PERFORMANCE OF A RESOURCE ALLOCATION STRATEGY FOR AN FH-OFDMA BASED SYSTEM IN A MULTI-CELL ENVIRONMENT

Sophie Gault(1), *Walid Hachem*(2) *and Philippe Ciblat*(3)

(1) Motorola Labs, Gif-sur-Yvette, France
(2) Département Télécom, Supélec, Gif-sur-Yvette, France
(3) Département Comélec, ENST, Paris, France
Emails: sophie.gault@motorola.com, walid.hachem@supelec.fr, philippe.ciblat@enst.fr

ABSTRACT

The paper deals with power and subcarrier allocation in a Frequency Hopping - Orthogonal Frequency Division Multiple Access (FH-OFDMA) based system. The problem is solved for the downlink of a cellular wireless transmission and the performance of the solution applied in two different frequency reuse schemes are compared. The proposed allocation strategy aims at minimizing the total power emitted by the base station (BS) and only requires the knowledge of the users' mean Signal to Interference plus Noise Ratios and rate requirements. We prove that in a universal frequency reuse scheme, the algorithm converges if the users' mean rate requirement is below a threshold which can be determined by the network designer. Moreover, numerical results show that this scheme outperforms the partial frequency reuse one under some cell size limit.

1. INTRODUCTION

The basic principle of cellular communications is to split an area into cells. The coverage of each cell is then ensured by one base station (BS) which communicates with its mobile terminals. In such systems, two main spectrum management strategies exist to perform a spectrally efficient coverage of an area. In the one hand, all BS and all associated mobile terminals may transmit on the entire available bandwidth, which is commonly known as *single frequency network* or *universal frequency reuse,* since the spectrum is *reused* from one cell to another. To carry out such a system, any spread-spectrum based multiple acces technique, such as DS-CDMA or FH-OFDMA, can be employed [1]. Systems such as UMTS or IS-95 have adopted this strategy. The other possibility is to assign distinct transmission bandwidths to adjacent cells and reuse patterns are defined so as to minimize the Multi-Cell Interference (MCI) effects: this is the case of GSM, where the coverage area is divided into hexagonal cells, and a 7 cell reuse pattern is defined such as a given cell uses 1/7 of the total bandwidth, and this portion of the bandwidth is not reused in the adjacent cells. These two solutions present both advantages and disadvantages w.r.t. intra-cell resource allocation, MCI, *etc.* For example, universal frequency reuse allows soft handover, macro-diversity and simplicity in the network deployment while partial frequency reuse requires frequency planning that implies new bandwidth assignments in case of incremental deployment of BS. On the other hand, partial frequency reuse provides "good" Signal to Interference plus Noise Ratio (SINR) transmissions while universal frequency reuse has to deal with more powerful MCI levels. This paper aims at giving a performance comparison of the two schemes for two basic scenarios. Since the performance clearly depends on the access technique which is used, we have chosen to focus on the downlink of a system using Frequency Hopping - Orthogonal Frequency Division Multiple Access (FH-OFDMA) [2]. This access method appears as a promising candidate in future wireless standards for many reasons: flexibility in subcarrier attribution, no multiuser interference, simplicity of the receiver, *etc.* Many papers exist in the literature, which deal with OFDMA resource allocation based on the optimization of various criteria ([3–5]). Our strategy is based on the minimization of the total power emitted by the BS under the constraint of users' rate requirements. The BS has to find the optimal proportion of subcarriers and the optimal power to attribute to the users in the cell. Due to a high degree of users' mobility, the BS is assumed to have only a statistical knowledge of the users' channels. The resource allocation is formulated as a convex optimization problem which is first solved in a single cell context. It is then extended to a multi-cell environment where all cells use the same frequency band and the performance is then compared with the other multi-cell scenario where different sets of subcarriers are assigned to adjacent cells and thus working with a frequency reuse factor η less than one.

The remainder of the paper is organized as follows. In section 2, we formulate the allocation issue as an optimization problem for the single cell context and derive the corresponding allocation algorithm. Section 3 is devoted to the extension to a multi-cell environment, for the two considered frequency reuse schemes. For the universal frequency reuse scheme, we especially study the impact of MCI on the network stability. Finally, in section 4, we present the numerical illustrations of the results. For reader's convenience, mathematical derivations of section 3 are carried out for linear (1D) cells whereas numerical results correspond to the more realistic case of 2D square cells.

In the sequel, $\mathbb{E}[.]$ will denote the expectation operator. The complex-valued circular Gaussian distribution with mean $\mathbf{a}$ and covariance matrix Σ will be denoted $\mathscr{CN}(\mathbf{a},\Sigma)$.

2. SINGLE CELL CONTEXT

2.1 Model

We consider a downlink transmission where a BS serves K users. The transmitted signals consist in OFDM symbols, each of duration T seconds. The users channels are time varying frequency selective but assumed invariant at the scale of an OFDM symbol. The channel impulse response (CIR) of user k is represented during OFDM symbol m by the vector $\mathbf{h}_k(m) = [h_k(m,0),\dots h_k(m,L-1)]^{\mathrm{T}}$ where L is an upper bound on the users channel lengthes. The equivalent transfer function is denoted $\mathbf{H}_k(m) = [H_k(m,0),\dots,H_k(m,N-1)]^{\mathrm{T}}$ where N is the number of OFDM subcarriers (equivalently the number of channel uses per OFDM symbol). In other words, $\mathbf{H}_k(m) = \sqrt{N}\mathbf{F}_{N,L}\mathbf{h}_k(m)$ where $\mathbf{F}_{N,L}$ is the $N\times L$ Fourier matrix which (n,l) entry is given by $[\mathbf{F}]_{n,l} = \frac{1}{\sqrt{N}}e^{(-2i\pi nl/N)}$ for $n = 0,\dots,N-1$ and $l = 0,\dots,L-1$. The signal $Y_k(m,n)$ received by user k at subcarrier n for the OFDMA symbol m after removing the cyclic prefix and applying the Discrete Fourier Transformation writes

$$Y_k(m,n) = H_k(m,n)S(m,n) + V_k(m,n) \tag{1}$$

where $S(m,n)$ is the signal transmitted by the BS in the discrete Fourier domain and $V_k(m,n)$ is the additive noise received by user k

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at subcarrier n of OFDM symbol m. The 2D noise process $V_k(m,n)$ is assumed white, and that a sample of this process has the distribution $\mathscr{CN}(0,\sigma^2)$ where σ^2 refers to the noise variance. We have $\sigma^2 = N_0 B$ where N_0 is the noise power spectral density and $B = N/T$ is the system bandwidth, or equivalently the number of channel uses per second.

As is usually done, a Rayleigh channel model is considered. The CIR coefficients $h_k(m,l)$ (for $l = 0,\dots,L-1$) thus are complex, circular Gaussian, independent, but do not necessarily have the same variances. It can then be shown that the transfer function coefficients in the discrete Fourier domain, have the distribution $\mathscr{CN}(0,\varsigma_k^2)$ with a variance $\varsigma_k^2 = \sum_{l=0}^{L-1} \varsigma_{k,l}^2$ where $\varsigma_{k,l}^2$ is the variance of channel l^{th} tap $h_k(m,l)$. Consequently, the Gain to Noise Ratios (GNR) $G_k(m,n) = |H_k(m,n)|^2/N_0$ of user k for $n \in \{0,\dots,N-1\}$ and $m \in \mathbb{Z}$ are identically distributed.

Because of users' mobility in the cell, we assume the BS only knows the K mean GNR a_k given by

$$a_k = \mathbb{E}[G_k(m,n)] = \varsigma_k^2/N_0 \tag{2}$$

As mentioned in section 1, the resource allocation problem we will consider is constrained by users' rate requirements. A relevant measure of those rates will be the ergodic Shannon capacities of the channels[1]. We denote by γ_k the sharing factor associated with user k, which is the proportion of time-frequency slots (m,n) for which $S(m,n)$ is allocated to user k: $\gamma_k \geq 0$ and $\sum_{k=1}^K \gamma_k \leq 1$. Once the sharing factors $\{\gamma_1,\dots,\gamma_K\}$ are chosen, the practical allocation is performed following some FH pattern (using *e.g.* latin squares [1]), designed in such a way that constraints associated with the sharing factors γ_k are respected.

Let $E_k = \mathbb{E}\left[|S(m,n)|^2\right]/B$ be the energy transmitted on the subcarrier n of OFDM symbol m when the slot (m,n) is allocated to user k. The ergodic capacity per channel use C_k given to user k is then

$$C_k = \gamma_k \mathbb{E}\left[\log\left(1 + \frac{|H_k(m,n)|^2 B E_k}{\sigma^2}\right)\right] = \gamma_k \mathbb{E}[\log(1+G_k E_k)] \ . \tag{3}$$

If Q_k is the mean energy per channel use sent to user k, we have $Q_k = \gamma_k E_k$. The mean energy per channel use Q transmitted by the BS writes

$$Q = \sum_{k=1}^K Q_k \ . \tag{4}$$

Our problem can then be formulated as follows: given a rate vector $\rho = [\rho_1,\dots,\rho_k]^{\mathrm{T}}$ where ρ_k is the capacity per channel use required by user k, find the energies $\{E_k\}$ and the sharing factors $\{\gamma_k\}$ such that the total transmitted energy Q is minimum.

Or more formally: minimize Q with the constraints

$$-C_k + \rho_k \leq 0 \quad \text{for } k = 1,\dots,K \tag{5}$$

$$\sum_{k=1}^K \gamma_k - 1 \leq 0 \ . \tag{6}$$

This problem is made convex by writing C_k as a function of Q_k instead of E_k.

2.2 The Allocation Algorithm

The convex constrained minimization problem (5-6) can be solved by writing the Lagrange-KKT conditions with respect to the vector parameter $\mathbf{x} = [Q_1,\dots,Q_K,\gamma_1,\dots,\gamma_K]^{\mathrm{T}}$:

$$\nabla_{\mathbf{x}} Q - \sum_{k=1}^K \lambda_k \nabla_{\mathbf{x}} C_k + \beta \nabla_{\mathbf{x}} \left(\sum_{k=1}^K \gamma_k\right) = 0 \tag{7}$$

[1] The ergodic capacity can be approached by coding schemes and appropriate FH patterns ([6, 7]) that exploit properly the channel coherence bandwidth and/or its coherence time which we shall assume in the sequel.

where $\nabla_{\mathbf{x}}$ denotes the gradient operator with respect to the vector $\mathbf{x}$, the positive real numbers $\lambda_1,\dots,\lambda_K$ are the Lagrange multipliers associated with constraints (5) and the positive number β is the Lagrange multiplier associated with the constraint (6). After partial derivation w.r.t. Q_k and γ_k, we obtain the $2K$ scalar equations

$$\lambda_k \mathbb{E}\left[\frac{G_k}{1+G_k E_k}\right] = 1 \tag{8}$$

$$\lambda_k \mathbb{E}\left[\log(1+G_k E_k) - \frac{G_k E_k}{1+G_k E_k}\right] = \beta \tag{9}$$

for $k = 1,\dots,K$. By plugging Equations (8) into (9) we have

$$f_1(E_1) = f_2(E_2) = \cdots = f_K(E_K) = \beta \tag{10}$$

where

$$f_k(x) = \frac{\mathbb{E}\left[\log(1+xG_k) - \frac{xG_k}{1+xG_k}\right]}{\mathbb{E}\left[\frac{G_k}{1+xG_k}\right]} = \frac{\mathbb{E}[\log(1+xG_k)]}{\mathbb{E}\left[\frac{G_k}{1+xG_k}\right]} - x \ . \tag{11}$$

It can be easily shown that the functions $f_k(x)$ increase from zero to infinity over the interval $[0,\infty)$. The allocation algorithm then naturally follows:

1. Initialize β to a value close to zero.
2. Compute the energies E_k by solving Equations (10).
3. Compute $\gamma_k(\beta)$ given by

$$\gamma_k(\beta) = \frac{\rho_k}{\mathbb{E}[\log(1+G_k E_k(\beta))]} \tag{12}$$

   where $E_k(\beta)$ are the solutions of Equations (10).
4. If β is too small, the energies $E_k(\beta)$ will be too small also and we will have $\sum_{k=1}^K \gamma_k(\beta) > 1$, therefore increase β until $\sum_{k=1}^K \gamma_k(\beta) = 1$ is satisfied.

Let us link these expressions with the GNR a_k. Since the random variables $H_k(m,n)$ are circular Gaussian, G_k has the exponential distribution with mean a_k. Let X_e be a positive random variable with pdf $t \mapsto e^{-t}$. The energies in Equations (10) can be computed as:

$$E_k(\beta) = \frac{1}{a_k} f^{(-1)}(a_k \beta) \tag{13}$$

where

$$f(x) := \frac{\mathbb{E}[\log(1+xX_e)]}{\mathbb{E}[X_e/(1+xX_e)]} - x = \frac{e^{1/x} x^2 \mathrm{Ei}(1/x)}{x - e^{1/x}\mathrm{Ei}(1/x)} - x$$

where Ei is the exponential integral function: $\mathrm{Ei}(x) = \int_x^\infty \frac{e^{-t}}{t} dt$ for $x > 0$, and $f^{(-1)}$ is the inverse of f (on $[0,\infty)$) with respect to composition.

Equation (12) can then be rewritten as follows

$$\gamma_k(\beta) = \frac{\rho_k}{F(a_k \beta)} \tag{14}$$

where $F(x)$ is the function defined on $\mathbb{R}_+$ by

$$F(x) = \mathbb{E}\left[\log\left(1 + X_e f^{(-1)}(x)\right)\right] \ .$$

Since $\sum_{k=1}^K \gamma_k(\beta) = 1$, the multiplier β is the unique solution to the following equation

$$\sum_{k=1}^K \frac{\rho_k}{F(a_k \beta)} = 1 \ . \tag{15}$$

And the minimal energy per channel use that enables us to ensure a rate ρ_k for user k in a single cell environment has the following closed-form expression

$$Q = \sum_{k=1}^K \frac{\rho_k}{a_k} \frac{1}{F(a_k \beta)} f^{(-1)}(a_k \beta) \ . \tag{16}$$

2.3 Asymptotic Analysis

We now consider the asymptotic regime where the number of users K in the cell grows toward infinity. This regime is also characterized by a bandwidth B growing toward infinity (unless the required rates could not be satisfied) while the ratio K/B tends to a positive constant α.
We need to define other quantities: the cell can be identified with a compact $\mathscr{C}$ included in $\mathbb{R}$ or in $\mathbb{R}^2$ according to whether the cell is one or two-dimensional. We model the GNR a_k as a function of the coordinate vector x_k of mobile k in the cell $\mathscr{C}$: $a_k = q(x_k)/N_0$ where q is the so-called path loss function. We need to define the joint distribution of rates and user locations $d\nu^{(K)}(u,x)$ when K users are active. We assume that this joint distribution converges to a limit distribution $d\nu(u,x)$. Moreover we assume that this limit distribution is the measure product of the limit rate distribution $d\zeta(u)$ times the limit location distribution $d\lambda(x)$, that is to say that users' rate requirements and locations are considered independent.
Then Equations (15) and (16) can be rewritten respectively as

$$\frac{K}{B}\int_{\Delta} \frac{u}{F(\pi(x)\beta)} d\nu^{(K)}(u,x) = 1 \tag{17}$$

and

$$Q^{(K)} = \frac{K}{B}\int_{\Delta} \frac{u}{\pi(x)} \frac{f^{(-1)}(\pi(x)\beta)}{F(\pi(x)\beta)}\, d\nu^{(K)}(u,x)\ . \tag{18}$$

where $\pi(x) = q(x)/N_0$.

Typically, the measure λ can be modeled as the uniform probability measure over $\mathscr{C}$, in other words $d\lambda(x) = (1/|\mathscr{C}|)\, dx$ where $|\mathscr{C}|$ is the cell volume. Concerning the limit rate distribution ζ, denote by $\overline{R} = \int_{\Delta_R} u d\zeta(u)$ its mean. A parameter that will be of prime importance in the following is the mean rate $\bar{r}$ per channel use and per cell volume unit. It is given by $\bar{r} = \alpha\overline{R}/|\mathscr{C}|$ nats per channel use and per (squared) meter.
The following theorem gives us the asymptotic expressions:

Theorem 1 *Assume $K \to \infty$ in such a way that $K/B \to \alpha > 0$. Assume that $\pi(x)$ is continuous and satisfies $\pi(x) > 0$ on $\mathscr{C}$. Then $Q^{(K)}$ converges to Q given by*

$$Q = \bar{r}\int_{\mathscr{C}} \frac{f^{(-1)}(\pi(x)\beta)}{\pi(x)\ F\left(\pi(x)\beta\right)}\ |\mathscr{C}| d\lambda(x) \tag{19}$$

where β is the unique positive number that satisfies

$$\bar{r}\int_{\mathscr{C}} \frac{|\mathscr{C}|}{F\left(\pi(x)\beta\right)} d\lambda(x) = 1. \tag{20}$$

For lack of space, the proof of this theorem is not detailed in this paper. An interesting implication of this theorem is that the energy per channel use in the asymptotic regime depends on the distribution of the required rates through its mean only.

3. MULTI-CELL ENVIRONMENT

In this section, we modify and analyze the behavior of the power allocation algorithm in a multi-cell environment, *i.e.* when, in addition to the background noise, the signal received by a user is corrupted by MCI.

3.1 Cell Model

For reader's convenience, we consider a cellular network that consists of a linear (1D) regular array of cells as shown on figure 1. Nevertheless, numerical illustrations of section 4 correspond to 2D cell model so as to reflect more realistic results. On this figure, D is the half distance between two neighboring BS, and if we identify its BS with the origin, the cells shown on this figure are of the type $\mathscr{C} = [-D,d] \cup [d,D]$, a model considered in general when the path loss is of the type $q(x) = |x|^{-r}$. Even if such a multi-cell model ([8], [9]) is an ideal model, it provides some interesting guidelines for implementing a practical cellular system. We furthermore assume that the MCI only comes from the adjacent cells.
To give the expression of the received signal in a multi-cell environment, we use the superscript notation $^{(c)}$ to refer to the quantities located to cell number c. For instance the signal transmitted by BS number c will be denoted $S^{(c)}(m,n)$ and $\mathbf{H}_k^{(c',c)}(m)$ is the transfer function of the channel that carries the signal of BS number c' to user k of cell c. The signal received by user k of cell c at OFDM symbol m and subcarrier n has the following expression:

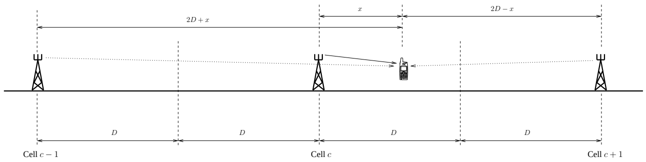

Figure 1: The multi-cell environment

$$\begin{aligned} Y_k^{(c)}(m,n) &= H_k^{(c)}(m,n)S^{(c)}(m,n) \\ &\quad + H_k^{(c-1,c)}(m,n)S^{(c-1)}(m,n) + H_k^{(c+1,c)}(m,n)S^{(c+1)}(m,n) \\ &\quad + V_k^{(c)}(m,n)\ . \end{aligned} \tag{21}$$

A remark which can be made is that if we had considered 2D square cells, we would have added the contribution of 9 adjacent cells instead of only 2 in the 1D cell model (see 2^{nd} line of Eq. (21)).
We assume that a FH algorithm ensures that the signal of any user is equally distributed on all subcarriers in every cell. Therefore we have $\mathbb{E}[|S^{(c)}(m,n)|^2] = BQ^{(c)}$ for all m and n. If we furthermore suppose that the channels $\mathbf{h}_k^{(c',c)}(m)$ satisfy the channel model described in 2.1, then the variance of $H_k^{(c-1,c)}(m,n)$ is independent of m and n. In this case, the GNR $a_k^{(c)}$ of user k in cell c is equal to

$$\frac{\mathbb{E}[|H_k^{(c)}(m,n)|^2]}{Q^{(c-1)}\mathbb{E}[|H_k^{(c-1,c)}(m,n)|^2] + Q^{(c+1)}\mathbb{E}[|H_k^{(c+1,c)}(m,n)|^2] + N_0}\ .$$

Since the noise $V_k(m,n)$ of Equation (1) is replaced with a non-Gaussian noise (because of MCI), the capacity derivation becomes a difficult problem. Nevertheless, if we consider $S^{(c)}(m,n)$ has a Gaussian distribution, it is still possible to have a lower bound on the true capacity ([10]) by assimilating MCI to a Gaussian noise and then computing the capacity expression in Eq. (3), the variance σ^2 being equal to the sum of the "real" AWGN variance plus the MCI power. As we constraint this lower bound to meet the required rates, the power solution of the optimization problem will represent an upper bound on the power necessary in theory to comply with these constraints.
In short, energies per channel use and shares are given by Equations (13–15) where the $\{a_k\}$ are replaced with the $\{a_k^{(c)}\}$, which can be consistently estimated by the BS number c.
In the asymptotic regime, the set of definition of the function $q(x)$ has to be extended to $\mathscr{C} \cup (2D + \mathscr{C}) \cup (2D - \mathscr{C})$ where $2D \pm \mathscr{C} = \{2D \pm x,\ x \in \mathscr{C}\}$. Let us denote by $\pi_{Q^{(c-1)},Q^{(c+1)}}(x)$ the GNR profile in the conditions described by Equation (21). This function is written

$$\pi_{Q^{(c-1)},Q^{(c+1)}}(x) = \frac{q(x)}{Q^{(c-1)}q(2D+x) + Q^{(c+1)}q(2D-x) + N_0}\ .$$

In a multi-cell setting, a lower bound on the total energy per channel use is therefore given by Equation (19) where β satisfies (20), and in both equations, $\pi(x)$ is replaced by $\pi_{Q^{(c-1)},Q^{(c+1)}}(x)$.

3.2 Universal frequency reuse $(\eta = 1)$

In a universal frequency reuse scheme, all BS transmit on the whole available bandwidth. We first assume that all cells have equivalent rate requirements. Our power allocation algorithm is based on the basic principle that a given BS combats the MCI coming from its neighbors by increasing its own transmitted power ([11]). By doing so, it will however increase the interference it produces with its neighboring cells, so that these cells will have to increase in turn their powers, and so forth. In this section, we study the stability of our system implementing this iterative algorithm, the question being whether the whole system can attain an equilibrium, and if it can, under which conditions. Here we consider an infinite array and we assume that each cell satisfies the conditions of the asymptotic regime.

Let $\beta(Q,\bar{r})$ be the unique solution to the equation

$$\bar{r}\int_{\mathscr{C}} \frac{|\mathscr{C}|}{F\left(\underline{\pi}_Q(x)\beta(Q,\bar{r})\right)} d\lambda(x) = 1 \tag{22}$$

where $\underline{\pi}_Q(x)$ is given by

$$\underline{\pi}_Q(x) = \pi_{Q,Q}(x) = \frac{q(x)}{Q\left(q(2D+x)+q(2D-x)\right)+N_0} . \tag{23}$$

By iterating, the energy Q_{n+1} delivered by each BS at moment n will be given w.r.t. Q_n by

$$Q_{n+1} = \xi(Q_n,\bar{r}) \tag{24}$$

where $\xi(Q,\bar{r})$ is the total energy per channel use a BS needs to transmit to attain the mean rate of $\bar{r}$ nats per channel use and cell volume unit when its neighboring cells transmit at energy Q

$$\xi(Q,\bar{r}) = \bar{r}\int_{\mathscr{C}} \frac{f^{(-1)}\left(\underline{\pi}_Q(x)\beta(Q,\bar{r})\right)}{\pi_Q(x)\, F\left(\underline{\pi}_Q(x)\beta(Q,\bar{r})\right)} |\mathscr{C}| d\lambda(x) . \tag{25}$$

The convergence of the sequence $\{Q_n\}$ is treated by the following theorem which is the main result of this section:

Theorem 2 *Let $t(x)$ be defined on $\mathscr{C}$ as*

$$t(x) = \frac{q(x)}{q(2D-x)+q(2D+x)} .$$

Assume that $t(x)$ is continuous and satisfies $t(x) > 0$ on $\mathscr{C}$. Define $\psi(r)$ on $\mathbb{R}_+^*$ as

$$\psi(r) = r\int_{\mathscr{C}} \frac{f^{(-1)}(t(x)b(r))}{t(x)F\left(t(x)b(r)\right)} |\mathscr{C}| d\lambda(x) \tag{26}$$

where $b(r)$ is the unique positive number that satisfies

$$r\int_{\mathscr{C}} \frac{|\mathscr{C}|}{F(t(x)b(r))}\, d\lambda(x) = 1 . \tag{27}$$

Then

1. *the equation $\psi(r) = 1$ admits an unique solution $r_0 > 0$.*
2. *for any initial value $Q_0 \geq 0$, if $\bar{r} < r_0$, then the sequence (Q_n) which elements are given by (24) converges, and if $\bar{r} \geq r_0$, then it grows to infinity.*

The reader interested in the proof can refer to [12]. Practically this theorem indicates that

- If the rate is less than a certain threshold r_0, then the multi-cell system can operate;
- For a given achievable rate, *i.e*, a rate less than the threshold r_0, the proposed allocation strategy converges and minimizes the power consumption.

3.3 Partial frequency reuse $(\eta < 1)$

We want to compare the universal frequency reuse scheme previously described with another scenario where the frequency reuse factor is less than one. For our 1D cell array model, the available bandwidth is split into two equal sub-bands and two adjacent cells are assigned different halves, therefore $\eta = 1/2$.
To remain coherent with the model described in subsection 3.2, we only take into account the MCI coming from adjacent cells. Actually, we consider a MCI power equal to zero since two adjacent cells transmit on distinct frequency bandwidths (no MCI coming from adjacent cells exists). The situation thus boils down to a single cell scenario where a given BS uses half the available spectrum. Therefore, Equations (19) and (20) can be reused but comparisons have to be performed for equivalent mean rates per cell volume unit $\overline{R}/|\mathscr{C}|$. In the same way, we have $\bar{r}' = \bar{r}/\eta$ and $\mathbb{E}[|S^{(c)}(m,n)|^2] = B'Q^{(c)}$ where $B' = B\eta$.

4. NUMERICAL ILLUSTRATIONS

As previously mentioned, simulations were carried out considering 2D cells and using two different path loss exponent values. We have considered a Free Space Loss (FSL) model characterized by a path loss exponent $s = 2$, and the so-called Okumura-Hata (O-H) model with $s = 3$ for open areas, which is widely used for predicting path loss in mobile wireless systems ([13]) for a carrier frequency $f_0 = 1.8$ GHz. The default value for the cell inner radius d is set to 150 m. Finally, the signal bandwidth is $B = 5$MHz and the noise power spectral density is $N_0 = -170$ dBm/Hz.

On Figure 2, we have plotted the power required by the BS to reach a mean rate $\bar{r}$ of 0.05 bit/s/Hz/km^2 versus the cell half size D for the two path loss exponent values in a single cell context. Notice that for a reasonable size of cell (*e.g.*, $|\mathscr{C}| = 20$ km^2), the spectral efficiency in each cell is equal to 1 bit/s/Hz which is a usual value. We have used the asymptotic expressions provided by Equations (19) and (20). For a given transmit power constraint, the maximum cell size can be simply deduced. For instance, be given a power constraint of 1 W, the maximum coverage area for a BS is characterized by $D_{max} = 5.9$ km and 5.6 km respectively, for path loss exponent values $s = 2$ and 3.

Figure 3 represents the corresponding quantities computed for a multi-cell environment and for the same mean rate requirement $\bar{r} = 0.05$ bit/s/Hz/km^2. Due to the presence of MCI, for a given power constraint, the maximum value of D is smaller than in the single cell case. For $s = 2$, this limit decreases from 5.9 km to 2.05 km and for $s = 3$, it decreases from 5.6 km to 2.5 km. Another but major difference w.r.t. the preceding curves is that when a certain value of D is reached, the required power tends to infinity. For the free space loss model, an asymptote exists at $D = 2.1$ km; it is shifted to $D = 2.6$ km for $s = 3$. These thresholds on the cell half size can be read on the curves of figure 4 where for a given distance (on x axis), the maximum mean rate requirement that can be satisfied is represented on y axis. In a dual manner, if one wants to design a network which is able to meet a certain mean rate requirement, we can deduce the maximum value of D, and thus the number of BS to deploy in a given area to cover.

Finally, on Figure 5, we compare the two frequency reuse strategies described in subections 3.2 and 3.3 by plotting the consumed power versus the size of the cell for “equivalent” mean rate requirements. The reference rate for the universal frequency reuse scheme is fixed to the same former value $\bar{r} = 0.05$ bit/s/Hz/km^2. Partial frequency reuse results are obtained by computing Equations (19) and (20) with $\eta = 1/9$. These curves show that for reasonable values of cell half size ($D \approx 2$ km for $s = 2$ and $D \approx 2.5$ km for $s = 3$), it is useless to implement frequency planning.

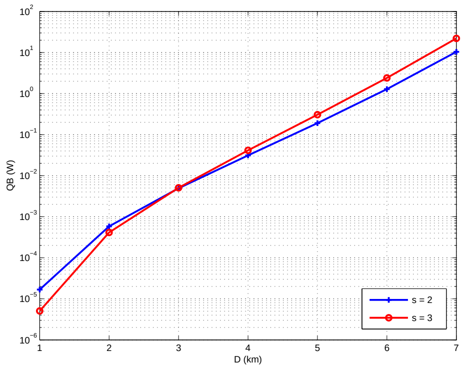

Figure 2: Required power vs. cell radius D in a single cell context

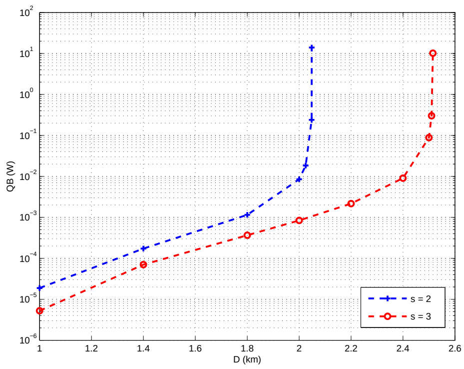

Figure 3: Required power vs. cell radius D in a multi-cell context

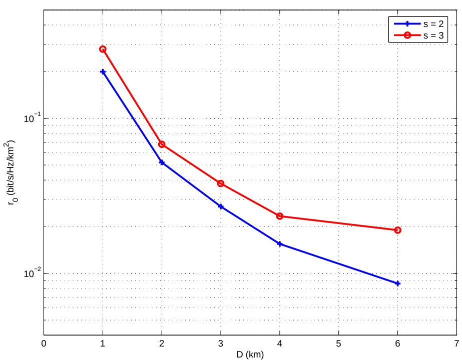

Figure 4: Limit on the mean rate R_0 vs. cell radius D

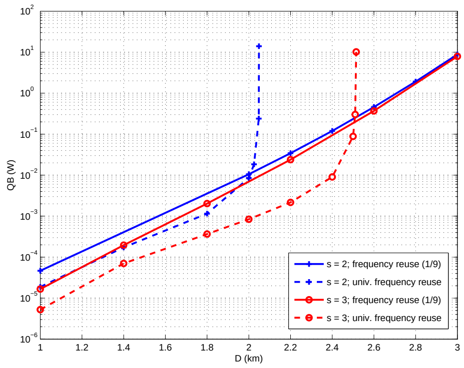

Figure 5: Comparison between universal frequency reuse and partial frequency reuse: required power vs. cell radius D

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