A BLIND FREQUENCY OFFSET ESTIMATOR FOR OFDM/OQAM SYSTEMS

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ABSTRACT

Like other OFDM-like systems, OFDM/OQAM systems are very sensitive to carrier frequency offset. A new blind carrier frequency estimator is introduced herein for OFDM-/OQAM systems by exploiting the non-circularity of the received signal. Since the received signal exhibits conjugate cyclic frequencies at twice the carrier frequency offset, a frequency estimator is proposed by maximizing a cost function expressed in terms of the sample conjugate cyclocorrelations. Computer simulations illustrate that the proposed estimator is very accurate whenever large observation windows are available, is robust to timing errors and frequency-selective fading effects.

1. INTRODUCTION

The Orthogonal Frequency Division Multiplexing (OFDM) system, which belongs to the family of multi-carrier transmission schemes, has been developed for combating efficiently the inter-symbol interference on multipath channels. Its main advantages are the very low computational cost (the receiver consists only of a Fast Fourier Transform (FFT) and, if necessary, of filterbank type devices) and the simplified equalization step [1]. For several years, the OFDM-like techniques have received increasing attention and are employed in the European digital radio broadcasting (DAB), digital terrestrial TV broadcasting (DVBT), indoor wireless systems (HIPERLAN), and Internet on twisted pair (ADSL).

However, it is well-known that OFDM-like techniques are more sensitive to carrier frequency offset than single carrier techniques [2]. The frequency offset (due to Doppler shifts and local oscillator drifts) gives rise to inter-carrier interference (ICI) which degrades dramatically the performance. Therefore, removing the frequency offset at the front-end of the receiver is a crucial task.

A lot of techniques based on the OFDM principle have been proposed in the literature. The standard OFDM system consists of the concatenation of an Fast Fourier transform and a guard interval. This simplified structure of the transmitter results from the choice of a linear modulation shaped by means of a rectangular window. As the time-frequency localization of such shaping windows is not compact, considerable research attention has been allocated to developing alternative modulations such as OFDM/OQAM systems, which consist of a combination between an offset quadrature-amplitude modulation and a square-root Nyquist pulse-shaping filter. For such systems, an equalization step becomes necessary because the guard interval is omitted. Indeed, this guard interval makes sense only in the case of simple FFT-based transmitters ([3, 4]).

This paper deals with the problem of frequency offset estimation in OFDM/OQAM systems. A non-data-aided (blind) approach is proposed to preserve the spectral efficiency of such modulation techniques.

Several works regarding the blind frequency offset estimation problem have been reported for the classical OFDM systems. Among these, the subspace based methods have been recently proposed ([5, 6]). Nevertheless, the validity domain of such methods is quite restrictive since the presence of virtual sub-carriers has to be assumed. Furthermore, a maximum likelihood based estimator has been introduced under the strong assumption of a flat-fading channel ([7]). Finally, this restrictive assumption is also required for the estimator introduced in [8], which relies on correlations of the oversampled received signal. This estimator is an extension of the approach proposed in [9] to the OFDM framework. It performs well for flat-fading channels and is just about robust over multi-path propagation channels. Recently, a new estimator that exploits the cyclostationary statistics of the received signal has been introduced in [10], under the following restrictive assumption: the symbol constellation is non-circular, i.e., the mathematical expectation of the square symbol is non-zero.

A natural extension of the estimator described in [8] to an OFDM/OQAM type system is proposed in [11]. One can observe that the performance and the properties of these estimators are closely similar. Similarly to the analysis performed in [11], we propose herein a generalization of the estimator introduced in [10] to OFDM/OQAM systems.

As the real and the imaginary parts of the transmitted signal in an OFDM/OQAM system are not statistically identical, such a modulating signal is intrinsically non-circular. Therefore, the estimator introduced in [10] can be extended without restriction to the considered OFDM system. Consequently, the purpose of the paper is to introduce and analyze this new estimator in the context of OFDM/OQAM systems.

2. NEW ESTIMATOR

For an OFDM/OQAM system, the transmitted continuous-time baseband signal $x_a(t)$ can be written as follows

$$x_a(t) = \sum_{q=0}^{Q-1} x_q(t)$$

where each component signal

$$x_q(t) = \sum_{l\in\mathbb{Z}} (a_{q,l} g_a(t-lT) + i b_{q,l} g_a(t-lT+\frac{T}{2})) e^{2i\pi \frac{q}{T}(t-lT)}$$

corresponds to a linear offset modulation based transmission translated to the subcarrier q/T. The sequences $a_{q,l}$ and $b_{q,l}$ are real-valued and belong to PAM-type modulations. Thus, the complex valued sequence $s_{q,l} = a_{q,l} + ib_{q,l}$ can be interpreted as a QAM-type modulation. Moreover, we assume that $a_{q,l}$ and $b_{q,l}$ are independently and identically distributed (i.i.d.) with unit-variance. The duration of a complete OFDM symbol which corresponds to the set $\{s_{0,l}, \cdots, s_{Q-1,l}\}$ is equal to T. Consequently, the period of each information symbol $s_{q,l}$ is $T_s = T/Q$. Furthermore, the pulse $g_a(t)$ is usually a square-root raised cosine (with roll-off ρ and built assuming the OFDM-rate $1/T$) instead of a rectangular window. Without any restriction, one can assume that the mapping $t \mapsto g_a(t)$ is time-limited with the time support $[-LT_s, LT_s]$. Lastly, no guard interval is inserted even in the presence of a frequency-selective channel context.

For sake of simplicity, we assume first that the transmitted signal passes through a flat-fading channel as in [11]. Hence, the continuous-time baseband received signal $y_a(t)$ takes the following form

$$y_a(t) = x_a(t) e^{2i\pi\delta f_0 t} + w_a(t) \quad (1)$$

where δf_0 is the carrier frequency offset, and $w_a(t)$ stands for the additive zero-mean Gaussian noise.

Our aim is to estimate the carrier frequency offset from the sole knowledge of the symbol-rate sampled received signal.

According to equation (1), the discrete-time signal $y(n) = y_a(nT_s)$ can be written as follows

$$y(n) = \left(\sum_{q=0}^{Q-1} x_q(n)\right) e^{2i\pi\Delta f_0 n} + w(n) \quad (2)$$

where

$$x_q(n) = \sum_{l\in\mathbb{Z}} \left(a_{q,l} g(n-lQ) + i b_{q,l} \tilde{g}(n-lQ)\right) e^{2i\pi\frac{q}{Q}(n-lQ)},$$

$g(m) := g_a(mT_s)$, $\tilde{g}(m) := g_a(mT_s + QT_s/2)$, $w(n) := w_a(nT_s)$. Finally, $\Delta f_0 = (\delta f_0 T_s \bmod 1)$ where $(a \bmod b)$ stands for a modulo b. By convention, $(a \bmod b)$ belongs to the interval $[0, b[$.

Before going further, we recall that a zero-mean discrete-time stochastic process $p(n)$ is said unconjugate (conjugate) cyclostationary if the unconjugate (conjugate) correlation coefficients $\mathbb{E}[p(m+n)\overline{p(n)}]$ [1] ($\mathbb{E}[p(m+n)p(n)]$, respectively) can be expressed in terms of a Fourier series expansion, i.e.,

$$\mathbb{E}[p(m+n)\overline{p(n)}] = \sum_{k=0}^{\infty} r^{(a_k)}(m) e^{2i\pi a_k n},$$

where $F := \{a_k\}_{k\geq 0}$ is the countable set of the so-called cyclic frequencies of $p(n)$. The sequence $\{r^{(a_k)}(m)\}_{m\in\mathbb{Z}}$ denotes the cyclocorrelation sequence at cyclic frequency a_k of $p(n)$.

In [11], it has been proved that the **unconjugate** correlations of the OFDM/OQAM signal $y(n)$ are cyclostationary with cycles $F = \{q/Q\}_{q=0,\cdots,Q-1}$. Also, it turns out that the phases of the associated unconjugate cyclocorrelations directly depend on the unknown carrier frequency offset. Based on this observation, [11] develops a frequency offset estimator whose performance is improved whenever the subcarriers are weighted with different factors. The weighting of the carriers obviously restricts the application field of this method.

We have observed that the **conjugate** correlations of the received signal are non-zero and are cyclostationary with the set of cycles $F = \{\alpha_0 + q/Q\}_{q=0,\cdots,Q-1}$, where $\alpha_0 = (2\Delta f_0 \bmod 1)$. Since Q is assumed to be known, estimating Δf_0 boils down to estimating α_0. Therefore, a frequency offset estimator can be developed by maximizing a certain cost function expressed in terms of the conjugate cyclocorrelations.

Let $r_c(n,\tau) = \mathbb{E}[y(n+\tau)y(n)]$ denote the conjugate correlation at time index n and lag τ of $y(n)$. After straightforward derivations, it turns out that

$$r_c(n,\tau) = \left(\sum_{q=0}^{Q-1} e^{2i\pi\frac{2q}{Q}(n+\tau/2)}\right) G(n,\tau) e^{2i\pi\alpha_0(n+\tau/2)}$$

where

$$\begin{aligned} G(n,\tau) \;=\; & \sum_{l\in\mathbb{Z}} (g(n+\tau-lQ)g(n-lQ) \\ & -\tilde{g}(n+\tau-lQ)\tilde{g}(n-lQ)) \end{aligned}$$

Obvious manipulations lead to

$$\begin{cases} r_c(n,\tau) = QG(n,\tau)e^{2i\pi\alpha_0(n+\frac{\tau}{2})} & \text{if } n = (-\frac{\tau}{2} \bmod \frac{Q}{2}) \\ r_c(n,\tau) = 0 & \text{otherwise} \end{cases} \quad (3)$$

[1] The overline stands for the complex-conjugate.

As g and $\tilde{g}$ are different, one can check that $n \mapsto G(n,\tau)$ is not reduced to zero and is periodic with period Q. This implies that the conjugate correlation of the received signal is non-zero, cyclostationary, and takes the following generic form

$$r_c(n,\tau) = \sum_{q=0}^{Q-1} r_c^{(\alpha_0+q/Q)}(\tau) e^{2i\pi(\alpha_0+q/Q)n}, \qquad (4)$$

where $r_c^{(\alpha_0+q/Q)}(\tau)$ is the conjugate cyclocorrelation at lag τ and can be expressed as

$$r_c^{(\alpha)}(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} r_c(n,\tau) e^{-2i\pi\alpha n} \qquad (5)$$

According to (3), we obtain

$$\begin{cases} r_c^{(\alpha_0+\frac{q}{Q})}(\tau) = -2G(\frac{Q-\tau}{2},\tau)e^{2i\pi(\frac{q}{2Q}+\frac{\alpha_0}{2})\tau} & \text{for } q \text{ odd} \\ r_c^{(\alpha_0+\frac{q}{Q})}(\tau) = 0 & \text{for } q \text{ even} \end{cases}$$

As soon as $|\Delta f_0| < 1/Q$, the knowledge of the set F provides exactly the value of α_0. Indeed, α_0 is the sole value in the interval $[-1/Q, 1/Q[$ for which $r_c^{(\alpha+q/Q)}(\tau)$ is non-zero, whenever q is an odd integer. Thus, α_0 satisfies the following equality

$$\alpha_0 = \arg \max_{\alpha\in[-\frac{1}{Q},\frac{1}{Q}[} J(\alpha), \quad J(\alpha) = \sum_{q \text{ odd}} w_q \left\| \mathbf{r}_c^{(\alpha+q/Q)} \right\|^2$$

with $\mathbf{r}_c^{(\alpha)} := [r_c^{(\alpha)}(-\Upsilon), \cdots, r_c^{(\alpha)}(\Upsilon)]^{\mathrm{T}}$, Υ is an integer, and the weights w_q are some positive scalars. In practice, the cyclocorrelation vector $\mathbf{r}_c^{(\alpha)}$ has to be estimated because only N observations are available. The sample estimate of $\mathbf{r}_c^{(\alpha)}$ is obtained by dropping the limit and the mathematical expectation in (5). This leads to the sample estimate:

$$\hat{\mathbf{r}}_{c,N}^{(\alpha)} := \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{z}(n) e^{-2i\pi\alpha n}$$

with $\mathbf{z}(n) := [y(n-\Upsilon)y(n), \cdots, y(n+\Upsilon)y(n)]^{\mathrm{T}}$. Then, the corresponding estimate of α_0, denoted by $\hat{\alpha}_N$, is defined by:

$$\hat{\alpha}_N = \arg \max_{\alpha\in[-\frac{1}{Q},\frac{1}{Q}[} J_N(\alpha), \; J_N(\alpha) = \sum_{q \text{ odd}} w_q \left\| \hat{\mathbf{r}}_{c,N}^{(\alpha+q/Q)} \right\|^2.$$

As the OFDM/OQAM signal is noncircular, we observe that the approach introduced in [10] for classical OFDM can be extended to the OFDM/OQAM context.

To analyze the influence of the weighting and of the number of considered cyclocorrelations, we inspect the two extreme cases. First, the *reduced* estimator associated with the criterion $J_r(\alpha) = |r_c^{(\alpha+1/Q)}(0)|^2$ only takes advantage of the cyclocorrelation coefficient at lag 0 around the first cyclic frequency. Second, the *complete* estimator is based on the criterion $J_c(\alpha) = \sum_{q \text{ odd}} \|\mathbf{r}_c^{(\alpha+q/Q)}\|^2$ and considers the entire cyclocorrelation vectors around all the cyclic frequencies. An asymptotic analysis, which will be introduced in the journal paper version of this work ([12]), can be performed to analyze the relative theoretical behavior of the two estimators. Nevertheless, in this conference paper, we are content with numerical simulations. In the next section, we show that the complete estimator numerically outperforms the reduced one.

3. NUMERICAL ILLUSTRATIONS

Let $\mathbf{e}(n) = \mathbf{z}(n) - \mathbb{E}[\mathbf{z}(n)]$ be a zero-mean noise-like process. Then one can remark ([10]) that

$$\mathbf{z}(n) = \sum_{q \text{ odd}} \mathbf{r}_c^{(\alpha_0+l/Q)} e^{2i\pi(\alpha_0+q/Q)n} + \mathbf{e}(n). \qquad (6)$$

Thus, $\mathbf{z}(n)$ can be interpreted as sum of harmonics embedded in additive noise. Moreover, one can check that the empirical criterion $J_N(\alpha)$ corresponds to the periodogram of $\mathbf{z}(n)$. This implies that the cost function $J_N(\alpha)$ should have the following shape: peaks around $(\alpha_0 + q/Q)$ (with q odd) and a ground noise like flat level otherwise.

In practice, it is well known that the cost function has several local maxima. Therefore, the computation of the estimate $\hat{\alpha}_N$ has to be performed into two steps. In the first step (referred to as the coarse search), the objective is to detect the peaks and the function $J_N(\alpha)$ is thus evaluated on an FFT grid. In the second step (fine search), a gradient minimization algorithm of $J_N(\alpha)$, initialized at the estimate provided by the first step, is performed in order to obtain the estimate $\hat{\alpha}_N$.

Both steps have to be analyzed separately. The first one, which is connected to the so-called *outliers* effect ([13]), is theoretically difficult to analyze. Indeed, the asymptotic analysis can not predict its performance. The relevant criterion is the probability of right detection of the peak around one harmonic. We assume that the coarse search leads to a right detection if it selects the closest frequency of the grid to the sought cyclic frequency. As for the second step, the mean square error (MSE) based criterion is as usual relevant.

The following simulation parameters are used throughout this section. The roll-off factor of the shaping filter $g_a(t)$ is equal to $\rho = 0.2$, and the noise $w(n)$ is white with the variance $\sigma^2 = \mathbb{E}[|w(n)|^2]$. The number of OFDM symbols is denoted by $M = N/Q$, and the Signal-to-Noise Ratio (SNR) is defined as $2/\sigma^2$. We average the experimental results over $MC = 500$ independent Monte-Carlo runs.

In Table 1, we plot the false detection probability of the peak versus M and SNR for the cases $Q = 4$ and $Q = 16$. In the reduced case, we observe that the peak is often not detected because it is hidden by the noise contribution. This is due to the numerical weakness of the cyclocorrelation at cyclic frequency $(\alpha_0 + 1/Q)$ and to the lack of information contained in the cyclocorrelation coefficient at lag 0. Moreover the false detection probability decreases very slowly as soon as either the SNR or the number of symbols strongly increases. In contrast, we note that the estimate provided by the complete criterion succeeds assuming much lower M or SNR. Furthermore, for larger values of Q, a better detection performance can be observed. Nevertheless, our estimation procedure (even with the complete estimate) requires large observation windows in order to provide reliable coarse estimates.

Table 1. False detection occurrence (in percent) for $Q = \{4, 16\}$

$Q = 4$						
	SNR	0	5	10	15	20
Reduced	$M = 256$	99	98	95	91	89
Complete		99	78	38	15	10
Reduced	$M = 512$	99	97	85	78	73
Complete		97	30	0.4	0	0
Reduced	$M = 1024$	99	80	75	62	55
Complete		95	8	0	0	0

$Q = 16$						
	SNR	0	5	10	15	20
Reduced	$M = 128$	99	99	98	98	98
Complete		99	92	71	53	47
Reduced	$M = 256$	99	99	98	98	98
Complete		98	70	15	2	1
Reduced	$M = 512$	99	99	98	98	98
Complete		95	8	0	0	0

In the next experiments, as the first step of the reduced estimate often fails, we only evaluate the empirical normalized Mean Square Error (MSE) for the complete estimate. The solid line is obtained for the entire algorithm (the first step followed by the second one) The dashed-curve corresponds to the complete estimate when the second step is initialized with a good (forced) estimate. In Figure 1, we depict the MSE versus SNR for $Q = 4$. As expected, the effect due to outliers appears only during the coarse search and appears at SNR levels below 10 dB. In Figure 2, we plotted the MSE versus M for $Q = 4$. The outliers effect vanishes when M increases. Moreover we observe that the convergence rate is equal to $1/M^3$. As our estimation problem refers to harmonic retrieval (cf. Eq. (6)), such a convergence rate is not surprising ([10, 12]).

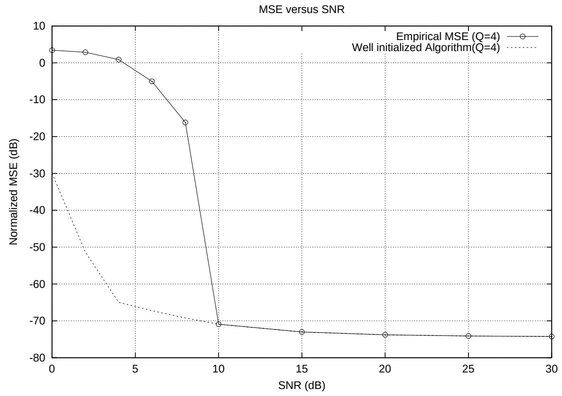

Fig. 1. MSE versus SNR (M=512)

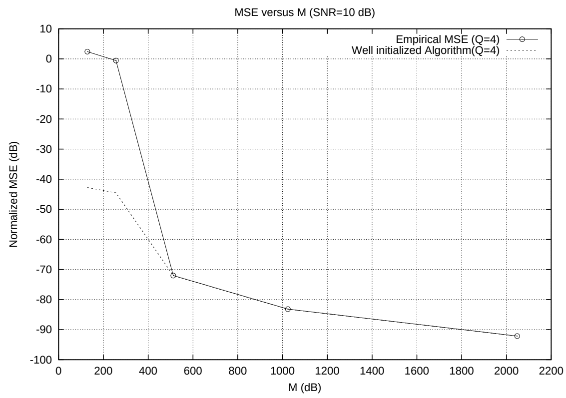

Fig. 2. MSE versus M (SNR=10dB)

We now consider the scenario when the transmitted signal passes through an unknown frequency-selective channel. The received signal is expressed as follows

$$y_a(t) = \sum_{k=1}^{K} \lambda_k x_a(t - \tau_k)$$

Two cases are studied: in the first one, only a timing error occurs, i.e., $K = 1$ and $\tau = \tau_1$. The timing error τ is assumed to be smaller than the sampling period $T_s = T/Q$. In the second case, we carry out an actual non-flat-fading channel. The complex amplitudes $\{\lambda_k\}_{k=1,\cdots,K}$ are Gaussian distributed and the time delays $\{\tau_k\}_{k=1,\cdots,K}$ are uniformly distributed in $[0, 3T_s]$. In a standard OFDM scheme with guard interval, the delay spread is assumed to be smaller than the OFDM symbol duration in order to ensure a reasonable length for the guard interval. Therefore, our assumption concerning the delay spread is not restrictive. In Figure 3, we plot the MSE versus SNR for the unsynchronized system (circle point) and for the filtered system (cross point). Note that the timing error and the channel are modified at each Monte Carlo trial. We observe that the estimator is quite robust to timing errors and multipath propagation effects. In the case of the multipath framework, the performance is worser than in the unsynchronized system. Lastly, the SNR threshold beyond which no outliers occur increases.

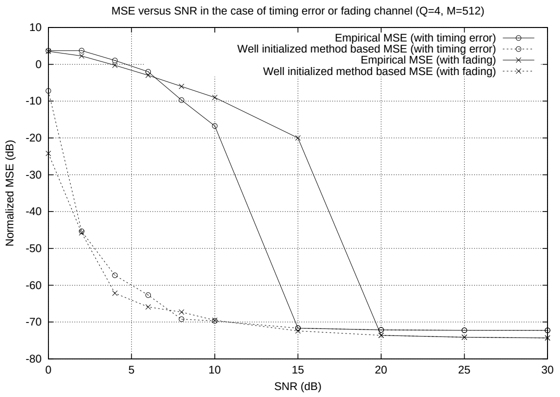

Fig. 3. Timing error case and non-flat-fading channel case: MSE versus SNR

4. CONCLUSION

We have investigated a new blind frequency offset estimator for OFDM/OQAM environments. The estimator performs well without being necessary to weight the subcarriers and is quite robust over frequency-selective channels. Nevertheless, it requires a large observation window in order to provide good initializations. Therefore, it is well adapted for a tracking mode (requiring only fine search) and not for an acquisition mode (requiring coarse search).

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