Cramer-Rao Bound for channel parameters in Ultra-Wide Band based system

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Abstract — **This paper focuses on channel estimation in standard Ultra-Wide Band (UWB) based system. We consider Impulse Radio scheme relying on a binary Pulse Position Modulation as well as on a Spread-Spectrum Time-Hopping multiple access. We address the closed-form expressions of the (resp. modified) Cramer-Rao bound for the multi-path channel parameters in Data-aided (resp. Non-data-aided) context. The derivations have been developed by assuming the multiple-access interference as an additive white Gaussian noise. Conversely the overlapping between signal echoes has been taken into account. Finally simplifications have been done by averaging the Fisher information matrix over the time-hopping random sequence as well as over the symbol random sequence.**

I. Introduction

For several years, the Ultra-Wide Band based communication has received a lot of attention, especially, in the context of short-range wireless communications. The main works focus essentially on the receiver design as well as on the multiple-access performance analysis ([1, 2]). The propagation channel parameters are still often assumed to be known at the receiver.

Therefore papers about channel parameter estimation concern are quite seldom ([3, 4]). In [3], Data-aided ML-like estimator is carried out in the case when a single mono-cycle is transmitted. In [4], ML estimator is introduced in the realistic scheme of Impulse Radio relying on a Pulse Position Modulation as well as on a Spread-Spectrum Time-Hopping multiple access for both Data-aided and Non-data-aided context. Moreover influence of estimation errors in the Rake receiver is performed numerically. One can notice that the overlapping between two signal echoes/paths are neglected in both papers, i.e., the authors consider that the signal provided by one path is orthogonal to each others. Furthermore one can remark that asymptotic theoretical performance of aforementioned estimators as well as Cramer-Rao bound have never been performed.

Therefore the purpose of this paper is to derive in closed-form expressions the (resp. Modified) Cramer-Rao Bound of the attenuations and delays of the different paths of the channel in Data-aided (resp. Non-data-aided) context. We consider the standard spread-spectrum multiple access (SSMA), time-hopping (TH), binary pulse-position modulated (PPM) based UWB system. At the receiver, we only concentrate on one user while the other users are viewed as noise.

The received signal $y(t)$ takes the following form

$$y(t) = \sum_{l=1}^{L} \gamma_l s(t-\tau_l) + w(t)$$

where $s(t)$ represents the UWB signal waveform of the user of interest. The noise $w(t)$ takes into account the thermal noise as well as the multiple-access interference, and is assumed to be white Gaussian with variance $\sigma^2 = \mathbb{E}[w(t)^2] = N_0/2$. The parameters γ_l and τ_l correspond to the attenuation and the delay of the l^{th} path respectively and need to be estimated. The attenuations and the delays are stacked in $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_L]$ and $\boldsymbol{\tau} = [\tau_1, \cdots, \tau_L]$ respectively.

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The UWB waveform can be defined as follows

$$s(t) = \sum_{i=0}^{M-1} b(t - iN_f T_f - a_i \Delta) \tag{1}$$

where M is the number of transmit symbols $\mathbf{a} = [a_0, \cdots, a_{M-1}]$. In the Data-aided context, data $\mathbf{a}$ are known at the receiver and thus refer to training sequence. In Non-data-aided context, data $\mathbf{a}$ are unknown and assumed to be i.i.d. with the following distribution function $p(a) = (\delta(a) + \delta(a-1))/2$ where $\delta(.)$ stands for the Dirac pulse. The term Δ represents the gap between two binary PPM symbols. Finally N_f is the number of frame per symbol and T_f is the duration of each frame. The super frame composed by N_f frames is structured as follows

$$b(t) = \sum_{j=0}^{N_f-1} g(t - jT_f - c_j T_c) \tag{2}$$

where T_c is the chip duration. We get $T_f = N_c T_c$ with N_c the number of chips in one frame. The time-hopping code in the j^{th} frame is given by $c_j \in \{0, \cdots, N_c - 1\}$. Finally $g(t)$ is the mono-cycle with the following temporal time-support $[0, T_g)$. Notice that we assume, as usual, $0 < \Delta < T_c - T_g$, i.e., the impulsion associated with the transmit bit (whatever its value) remains inside the current chip.

II. Cramer-Rao Bound

In the sequel, we treat the Data-aided case and the Non-data-aided case within the same framework. For this reason, the notation $\mathbb{E}_{\mathbf{a}}[f(\mathbf{a})]$ (where $f(.)$ is a certain mapping) stands either for $f(\mathbf{a})$ if $\mathbf{a}$ is a known deterministic sequence like a training sequence or for the standard statistical mean over $\mathbf{a}$ if $\mathbf{a}$ is an unknown random sequence.

The likelihood function of $\boldsymbol{\theta} = [\boldsymbol{\gamma}, \boldsymbol{\tau}, \mathbf{a}]$ is given by

$$\Lambda(\boldsymbol{\gamma}, \boldsymbol{\tau}, \mathbf{a}) \propto \exp\left\{ -\frac{1}{N_0} \int_{\mathcal{I}} [y(t) - \sum_{l=0}^{L} \gamma_l s(t-\tau_l)]^2 dt \right\} \tag{3}$$

where $\mathcal{I} = [0, MN_f T_f)$ represents the duration of the observation window. We now define the following term

$$J(\theta_l, \theta_k) = -\mathbb{E}_{y,\mathbf{a}} \left[\frac{\partial^2 \ln\Lambda(\boldsymbol{\gamma}, \boldsymbol{\tau}, \mathbf{a})}{\partial\theta_l \partial\theta_k} \right]. \tag{4}$$

The previous term represents the *true* Fisher information component for parameters (θ_l, θ_k) if Data-aided context is assumed. In contrast, Eq. (4) only provides the *modified* Fisher information component for parameters (θ_l, θ_k) if Non-data-aided context is considered [5].

Putting Eq. (3) back in Eq. (4) leads to the following result after straightforward algebraic manipulations.

$$\begin{aligned} J(\gamma_l,\gamma_k) &= \frac{2}{N_0} f_1^{(k,l)} \\ J(\gamma_l,\tau_k) &= -\frac{2\gamma_k}{N_0} f_2^{(l,k)} \\ J(\tau_l,\tau_k) &= \frac{2\gamma_k\gamma_l}{N_0} f_3^{(k,l)} \end{aligned}$$

where

$$\begin{aligned} f_1^{(k,l)} &= \mathbb{E}_{\mathbf{a}}\left[\int_{\mathcal{I}} s(t-\tau_k)s(t-\tau_l)dt\right] \\ f_2^{(k,l)} &= \mathbb{E}_{\mathbf{a}}\left[\int_{\mathcal{I}} s(t-\tau_k)s'(t-\tau_l)dt\right] \\ f_3^{(k,l)} &= \mathbb{E}_{\mathbf{a}}\left[\int_{\mathcal{I}} s'(t-\tau_k)s'(t-\tau_l)dt\right] \end{aligned}$$

with $s'(t) = ds(t)/dt$.

We hereafter wish to provide closed-form expressions for $f_m^{(k,l)}$. In [4], the previous term has been assumed to be null as soon as $k \neq l$. This means that they assume the signal echoes are orthogonal and overlapping do not exist between two paths. The assumption obviously simplifies the derivations, but does not necessary hold in realistic situation. One can notice that doing such an assumption for achieving simple ML-like estimate is justified but the approach is *a priori* flawed when the aim is to derive exact CRB and not an approximation. In subsection A, we however derive CRB for the non-overlapping case since the CRB is not available in the literature even in this simplified configuration. The subsection B introduces the main contribution of this paper. Indeed we derive closed-from expressions for $f_m^{(k,l)}$ even when $k \neq l$. Finally, in section IV, we compare the numerical values of both CRB in order to observe the influence of the overlapping paths on the performance. According to the shape of the Fisher information matrix, one can already assert that the Cramer-Rao bound in presence of overlapping is larger that the Cramer-Rao bound in absence of overlapping [14] but numerical simulations are necessary for evaluating the gap between both Cramer-Rao bounds.

A. Absence of overlapping

In case of absence of overlapping, the derivations boil down to those performed for amplitude and symbol timing estimation issue in the context of linearly modulated signal ([6, 5]). This leads to

$$f_m^{(l,l)} = MN_f E_m \tag{5}$$

with

$$\begin{aligned} E_1 &= \int g(t)g(t)dt, \\ E_2 &= \int g(t)g'(t)dt, \\ E_3 &= \int g'(t)g'(t)dt. \end{aligned}$$

Notice that the previous expression (5) holds for DA as well as for NDA context. One can especially remark that the performance does not depend on the training sequence in DA scheme. Consequently, we get

$$\mathrm{CRB}_{\mathrm{DA}}(\gamma_l) = \mathrm{MCRB}_{\mathrm{NDA}}(\gamma_l) = \frac{N_0}{MN_f}\frac{E_3}{2(E_1E_3-E_2^2)}$$

$$\mathrm{CRB}_{\mathrm{DA}}(\tau_l) = \mathrm{MCRB}_{\mathrm{NDA}}(\tau_l) = \frac{N_0}{MN_f}\frac{E_1}{2\gamma_l^2(E_1E_3-E_2^2)}.$$

The above formulae are the same as in the context of single-path ($L = 1$). This is not a surprising result since each path is not disturbed by the others ones as its echoes are orthogonal. Therefore the estimation step of path of interest is not influenced by its echoes.

At low SNR, derivations for $\mathrm{CRB}_{\mathrm{NDA}}$ can be achieved by the well-known approach mentioned in [6] and [4]. Nevertheless, due to the lack of space, the corresponding expressions are omitted in this communication.

B. Presence of overlapping

In this subsection, we do not neglect the overlapping paths anymore. We will see that the obtained expressions for $f_m^{(k,l)}$ are simple thanks to the so-called *developed code* which has been introduced in [7]. The developed code is a nice tool and a relevant way for describing the time-hopping code.

Let us now recall the notion of developed code. The integer c_j represents the chip number in which the signal has been put in the j^{th} frame, and belongs to the set $\{0,\cdots,N_c-1\}$. We now consider the vector $\tilde{\mathbf{c}}_j = [\tilde{\mathbf{c}}_j(0),\cdots,\tilde{\mathbf{c}}_j(N_c-1)]$ of size $1\times N_c$ described as follows [7]

$$\tilde{\mathbf{c}}_j(i) = \begin{cases} 1 & \text{if } i = c_j \\ 0 & \text{otherwise} \end{cases}.$$

Obviously c_j and the so-called developed code $\tilde{\mathbf{c}}_j$ provides the same information. Instead of providing the number of the occupied chip, the developed code indicates either the chip is occupied if the value is 1 or the chip is empty if the value is 0. Finally we concatenate all the vectors $\tilde{\mathbf{c}}_j$ into the following $1\times N_fN_c$ vector $\tilde{\mathbf{c}} = [\tilde{\mathbf{c}}_0,\cdots,\tilde{\mathbf{c}}_{N_f-1}]$. The components of $\tilde{\mathbf{c}}$ are defined as follows $(\tilde{c}(j))_{0\le j<N_fN_c}$. According to the developed code, one can remark that $b(t)$ can takes the following form

$$b(t) = \sum_{j=0}^{N_fN_c-1} \tilde{c}(j)g(t-jT_c).$$

This formulation of $b(t)$ compared to that given in Eq. (2) is more advantageous because the information about the status of the chip (occupied or free) is outside $g(t)$.

Before going further, we need to decompose the path difference as follows

$$\delta\tau_{k,l} = \tau_k - \tau_l = q_{k,l}T_c + \varepsilon_{k,l} \tag{6}$$

where $q_{k,l}$ is the floor integer part of $\delta\tau_{k,l}/T_c$ denoted by $\mathrm{int}[\delta\tau_{k,l}/T_c]$ and defined by $\mathrm{int}[x] \le x < \mathrm{int}[x]+1$. Moreover $\varepsilon_{k,l}$ represents the remainder. As usually done ([2, 4]), we consider that the maximum delay $\tau_{\max}$ is less that N_fT_f, the duration of a superframe. This implies that $q_{k,l} \in \{-N_fN_c,\cdots,N_fN_c-1\}$. By construction, we get $\varepsilon_{k,l} \in [0,T_c)$. The integer $q_{k,l}$ represents the number of shifted entire chips between two echoes. For instance, if $\delta\tau_{k,l}$ is positive, the beginning of the first chip of the signal $s(t-\tau_k)$ is coming during the $q_{k,l}^{\text{th}}$ chip of the signal $s(t-\tau_l)$. The real $\varepsilon_{k,l}$ represents the relative position of the beginning of both echoes once chip desynchronization has been corrected. For instance, if $\varepsilon_{k,l}$ is large, then the beginning of the chip of the signal $s(t-\tau_k)$ is coming at the end of the $q_{k,l}^{\text{th}}$ chip of the signal $s(t-\tau_l)$. This enable us to treat the problem of the overlapping more easily. Indeed our problem is now split into two easier problems : the first one deals with the overlapping due to the shift between whole chips (given by $q_{k,l}$), and the second one deals with the position (given by $\varepsilon_{k,l}$) inside the considered chips.

After straightforward algebraic manipulations relying on Eq. (1), we obtain

$$\begin{aligned} f_m^{(k,l)} = & \sum_{i_1,i_2=0}^{M-1}\sum_{j_1,j_2=0}^{N_fN_c-1} \tilde{c}(j_1)\tilde{c}(j_2) \\ & \mathbb{E}_{\mathbf{a}}[r_m(\delta iN_fT_f + \delta jT_c + \delta a\Delta + \delta\tau_{k,l})] \end{aligned}$$

with $\delta i = i_1 - i_2$, $\delta j = j_1 - j_2$, $\delta a = a_{i_1} - a_{i_2}$, and

$$r_1(\tau) = \int g(t-\tau)g(t)dt,$$

$$r_2(\tau) = \int g(t-\tau)g'(t)dt,$$

$$r_3(\tau) = \int g'(t-\tau)g'(t)dt.$$

By using Eq. (6), we get

$$f_m^{(k,l)} = \sum_{i_1,i_2=0}^{M-1} \sum_{j_1,j_2=0}^{N_fN_c-1} \tilde{c}(j_1)\tilde{c}(j_2)\mathbb{E}_{\mathbf{a}}[r_m(\alpha T_c + \beta)] \quad (7)$$

with the integer $\alpha = \delta i N_f N_c + \delta j + q_{k,l}$ and the real $\beta = \delta a \Delta + \varepsilon_{k,l}$.

As the support of $\tau \mapsto r_m(\tau)$ is $(-T_g, T_g)$, the number of terms in Eq. (7) can be strongly reduced. For proving that, we firstly consider that $\tau_k - \tau_l \geq 0$. The proof for the case $\tau_k - \tau_l < 0$ can be achieved in a similar way, and is omitted due to the lack of space.

The term $\mathbb{E}_{\mathbf{a}}[r_m(\alpha T_c + \beta)]$ occurring in the sum of Eq. (7) is different from 0 if and only if $-T_g < \alpha T_c + \beta < T_g$. As $-\Delta \leq \beta < \Delta + T_c$, we obtain that

$$-2T_c < \alpha T_c < T_c.$$

This implies that only the terms corresponding to either $\alpha = 0$ or $\alpha = -1$ occur in the sum of Eq. (7). In the sequel, we will derive the term corresponding to $\alpha = 0$. The term corresponding to $\alpha = -1$ can be treated similarly. For sake of space, we also remove the index (k, l).

The property $\alpha = 0$ leads to

$$\delta j = -q - \delta i N_f N_c. \quad (8)$$

As $\tau_k - \tau_l \geq 0$, q is positive and bounded as follows $0 \leq q \leq N_f N_c - 1$. By construction, we get $|\delta j| \leq N_f N_c - 1$. Gathering both previous inequality enable us to prove that Eq. (8) holds if and only if either $\delta i = 0$ or $\delta i = -1$. If $\delta i = 0$, then $j_1 = -q + j_2$. This implies that the term appearing in Eq. (7) for $\alpha = 0$ and $\delta i = 0$ writes as follows

$$M \left(\sum_{j=0}^{N_fN_c-q-1} \tilde{c}(j)\tilde{c}(j+q) \right) r_m(\varepsilon).$$

The case $\alpha = 0$ and $\delta i = -1$ can be obtained thanks to a similar procedure.

Finally, by adding the terms obtained for $\alpha = 0$ and $\alpha = -1$, Eq. (7) can be simplified as follows

$$\begin{aligned} f_m^{(k,l)} &= M\left[\mathcal{C}(|q|)A_\varepsilon + \mathcal{C}(|q+1|)A_{\varepsilon - T_c} \right. \\ &\quad \left. + \mathcal{D}(|q|)B_\varepsilon + \mathcal{D}(|q+1|)B_{\varepsilon - T_c}\right] \end{aligned} \quad (9)$$

where

$$\mathcal{C}(q) = \sum_{j=0}^{N_fN_c-q-1} \tilde{c}(j)\tilde{c}(j+q)$$

and

$$\mathcal{D}(q) = \sum_{j=0}^{q-1} \tilde{c}(j)\tilde{c}(j + N_f N_c - q)$$

with

$$A_\varepsilon = r_m(\varepsilon)$$

and

$$B_\varepsilon = \frac{1}{M} \sum_{i=0}^{M-1} \mathbb{E}_{\mathbf{a}}[r_m((a_{i\pm 1} - a_i)\Delta + \varepsilon)].$$

Note that Eq. (9) also holds for $\delta\tau$ negative. Besides the notation $'\pm'$ stands either for $'+'$ if $\delta\tau$ negative or for $'-'$ if $\delta\tau$ positive.

Eq. (9) enables us to put several comments. Obviously, according to the expressions of $\mathcal{C}$ and $\mathcal{D}$, the number of collisions between shifted chips significantly influences the performance. However these collisions can be canceled or enforced according to the relative position of both echoes given by $\varepsilon_{k,l}$ within the chip. As the area occupied by non-null signal inside the chip is very short (T_g) compared to the duration of an entire chip (T_c), the collision between both occupied chips is often null because both signals do not lie in the same area of the chip.

Let $\mathcal{I}_1 = [T_g, \Delta - T_g]$, $\mathcal{I}_2 = [T_g + \Delta, T_c - T_g - \Delta]$, and $\mathcal{I}_3 = [T_c + T_g - \Delta, T_c - T_g]$ be three intervals. After simple algebraic manipulations, one can proven that, if $\varepsilon_{k,l} \in \mathcal{I}_1$, or if $\varepsilon_{k,l} \in \mathcal{I}_2$, or if $\varepsilon_{k,l} \in \mathcal{I}_3$, for all (k, l), then we do not encounter any overlapping and thus results drawn in subsection A hold. Notice that, when $\Delta < T_g$, the above condition for non-overlapping simplifies because intervals $\mathcal{I}_1$ and $\mathcal{I}_3$ become empty. One can also remark that if we consider $q_{k,l} = 0$ and $\varepsilon_{k,l} = 0$ (i.e. , we compare the same path and $k = l$), then Eq. (9) boils down to Eq. (5) hopefully.

III. Average Fisher information matrix

In this section, we wish to obtain simplified expressions for the terms $\mathcal{C}$,$\mathcal{D}$ (resp. A,B) occurring in Eq. (9) by averaging them over the time-hopping code (resp. symbol sequence).

First of all, assume that the CRB, as well as, the associated Fisher information matrix J depends on one parameter $\mathbf{x}$ over which we wish to average the performance. Consequently, it is equivalent to derive the following term $\mathbb{E}_{\mathbf{x}}[\text{CRB}(\mathbf{x})]$. Due to the inversion of the Fisher information matrix (FIM), the previous term is often intractable. Therefore we replace the previous term with the average of the FIM following by its inversion. This operation makes sense since the Jensen's inequality leads to

$$\mathbb{E}_{\mathbf{x}}[\text{CRB}(\mathbf{x})] = \mathbb{E}_{\mathbf{x}}[J(\mathbf{x})^{-1}] \geq \mathbb{E}_{\mathbf{x}}[J(\mathbf{x})]^{-1}.$$

In the sequel, the right hand side of the above equation will be called "modified-type CRB". Obviously this CRB is looser than the true CRB but the conclusions that we will draw are nevertheless relevant.

A. Average over symbol sequence

Averaging over the symbol sequence makes sense in NDA mode as well as in DA mode. For the NDA mode, the symbol sequence is random sequence by definition. For the DA mode, it is convenient to consider the training sequence as a realization of binary pseudo-random process. Then in order to obtain performance regardless of the selected training sequence, it is usual to average the CRB over the model followed by the training sequence [8]. Unlike [8], we hereafter obtain the modified-type CRB and not the exact average CRB. Then, both modes (NDA/DA) lead to the same stochastic model for which the sequence is i.i.d. with the pdf $p(a)$ given in the Introduction.

As A_ε does not depend on $\mathbf{a}$, we only focus on the term B_ε. We finally obtain that

$$B_\varepsilon = \frac{1}{2}A_\varepsilon + \frac{1}{4}A_{\varepsilon-\Delta} + \frac{1}{4}A_{\varepsilon+\Delta}.$$

Notice that, for the NDA scheme, we provide the so-called modified Cramer-Rao Bound [5].

B. Average over time-hopping code

As done for the training sequence in previous subsection, it is usual to assume that each vector $\tilde{\mathbf{c}}_j$ is the realization of i.i.d. random vector whose each component admits the following distribution $p(c) = ((N_c - 1)\delta(c) + \delta(c-1))/N_c$. Consequently the whole vector $\tilde{\mathbf{c}}$ is a realization of i.i.d. random vector, and the probability for its component $\tilde{c}(j)$ to be equal to 0 is $(N_c - 1)/N_c$ and to be equal to 1 is $1/N_c$. According to such a distribution, we get that $\mathcal{C}(q)$ and $\mathcal{D}(q)$ satisfy a binomial distribution with the following mean [9]

$$\begin{cases} \mathbb{E}_{\mathbf{c}}[\mathcal{C}(q)] = \frac{N_f N_c - q}{N_c^2} & \text{if } q \neq 0 \\ \mathbb{E}_{\mathbf{c}}[\mathcal{C}(0)] = N_f & \text{if } q = 0 \end{cases}$$

and

$$\begin{cases} \mathbb{E}_{\mathbf{c}}[\mathcal{D}(q)] = \frac{q}{N_c^2} & \text{if } q \neq N_f N_c \\ \mathbb{E}_{\mathbf{c}}[\mathcal{D}(N_f N_c)] = N_f & \text{if } q = N_f N_c. \end{cases}$$

IV. Simulations

In this section, we just evaluate the Cramer-Rao bound because we only focus on the performance of the estimates. In a further journal version, Bit Error Rate (BER) will be computed at the output of standard Rake receiver. Secondly we consider the Non-data-Aided (NDA) mode. Therefore the displayed CRB refers to the standard modified CRB for which the associated FIM has been averaged over time-hopping code sequence as explained in section III.

For each figure, we plot i) the CRB for various model of channels and by taking into account the possible paths overlapping and ii) the (simplified) CRB which does not take into account the overlapping.

In order to handle all the paths, we sum the Cramer-Rao bound in the following way

$$\text{MCRB}(\boldsymbol{\gamma}) = \frac{1}{L}\sum_{l=1}^{L}\text{MCRB}(\gamma_l) \quad \text{MCRB}(\boldsymbol{\tau}) = \frac{1}{L}\sum_{l=1}^{L}\text{MCRB}(\tau_l).$$

The design parameters of the UWB system are chosen as in [12] : $T_c = 0.9$ns, $T_g = 0.2877$ns, $\Delta = 0.15$ns, $N_c = 8$, and $N_f = 4$. The pulse shape is the second derivative of the Gaussian function [4].

In Figures 1 and 2, we consider an academic context : two paths with amplitudes $\gamma_1 = 1$ and $\gamma_2 = 0.5$, and with delays $\tau_1 = 0$ and $\tau_2 = \delta\tau$. The modulus of $\delta\tau$ is fixed to be less than T_c. The SNR and the number of superframe M are equal to 20dB and 100 respectively.

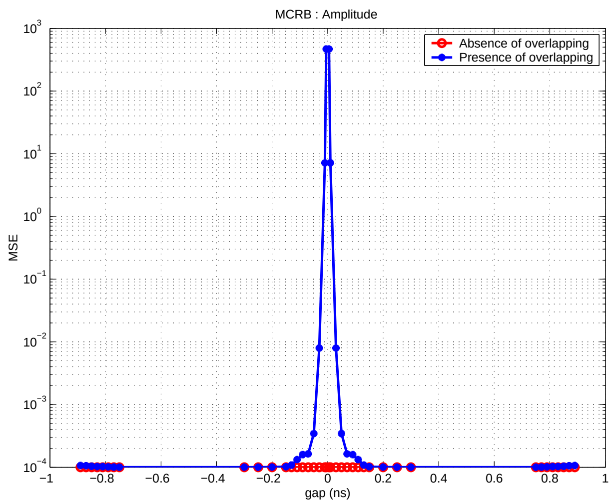

Figure 1: MCRB($\boldsymbol{\gamma}$) versus $\delta\tau$

As $\Delta < T_g$, we know that there is overlapping if $\delta\tau$ belongs to $[-T_c, -(T_c - T_g - \Delta)] \cup [-(T_g + \Delta), T_g + \Delta] \cup [T_c - T_g - \Delta, T_c]$ (cf. the end of section II). We observe that the overlapping has a numerical influence for much smaller interval. Actually the overlapping degrades dramatically the performance if the delay gap is less than T_g and so very close to 0.

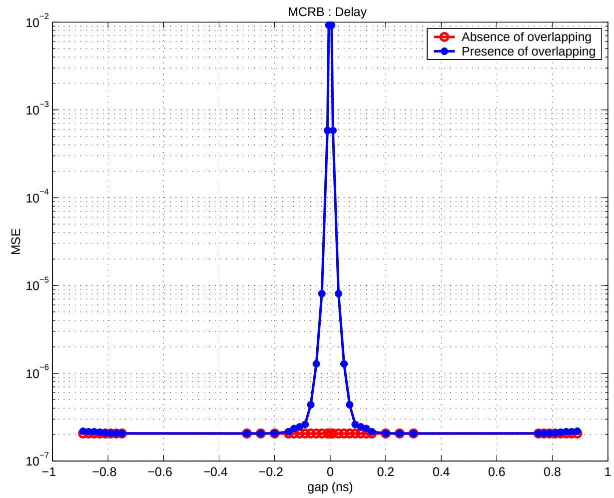

Figure 2: MCRB($\boldsymbol{\tau}$) versus $\delta\tau$

We now want to know if the situation for which the overlapping can not be neglected (i.e., for which there exits small gap between two paths) appears sufficiently often in a realistic model of channel for disturbing notably the performance. Therefore, in the rest of this section, we consider the standard model of Saleh & Valenzuela [10]. For sake of simplicity, we consider only one cluster : then the difference between two consecutive delays satisfies an exponential distribution with parameter λ. Finally the amplitudes are obtained as $\gamma_l = g * e^{-\tau_l/\gamma}$ where g is a Gaussian distributed random variable and where γ is a constant. Moreover the paths obey the following normalisation condition $\sum_{l=1}^{L}\gamma_l^2 = 1$. As done Saleh & Valenzuela ([10]), we put $\gamma = 20$ns and $1/\lambda = 5$ns. We inspect also the case where $\gamma = 5$ns and $1/\lambda = 0.5$ns introduced in Lee ([11]). The number of paths is limited to 3. The curves are averaged over 120 Monte-Carlo trials for which the symbols and the paths are modified. Finally, in order to exhibit reasonable performance, we have canceled too close adjacent paths and the paths associated with too small magnitude. Thus if $|\delta\tau_{k,l}| < 10^{-2}$ns and/or if $\gamma_l < 10^{-2}$, the l^{th} path is dropped.

In Figures 3 and 4, the Mean-Square Error (MSE) are plotted versus E_b/N_0. The number of superframe is equal to $M = 100$. We notice that the performance for Saleh & Valenzuela model is insensitive to overlapping. In contrast, there is a gap, for the Lee model, because overlapping can not be neglected. Actually, according to the value of (γ, λ), Lee model leads to several close paths (e.g., a classical gap between two paths is of order 0.1ns) although for Saleh & Valenzuela model, the gap between two consecutive paths is much larger. In Figures 1 and 2, notice that the overlapping has to be taken into account if the difference between two paths is of order 0.2ns.

In Figures 5 and 6, the MSE is plotted versus M with SNR $= 20$dB.

V. Perspectives

In a further journal version, we will extend this work in various ways : i) The nature of multiple access interference should be taken into account. ii) The Maximum-Likelihood estimator introduced in [4] and performed for a non-overlapping context should be computed in an overlapping environment and then compared to the CRB. iii) In a few works [13], channel estimation is done by using an undersampled UWB signal (with respect to Nyquist rate). It should be worthy to evaluate the loss in performance due to this operation. iv) As done in DOA, we should derive the resolution beyond which distinguish two paths makes sense.

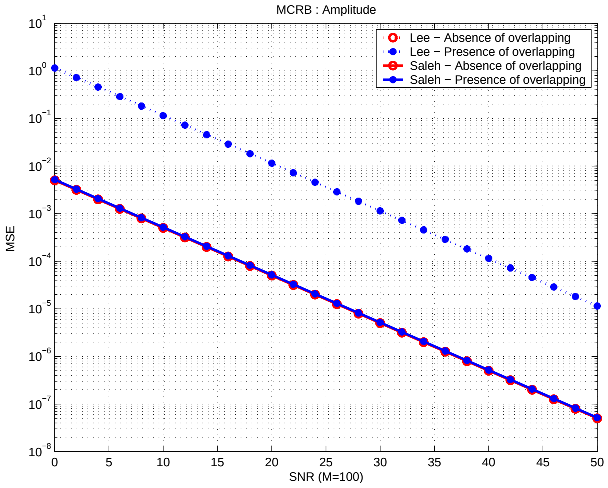

Figure 3: MCRB(γ) versus E_b/N_0

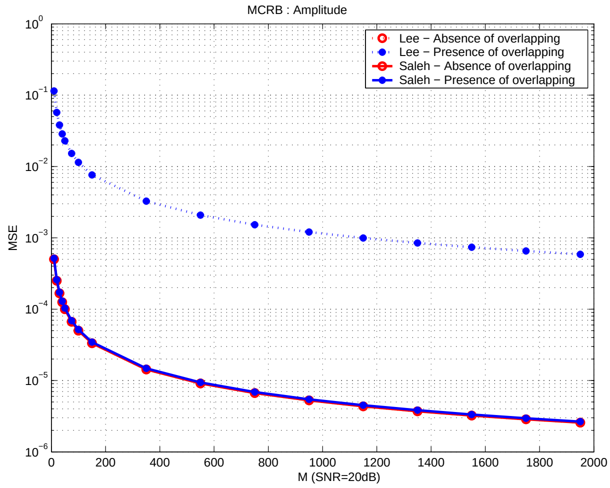

Figure 5: MCRB(γ) versus M

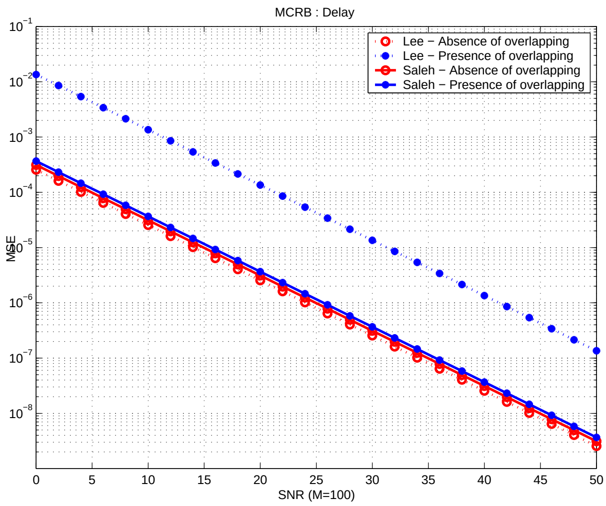

Figure 4: MCRB(τ) versus E_b/N_0

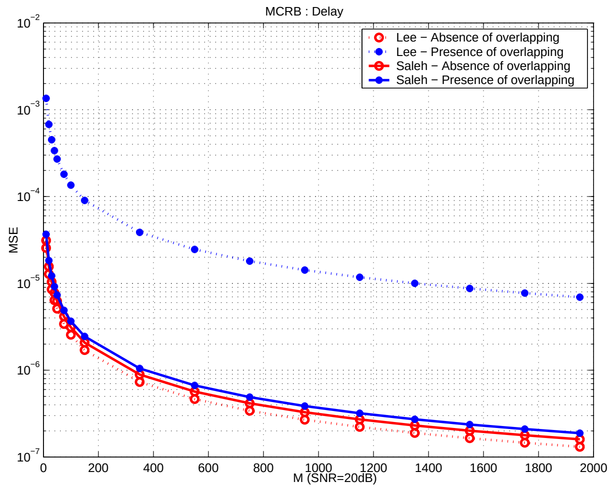

Figure 6: MCRB(τ) versus M

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