \end{cases}$$

As $\hat{P} = \lfloor \widehat{NT_c}/T_e \rfloor$, we have that $\hat{P}T_e/NT_c$ is close to $\widehat{NT_c}/NT_c$ if N is large enough. One can see that the last property is satisfied if and only if $\widehat{NT_c} = NT_c$ and $n_0 = v$, i.e., if $\hat{a}_{0,v}^{(l)}$ is proportional to $\tilde{a}_{k_0,\nu}^{(l)}$ and so to $a_{k_0,v}$. Due to Eq. (9), this implies that $\hat{a}_{0,v}$ is proportional to $a_{k_0,v}$. ■

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