FRACTIONALLY SPACED BLIND EQUALIZATION: CMA VERSUS SECOND ORDER BASED METHODS.

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ABSTRACT

It is now well established that most of the blind fractionally spaced channel estimates based on the exclusive use of the second order statistics of the observation have poor performance when the excess bandwidth of the received signal is reduced. Recent papers proposed to use covariance matching approaches, and showed that a significant improvement of the mean square error of the channel estimate is possible. The purpose of this paper is to evaluate more precisely the potential of these methods. For this, we propose to compare the performance of a Wiener equalizer based on the optimally weighted covariance matching estimate of the channel (known as the best second order statistics based estimate) with the most standard higher order statistics based method, i.e. the (block) fractionally spaced CMA equalizer. It is shown that the CMA outperforms quite significantly the second order statistics based Wiener equalizer.

1. INTRODUCTION

Let $\{v_n\}_{n\in Z}$ be a zero mean unit variance i.i.d symbol sequence to be transmitted through a linear channel at the baud rate $1/T_s$. The continuous time received signal $\tilde{y}(t)$ can be written as:

$$\tilde{y}(t) = \sum_{n\in Z} v_n \tilde{h}(t - nT_s) + \tilde{w}(t)$$

where the filter $\{\tilde{h}(t)\}$ results from the emission and the reception filters and from the multi-path effects and where $\tilde{w}(t)$ is a additive white noise. In this paper, we assume without restriction that $\tilde{h}(t)$ is causal and time limited. Generally, $\{\tilde{h}(t)\}$ is unknown, and has therefore to be estimated in order to retrieve the symbols from the received signal. In most communication systems, the emitter sends periodically a training sequence known from the receiver, and which allows to estimate the unknown channel. However, the use of a training sequence has certain well known drawbacks. Therefore, a number of works have been devoted to the so-called blind equalization problem consisting in identifying the channel from the sole knowledge of the received signal $\tilde{y}(t)$. Gardner ([5]) and Tong et al ([11]) were the first to remark that it is possible to use the cyclostationarity of $\tilde{y}(t)$ in order to identify the channel from the second order statistics of the observations. For this, they proposed to sample $\tilde{y}(t)$ at rate $2/T_s$ (or more generally to q/T_s for $q > 1$; we just consider $q = 2$ in this paper), and to consider the 2–variate discrete time signal $\mathbf{y}(n)$ defined by $\mathbf{y}(n) = [\tilde{y}(2nT_s/2), \tilde{y}((2n+1)T_s/2)]^T$. $\mathbf{y}(n)$ can be written as

$$\mathbf{y}(n) = [\mathbf{h}(z)]\, v_n + \mathbf{w}(n) \quad (1)$$

where $\mathbf{w}(n) = \left[\tilde{w}(2n\frac{T_s}{2}), \tilde{w}((2n+1)\frac{T_s}{2})\right]^T$ and $\mathbf{h}(z)$ is defined by $\sum_{k=0}^{M} \mathbf{h}_k z^{-k}$, where $\mathbf{h}_k = \left[\tilde{h}(2k\frac{T_s}{2}), \tilde{h}((2k+1)\frac{T_s}{2})\right]^T$ and where M is the degree of the filter $\mathbf{h}(z)$. In this framework, the blind identification of the channel is equivalent to the identification of the 1 input / 2 outputs FIR transfer function $\mathbf{h}(z)$. It is now well established that if the components of $\mathbf{h}(z)$ are coprime, then $\mathbf{h}(z)$ can be consistently estimated from the sole use of the second order statistics of the observation $\mathbf{y}(n)$. For this, a number of approaches have been proposed (see ([11], [8], [10], [13], [7], [2] among others). One of the common features of theses works is to lead to a closed form expression of the channel estimate. However, it has been remarked that if the excess bandwidth of the received signal is reduced, then the performance provided by these schemes are very poor (see e.g. [12], [3]). On the other hand, it is known ([9]) that the so-called optimally weighted covariance matching estimate is the best second order statistics based estimate of the channel. Therefore, Zheng and Tong ([14], [15]) and Giannakis and Halford ([6]) proposed independently to

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use covariance matching estimators. They essentially showed that the statistical performance of the channel estimates can be improved very significantly. However, the practical calculation of the covariance matching estimate needs to solve a difficult non convex optimization problem. It is therefore relevant to know if the performance improvement provided by these methods justifies further researches aiming at implementing efficiently the covariance matching approaches. In order to answer this question, we propose to compare the reconstruction error provided by a fixed length Wiener equalizer based on the optimally weighted covariance matching channel estimate with the reconstruction error of a same length fractionally spaced CMA equalizer. The variance of the reconstruction error based on the optimal second order scheme is calculated in closed form in the case of large sample size by using standard perturbation analysis. For the CMA, the calculations are too heavy, and the reconstruction error variances are evaluated by Monte Carlo simulations. This paper is organized as follows. In section 2, we briefly recall the principle of the optimally weighted covariance matching channel estimate, and give the closed form expression of the corresponding asymptotic covariance matrix estimate. Next, we study in section 3 the statistical properties of the Wiener equalizer based on the channel estimate, and give the expression of the reconstruction error. We finally present in section 4 a lot of numerical evaluations showing that the CMA equalizer outperforms quite significantly the second order statistics based Wiener equalizer.

2. THE COVARIANCE MATCHING ESTIMATE.

Denote by $\mathbf{R}_N$ the covariance matrix of the stacked inputs $\mathbf{Y}_N(n) = [\mathbf{y}^T(n) \ldots \mathbf{y}^T(n-N)]^T$, and by $\mathbf{h}$ the $2(M+1)$–dimensional vector defined by $\mathbf{h} = (\mathbf{h}_0^T, \ldots, \mathbf{h}_M^T)^T$. Then, it is well established that $\mathbf{R}_N$ is given by

$$\mathbf{R}_N = \mathcal{T}_N(\mathbf{h})\mathcal{T}_N(\mathbf{h})^* + \sigma^2 I \qquad (2)$$

where σ^2 is the noise variance and where $\mathcal{T}_N(\mathbf{h})$ is the so-called generalized Sylvester $2(N+1) \times (M+N+1)$ matrix associated to $\mathbf{h}$. In this paper, we assume for the sake of simplicity that the noise variance σ^2 is known. The principle of the covariance matching estimation consists in looking for a filter $\mathbf{f}(z)$ for which the matrix $\mathbf{R}_N(\mathbf{f}) = \mathcal{T}_N(\mathbf{f})\mathcal{T}_N(\mathbf{f})^* + \sigma^2 I$ is as close as possible from the empirical estimate $\tilde{\mathbf{R}}_N$ of $\mathbf{R}_N$ defined by

$$\tilde{\mathbf{R}}_N = \frac{1}{T}\sum_{n=0}^{T-1} \mathbf{Y}_N(n)\mathbf{Y}_N(n)^* \qquad (3)$$

where T denotes the observation sample size. More precisely, denote by $\mathbf{U}\left(\tilde{\mathbf{R}}_N, \mathbf{f}\right)$ the $8(N+1)^2$ dimensional vector defined by

$$\mathbf{U}\left(\tilde{\mathbf{R}}_N, \mathbf{f}\right) = \begin{bmatrix} \mathrm{vec}(\tilde{\mathbf{R}}_N) - \mathrm{vec}(\mathbf{R}_N(f)) \\ \mathrm{vec}(\overline{\tilde{\mathbf{R}}}_N) - \mathrm{vec}(\overline{\mathbf{R}}_N(f)) \end{bmatrix}^{1}$$

(if x is a vector, $\overline{x}$ stands for the complex conjugate of x) and by $\mathbf{W}$ a positive $8(N+1)^2 \times 8(N+1)^2$ matrix. Then, the $\mathbf{W}$–weighted covariance matching estimate of $\mathbf{h}$ is defined as the argument $\hat{\mathbf{h}}_W$ of the minimization problem

$$\min_{\mathbf{f}} \mathbf{U}\left(\tilde{\mathbf{R}}_N, \mathbf{f}\right)^* \mathbf{W} \mathbf{U}\left(\tilde{\mathbf{R}}_N, \mathbf{f}\right) \qquad (4)$$

[1] vec() is the operator which reshapes any matrix into a column vector

It is well known that under suitable hypotheses, the column vector $\mathrm{vec}(\tilde{\mathbf{R}}_N)$ converges in distribution when $T \to \infty$ to a non circular Gaussian random vector. This limiting distribution is given by the asymptotic covariance matrix $\mathcal{C}_{\mathbf{R}_N}$ of the vector $\left[\mathrm{vec}(\tilde{\mathbf{R}}_N)^T, \mathrm{vec}(\overline{\tilde{\mathbf{R}}}_N)^T\right]^T$, i.e. the matrix

$$\lim_{T\to\infty} T\mathrm{cov}\left(\mathrm{vec}(\tilde{\mathbf{R}}_N)^T, \mathrm{vec}(\overline{\tilde{\mathbf{R}}}_N)^T\right)^T$$

This matrix can be easily calculated in closed form (see e.g. [1] or [6]). Moreover, if $\mathbf{W}$ is properly chosen (see [1]), $\hat{\mathbf{h}}_W$ is a consistent and asymptotically non circular Gaussian estimate of $\mathbf{h}$. The asymptotic covariance matrix $\Sigma_{\mathbf{W}}$ of $(\hat{\mathbf{h}}_W^T, \hat{\mathbf{h}}_W^*)^T$ is given by

$$\Sigma_{\mathbf{W}} = [\mathbf{G}^*\mathbf{W}\mathbf{G}]^{\#}\, \mathbf{G}^*\mathbf{W}\mathcal{C}_{\mathbf{R}_N}\mathbf{W}\mathbf{G}\,[\mathbf{G}^*\mathbf{W}\mathbf{G}]^{\#} \qquad (5)$$

where the matrix $\mathbf{G}$ is given by

$$\mathbf{G} = \begin{bmatrix} \left.\frac{\partial \mathrm{vec}(\mathbf{R}_N(\mathbf{f}))}{\partial \mathrm{vec}(\mathbf{f})}\right|_{\mathbf{f}=\mathbf{h}} & \left.\frac{\partial \mathrm{vec}(\mathbf{R}_N(\mathbf{f}))}{\partial \mathrm{vec}(\overline{\mathbf{f}})}\right|_{\mathbf{f}=\mathbf{h}} \\ \left.\frac{\partial \mathrm{vec}(\overline{\mathbf{R}}_N(\mathbf{f}))}{\partial \mathrm{vec}(\mathbf{f})}\right|_{\mathbf{f}=\mathbf{h}} & \left.\frac{\partial \mathrm{vec}(\overline{\mathbf{R}}_N(\mathbf{f}))}{\partial \mathrm{vec}(\overline{\mathbf{f}})}\right|_{\mathbf{f}=\mathbf{h}} \end{bmatrix}$$

It can be shown ([9], [6]) [2] that if $\mathbf{W} = \mathbf{W}_{opt} = \mathcal{C}_{\mathbf{R}_N}^{\#}$, then the $\mathbf{W}$–weighted estimate has the smallest asymptotic covariance matrix over the set of all estimates of $\mathbf{h}$ based on the column vector $(\mathrm{vec}(\tilde{\mathbf{R}}_N)^T, \mathrm{vec}(\overline{\tilde{\mathbf{R}}}_N)^T)^T$. This result shows that the covariance matching estimation approaches are interesting in our context. However, the practical computation of the estimate is not so easy. First, the optimal weighted matrix can be shown to depend on the channel to be estimated, and is therefore unknown. However, the performance of the estimate is unchanged if $\mathbf{W}_{opt}$ is replaced by a consistent estimate (see [6]) for more details). Second, the cost function (4) to be minimized is non convex, and shows a number of spurious local minima. Therefore, it seems difficult to extract its global minimum by using standard approaches. This problem may however be partially overcome by using the so-called JOSC approaches presented in [15] consisting in using a priori informations on the channel to reduce the dimension of the minimization problem.

3. ANALYSIS OF THE RECONSTRUCTION ERROR PROVIDED BY A WIENER EQUALIZER BASED ON THE COVARIANCE MATCHING ESTIMATE.

The channel being estimated, the emitted symbols have to be retrieved. For this, one can use a degree N FIR linear Wiener equalizer based on the channel estimate. In order to precise this, we first assume that the true channel is known, i.e. that the vector $\mathbf{h}$ is known. The Wiener equalizer is the degree N 1×2 FIR filter $\mathbf{g}(z) = \sum_{k=0}^{N} \mathbf{g}_k z^{-k}$ defined by the fact that

$$\Gamma = \mathbf{E}\left[\|v_{n-d} - [\mathbf{g}(z)]\mathbf{y}(n)\|^2\right] \qquad (6)$$

is minimum. Here, d is a delay. The optimal filter $\mathbf{g}(z)$ corresponds to the $2(N+1)$–dimensional row vector $\mathbf{g} = (\mathbf{g}_0, \ldots, \mathbf{g}_N)$ given by

$$\mathbf{g} = \mathbf{h}^* P \mathbf{R}_N^{-1} \qquad (7)$$

[2] as the matrix $\mathcal{C}_{\mathbf{R}_N}$ is in general non invertible, this result is not an obvious consequence of [9] and [6]. Some results of [1] have also to be used

where P is a certain $2(M+1) \times 2(N+1)$ selection/permutation matrix depending on the value of d. In practice, $\mathbf{h}$ and $\mathbf{R}_N$ are of course unknown, and have to be replaced by estimates. The resulting estimated Wiener filter is denoted by $\hat{\mathbf{g}}(z)$, and one estimates the symbol sequence by the signal $\hat{v}(n) = [\hat{\mathbf{g}}(z)]\mathbf{y}(n+d)$. It is clear that the reconstruction error provided by this equalizer depends in a crucial way from the considered channel estimate. This error characterizes in a quite relevant way the performance of the channel estimate in the sense that it measures the quality of the equalization, which is the ultimate goal of the channel estimation procedure. Therefore, in order to study the performance of the optimally weighted covariance matching estimate, we are going to evaluate its associated reconstruction error.

In order to simplify the notations, we denote by $\hat{\mathbf{h}}$ the optimally weighted covariance matching estimate of the channel. The estimated Wiener filter is therefore given by

$$\hat{\mathbf{g}} = \hat{\mathbf{h}}^* P \hat{\mathbf{R}}_N^{-1}$$

where the covariance matrix $\mathbf{R}_N$ is estimated by $\hat{\mathbf{R}}_N = \mathcal{T}_N(\hat{\mathbf{h}})\mathcal{T}_N(\hat{\mathbf{h}})^* + \sigma^2 I$ (remember that σ^2 is assumed to be known). An important point is that it is possible to evaluate in closed form the reconstruction error:

$$\Gamma = \mathbf{E}\left[\|v_{n-d} - [\hat{\mathbf{g}}(z)]\,\mathbf{y}(n)\|^2\right]$$

if the sample size T is large enough and if one assumes that the Wiener filter estimate is independent from the data on which it is applied to reconstruct the symbol sequence. This last hypothesis is in particular verified if the Wiener filter is estimated on a different slot than the data on which it is applied. In order to evaluate Γ, we put $\hat{\mathbf{g}}(z) = \mathbf{g}(z) + \Delta\hat{\mathbf{g}}(z)$, and $\hat{\mathbf{g}} = \mathbf{g} + \Delta\hat{\mathbf{g}}$. As we assume T large enough, and the correspondence $\hat{\mathbf{h}} \to \hat{\mathbf{g}}$ is differentiable, $\Delta\hat{\mathbf{g}}$ is asymptotically centered Gaussian (non circular), and its covariance matrix Cov $(\Delta\hat{\mathbf{g}})$ is given by

$$\text{Cov}\,(\Delta\hat{\mathbf{g}}) = \frac{1}{T}\mathcal{D}_{\mathbf{g}}\Sigma_{\mathbf{W}_{\text{opt}}}\mathcal{D}_{\mathbf{g}}^* + o(\frac{1}{T}) \qquad (8)$$

where we set

$$\mathcal{D}_{\mathbf{g}} = \left[\frac{\partial\hat{\mathbf{g}}}{\partial\begin{bmatrix}\hat{\mathbf{h}}\\ \bar{\hat{\mathbf{h}}}\end{bmatrix}}\right]_{\hat{\mathbf{h}}=\mathbf{h}}$$

On the other hand, as we assume $\Delta\hat{\mathbf{g}}$ independent from the current data, Γ can be written as:

$$\Gamma = \mathbf{E}\left[\|v_{n-d} - [\mathbf{g}(z)]\,\mathbf{y}(n)\|^2\right] + \mathbf{E}\left[\|[\Delta\hat{\mathbf{g}}(z)]\,\mathbf{y}(n)\|^2\right]$$

The first term is the inherent Wiener filter error and is equal to $1 - \text{vec}(\mathbf{h})^* P\mathbf{R}_N^{-1}P^*\text{vec}(\mathbf{h})$. The second part, which is due to the error estimation $\hat{\mathbf{h}} - \mathbf{h}$, can be rewritten as Trace $\{\text{Cov}\,(\Delta\hat{\mathbf{g}})\,\mathbf{R}_N\}$. Therefore,

$$\Gamma = 1 - \text{vec}(\mathbf{h})^* P\mathbf{R}_N^{-1}P^*\text{vec}(\mathbf{h}) + \text{Trace}\,\{\text{Cov}\,(\Delta\hat{\mathbf{g}})\,\mathbf{R}_N\} \quad (9)$$

We note that it is possible to evaluate similarly the reconstruction error provided by an estimated Wiener filter based on every consistent and asymptotically Gaussian estimate of the channel: in the expression (8), one has to replace the matrix $\Sigma_{\mathbf{W}_{\text{opt}}}$ by the corresponding asymptotic covariance matrix. In the following section, we compare the reconstruction error provided by the present optimally weighted covariance matching estimate with the reconstruction error based on the subspace channel estimate proposed in [8].

4. NUMERICAL EVALUATIONS

In this section, we finally compare the reconstruction errors of the Wiener equalizer corresponding to the channel estimate $\hat{\mathbf{h}}$ with the errors provided by a block fractionally spaced CMA equalizer of the same size. The reconstruction errors corresponding to a subspace channel estimate are also given to evaluate the improvement provided by the second order statistics based optimal estimate $\hat{\mathbf{h}}$.

We first recall that the fractionally spaced CMA equalizer consists in looking for the 1×2 FIR degree N filter $\mathbf{g}(z)$ for which $\mathbf{E}\left[\left| |[\mathbf{g}(z)]\,\mathbf{y}(n)|^2 - 1\right|^2\right]$ is minimum. An important point is that, in the noiseless case, this cost function does not show spurious local minimum ([4]). Its minimization is therefore easy. In practice, this cost function is of course unknown, and one minimizes w.r.t. $\mathbf{g}$ its empirical estimate given by:

$$\frac{1}{T}\sum_{n=0}^{T-1}\left| |[\mathbf{g}(z)]\,\mathbf{y}(n)|^2 - 1\right|^2$$

The minimization is achieved by a gradient algorithm. The calculation of the closed form asymptotic expression of the reconstruction error Γ associated to the CMA equalizer is very tedious. Therefore, we evaluated it by using Monte Carlo simulations.

We completed a set of test on various channels. We set $T = 1000$ and $N = 2M$ and $d = M + 1$. In the following tables we give the value of the asymptotic reconstruction error 9 for the optimal covariance matching based equalizer, and for the subspace channel estimate based equalizer. We also give the reconstruction error of the CMA equalizer (evaluated by Monte Carlo simulation) for $T = 1000$ and $T = 200$. Finally, we give the variance of the reconstruction error of the exact Wiener equalizer to plot the absolute error limit.

4.1. A random channel

We first consider a PSK4 modulation and a random channel filter (i.e. the various coefficients are generated randomly) with 7 coefficients. In this purely academical situation, all the value schemes have approximatively the same performance.

SNR (dB)	5.0	10.0	15.0	20.0	25.0	30.0
Wiener (dB)	-5.0	-8.2	-11.9	-16.1	-20.6	-25.4
CMA (dB) (1000)	-4.3	-8.1	-12.1	-16.4	-20.7	-25.2
CMA (dB) (200)	0.3	-3.6	-8.9	-16.2	-20.4	-24.7
CM opt. / Wiener (dB)	-4.9	-8.1	-11.7	-15.9	-20.3	-24.8
SSM / Wiener (dB)	-4.2	-7.7	-11.5	-15.8	-20.4	-25.1

Figure 1: Random channel.

We note that the optimally weighted covariance matching performance is a bit less efficient that the subspace method. It is because the asymptotic covariance matrix subspace estimate we used assume the knowledge of the norm of the channel while the covariance matching estimates the norm of the channel.

4.2. Two realistic channels

To truly compare the performance of those methods, we consider two more realistic channel with three paths shaped a square root raised cosine filter with roll-off 0.7.

4.2.1. Constant module modulation

The modulation is a PSK4 and the channel is given by the following table.

Delay (Ts)	-0.300	1.979	3.013
Amplitude	0.424+0.886i	0.279+0.393i	-0.220+0.194i

SNR (dB)	5.0	10.0	15.0	20.0	25.0	30.0
Wiener (dB)	-5.2	-8.5	-12.3	-16.3	-20.6	-25.2
CMA (dB) $T = 1000$	-4.4	-8.3	-12.1	-16.3	-20.0	-23.5
CMA (dB) $T = 200$	-0.8	-7.1	-11.7	-15.9	-19.7	-22.8
CM opt. / Wiener (dB)	0.5	-4.0	-8.4	-12.7	-16.8	-20.4
SSM / Wiener (dB)	64.7	56.8	47.8	38.2	28.3	18.4

Figure 2: Asymptotic performance of an usual channel with a constant module modulation.

We first remark that the subspace channel estimate gives extremely poor performance. Moreover, the CMA performance outperforms the optimal second order scheme. The CMA performance is, also, very close from the lower bound corresponding to the exact Wiener filter, even for $T = 200$.

4.2.2. Non-constant module modulation

We finally consider a QAM16 modulation and the following channel.

Delay (Ts)	0.151	2.093	3.046
Amplitude	0.480+0.912i	0.297+0.373i	0.202-0.270i

SNR (dB)	5.0	10.0	15.0	20.0	25.0	30.0
Wiener (dB)	-5.2	-8.6	-12.3	-15.5	-17.6	-18.9
CMA (dB) (1000)	-1.1	-7.0	-10.9	-13.7	-15.4	-16.3
CMA (dB) (200)	0.9	-0.9	-3.8	-7.1	-8.6	-8.8
CM opt. / Wiener (dB)	4.2	-0.5	-5.1	-9.6	-13.4	-16.2
SSM / Wiener (dB)	49.5	41.5	32.5	23.0	13.3	3.7

Figure 3: Asymptotic performance of an usual channel with a nonconstant module modulation.

Of course, the CMA performance fall dowm because the symbol sequence has not constant modulus (in perticular for $T = 200$). However, for $T = 1000$, the CMA still outperforms the optimally weighted covariance matching approach.

5. CONCLUSION

In this paper, we have evaluated precisely the potential of the optimally weighted covariance matching channel estimate. For this, we have evaluated in closed form the reconstruction error provided by a Wiener equalizer based on the channel estimate. These results have been compared with the quite standard CMA equalizer. Although the covariance matching approach allows to outperform considerably the performance provided by a subspace channel estimate, it appears that a standard CMA equalizer produces better reconstruction errors.

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