# POWER AND DIRECTION OF TRANSMISSION ESTIMATION FOR A DIRECTIVE SOURCE: IDENTIFIABILITY ANALYSIS AND ESTIMATION ALGORITHM

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**ABSTRACT**

Reliable spectrum cartography of directive sources depends on an accurate estimation of the direction of transmission (DoT) as well as the transmission power. Joint estimation of power and DoT of a directive source using ML estimation techniques is considered in this paper. We further analyze the parametric identifiability conditions of the problem, develop the estimation algorithm, and derive the Cramer-Rao-Bound (CRB).

***Index Terms*—** Direction of transmission (DoT), ML estimation, spectrum cartography, cognitive radio, directive source

## 1. INTRODUCTION

Spectrum cartography or radio environment mapping is considered as an efficient technique to produce a dynamic database of the incumbent users. This database can enable a network level deployment of cognitive radios [1]. However, spectrum cartography has plethora of other applications, e.g. network monitoring, malicious user detection, interference monitoring, and etc. The cornerstone of any spectrum cartography technique is a collaboration of sensors to estimate source parameters, e.g. location and power [2, 3, 4, 5, 6]. Most of these works provide efficient tools for spectrum cartography of omni-directional sources which is a valid assumption for lower parts of the frequency spectrum. However, considering the highly directive nature of wireless communications in higher parts of spectrum (e.g. Ka band, mmWave, etc. [7]), estimation of direction of transmission (DoT) becomes an essential component of spectrum cartography in order to obtain accurate results.

There are few works which touch the problem of DoT estimation for spectrum cartography, [5, 6]. However, they do not consider the joint estimation of power and DoT, and further, the developed techniques only consider the case with Gaussian shaped antenna radiation patterns and can not be applied to a generalized antenna pattern. In [8], we developed a joint power and DoT estimation for a directive source, considering the source signal to be known to the sensors. The developed algorithm of [8] can be applied to any antenna radiation pattern with a single main lobe. However, in most cases the source signal is not known, and further the algorithm of [8] incurs a high complexity in terms of synchronization between the sensors and the source, and among the sensors.

Here, the joint estimation of power and DoT is investigated by considering the source signal to be unknown but random with a known distribution. A number of sensors collect observations, and transmit their observations to a fusion center (FC). Unlike the setup in [8], the sensors are not synchronized in sampling. The FC is responsible to infer the received data and globally estimate the power and DoT. Further, we provide a set of sufficient conditions for the problem to be identifiable in this paper which are not presented in [8]. Following the introduction of the signal model, the underlying parameter identifiability conditions of the model are derived in Section 2. Afterward, we develop the estimation algorithm by employing ML estimation techniques in Section 3. Furthermore, to achieve a theoretical benchmark for performance comparison, we derive the Cramer-Rao-Bound (CRB) in this section. As shall be shown in Section 4, where a set of simulations results are depicted, the developed algorithm performs close to the CRB. Finally, we draw our conclusions in Section 5.

## 2. SYSTEM MODEL AND IDENTIFIABILITY ANALYSIS

We consider a source which employs a directive antenna with a known radiation pattern, and a single main lobe. The transmission occurs in a deterministic but unknown direction. The DoT is denoted by angle $\phi$ towards a specific reference line and represents the direction of the main lobe. We denote $P_s$ as the source transmission power, and $M > 1$ as the number of sensors which are located at different angles towards the reference line denoted by $\theta_i,\ i = 1, \cdots, M$. A schematic figure for the considered model of the source and the sensors is depicted in Fig. 1. The sensors collect the samples and send them sequentially to the FC for global data fusion. To reduce the overhead and complexity, we assume sensors are not synchronized to each other for sampling. Further, we consider a scenario where the FC is aware of the sensors locations (and thus the angles $\theta_i, i = 1, \cdots, M$) as well as the location of the source. This information can be obtained either through a database or estimated using localization techniques, e.g. [9, 10, 11]. However, the FC is not aware of the transmission power $P_s$ and $\phi$. The goal of the FC is to jointly estimate $P_s$ and $\phi$ based on sensors observations. Further, we assume that the sensors and the source are fixed during the estimation period.

Denoting $x_i[n],\ i = 1, \cdots, M$ to be the received signal at time $n$ and sensor $i$, and assuming an additive-white-Gaussian-noise (AWGN) channel model, we have

$$x_i[n] = \sqrt{P_s G(\phi, \theta_i) h(d_i)}\, s_i[n] + w_i[n], \qquad (1)$$

where $h(d_i)$ is the path-loss gain over source to sensor distance of $d_i$, $G(\phi, \theta_i)$ is the antenna gain in the direction of sensor $i$, $s_i[n]$ is the real-valued source signal received at sensor $i$ at its $n$-th sampling instance, and $w_i[n]$ is the i.i.d. AWGN with zero-mean and variance $\sigma_w^2$ which is assumed to be known. The path-loss gain is

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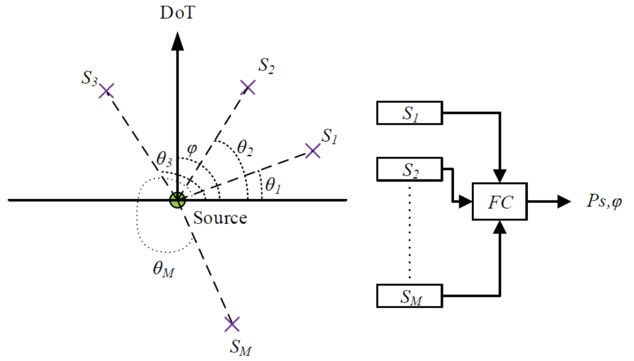


**Fig. 1**. Schematic configuration of the considered system model.

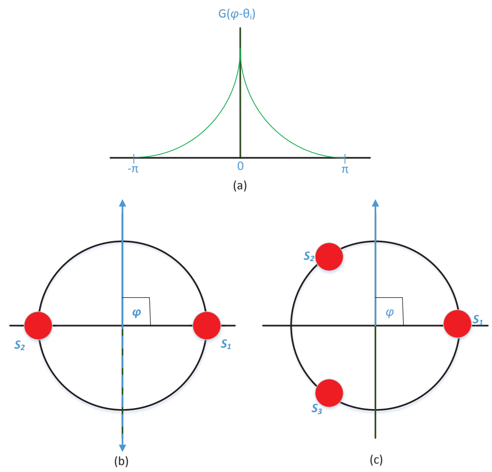


**Fig. 2**. (a): A symmetric antenna pattern example, (b) and (c): a *not identifiable* and an *identifiable* setup example with $\phi = \frac{\pi}{2}$ in both, and $\theta_1 = 0,\ \theta_2 = \pi$ in (b), and $\theta_1 = 0,\ \theta_2 = \frac{2\pi}{3},\ \theta_3 = \frac{4\pi}{3}$ in (c). The solid blue line shows the true DoT, and the dashed blue line in (b) depicts the ambiguity. It is clear that in (b) both $\phi = \frac{\pi}{2}$ and $\phi = \frac{3\pi}{2}$ leads to the same power and gain product, thus the problem is not identifiable. This ambiguity is resolved in (c), because of addition of one more sensor.

obtained by $h(d_i) = (4\pi d_i/\lambda)^{-\gamma}, d \neq 0$, where $\lambda$ is the source signal wavelength, and $\gamma$ is the path-loss exponent. Since the sensors sampling instances are not synchronized, the source signal received at them at the $n$-th sampling instance is not the same, that is why we use the index $i$ for $s_i[n]$. Further, the signal $s_i[n]$ is usually unknown, therefore, one way of modeling it is to assume a random variable following a zero mean i.i.d. Gaussian distribution with variance $\sigma_s^2$. This is the model we follow in order to develop the estimation algorithm. Note that under this model and the fact that sampling is performed asynchronously among the sensors and the fact that we assume the sensors are sufficiently far from each other, $x_i[n]$s are independent both over time and space. However, if the sensors sampling is synchronous, then the observations are spatially correlated which needs a different analysis, and is a subject of further study.

Before going through the detail of the estimation problem and its corresponding algorithm, in the following theorem, we establish the sufficient conditions for the considered model to be parametrically identifiable. In this theorem $\forall$ denotes "for all", and $\exists$ denotes "there is".

*Theorem 1*. The model in (1) is identifiable, if the following conditions are satisfied,

1. $\forall \phi \neq \phi^t : \exists \theta_i : G(\phi, \theta_i) \neq G(\phi^t, \theta_i)$.
2. $\forall \Delta \neq 1$ and $\phi \neq \phi^t : \exists \theta_i : G(\phi, \theta_i) \neq \frac{1}{\Delta} G(\phi^t, \theta_i)$, where $\Delta = \frac{P_s}{P_s^t}$.

With $\phi^t$ and $\phi$ denoting the true and estimated DoT, respectively, and $P_s^t$ and $P_s$ denoting the true and estimated transmission power.

**Proof**. Parameter identifiability means that model parameters can be uniquely determined from a set of noise and error free observations [13, 14]. Hence, in our case, we need to show that the set of equations $\forall i : s_i[n]\sqrt{P_s G(\phi, \theta_i) h(d_i)} = s_i[n]\sqrt{P_s^t G(\phi^t, \theta_i) h(d_i)}$ results in $P_s = P_s^t$ and $\phi = \phi^t$, with $P_s^t$ and $\phi^t$ denoting the true $P_s$ and $\phi$. Therefore, the problem boils down to finding the conditions under which no other $P_s \neq P_s^t$ or $\phi \neq \phi^t$ can result in $P_s G(\phi, \theta_i) = P_s^t G(\phi^t, \theta_i)\ \forall i$.

First, we start with the case where $\phi = \phi^t$ but $P_s \neq P_s^t$. In this case, it is clear that there is no $P_s \neq P_s^t$ for which $P_s G(\phi, \theta_i) = P_s^t G(\phi^t, \theta_i)$. Therefore, if $\phi = \phi^t$, the problem is always identifiable.

Now, we consider the case where $P_s = P_s^t$, but $\phi \neq \phi^t$. This way, the problem is identifiable if $\forall i, \phi \neq \phi^t : G(\phi \neq \phi^t, \theta_i) \neq G(\phi^t, \theta_i)$. This condition does not hold for a general antenna pattern, all the time, e.g. symmetric antenna patterns as in Fig. 2a. In this case, the problem is identifiable if the common solution of the set $G(\phi, \theta_i) = G(\phi^t, \theta_i), i = 1, \cdots, M$, is unique. It is clear that all the equations have at least a common solution which is $\phi = \phi^t$, and further, the uniqueness can be satisfied if $\forall \phi \neq \phi^t : \exists \theta_i : G(\phi, \theta_i) \neq G(\phi^t, \theta_i)$.

Finally, we look into the case where $P_s \neq P_s^t$, and $\phi \neq \phi^t$. Assuming $P_s = \Delta P_s^t$, the problem in this case is unidentifiable if $\exists \phi \neq \phi^t : G(\phi \neq \phi^t, \theta_i) = \frac{1}{\Delta} G(\phi^t, \theta_i)$ for all $i$s. Therefore, the problem becomes identifiable if $\forall \Delta \neq 1, \phi \neq \phi^t : \exists \theta_i : G(\phi \neq \phi^t, \theta_i) \neq \frac{1}{\Delta} G(\phi^t, \theta_i)$. And this concludes our proof. ■

From Theorem 1, we can see that the parameter identifiability of (1) depends on the proper selection of the sensors, which in turn depends on the specific $G(\phi, \theta_i)$ function of the source. Below, we outline the proper selection/placement of the sensors for the specific case of symmetric antenna patterns (e.g. Horn antennas) in order to gain additional insight into the conditions outlined in Theorem 1.

In the symmetric antenna patterns, the gain function only depends on $|\phi - \theta_i|$ where $|\cdot|$ denotes the absolute value, and thus $G(\phi - \theta_i) = G(\theta_i - \phi) = G(\phi - \theta_i + \pi), 0 \leq \phi \leq 2\pi$. Note, for this discussion, we consider a symmetric antenna pattern which is a one-to-one monotonically decreasing function over $|\phi - \theta_i| \in [0, \pi]$, e.g. Fig. 2a. Since, we are not aware of the specific value of $(P_s^t, \phi^t)$, we need to select the sensors such that irrespective of $\phi^t$, the identifiability conditions in Theorem 1 always hold.

For the first condition in Theorem 1, assuming $P_s^t$ to be known, it is easy to show that this condition is satisfied, if at least three of the sensors are located on both sides of $\phi^t$ at different $\theta_i$s (e.g. Fig. 2c). Note that two sensors located on both sides of $\phi^t$ is not sufficient for identifiability as in Fig. 2b. Further, in order to make sure that irrespective of $\phi^t$, the the selected sensors ($M \geq 3$) make the problem identifiable, one of the possibilities is to choose/place the sensors with equal angular distance to each each other, e.g. $\theta_i = (i-1)\frac{2\pi}{M}$ as in Fig. 2c.

To satisfy the second condition in Theorem 1, one approach could be to select the sensors such that $\forall \phi^t, \Delta \neq 1 \;:\; \exists \theta_i \;:\; \frac{\partial G(\phi^t,\theta_i)}{\partial \phi^t} \neq \frac{1}{\Delta}$. Assuming a non-linear gain pattern as in Fig. 2a (which is mostly the case), again, one approach can be to select/place the sensors such that $\theta_i = (i-1)\frac{2\pi}{M}$ (e.g. Fig. 2c). In this case, for all possible $\phi^t$ and $\Delta$, there is always at least one sensor $i$ for which $\frac{\partial G(\phi^t,\theta_i)}{\partial \phi^t} \neq \frac{1}{\Delta}$. This is an important result for identifiable estimation setup of symmetric antenna patterned sources. Hence, we highlight a generalized description of this discussion in the following proposition which can be proved easily following the same way as discussed above.

*Proposition 1:* If the source is equipped with a non-linear symmetric antenna pattern which is a one-to-one non-linear decreasing function over $|\phi-\theta_i| \in [0,\omega]$, the model parameters are identifiable if $\theta_i = (i-1)\frac{2\pi}{M}, i = 1, \cdots, M$, with $M > \frac{2\pi}{\omega}$, and $\omega \leq \pi$.

In the following section, we provide the required algorithms in the FC to estimate the power and DoT of the source using maximum likelihood (ML) estimation technique.

## 3. ANALYSIS AND PROBLEM FORMULATION

In this section, ML estimation of $P_s$ and $\phi$ is considered. Without loss of generality, we assume $\sigma_s^2 = 1$. Therefore, for the required probability distribution function of $x_i[n]$, we obtain

$$p(x_i[n]) = \frac{1}{\sqrt{2\pi[P_s G(\phi,\theta_i)h(d_i) + \sigma_w^2]}} \exp\Big(-\frac{1}{2}\frac{x_i^2[n]}{P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2}\Big), \tag{2}$$

and after calculating the joint likelihood of $\{x_i[n]\}$s and applying the natural logarithm, we obtain the log-likelihood (LL) function of $P_s, \phi$ denoted by $LL(P_s,\phi)$ as follows

$$LL(P_s,\phi) = \sum_{i=1}^{M}\Bigg[-\frac{N}{2}\ln\Big(2\pi[P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2]\Big) - \frac{1}{2}\frac{\sum_{n=1}^{N} x_i^2[n]}{P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2}\Bigg]. \tag{3}$$

where we used the fact that $\{x_i[n]\}$s are independent in time and between the sensors. We then estimate $P_s$ and $\phi$ by maximizing the function in (3) as follows,

$$\max_{P_s,\phi} LL(P_s,\phi) \text{ s.t. } P_s \geq 0,\ 0^\circ \leq \phi \leq 360^\circ. \tag{4}$$

To solve the problem, first we obtain the ML of $P_s$ assuming a given $\phi$, and then we insert the obtained result in (4) and perform a grid search to find the optimal $\phi$ and consequently $P_s$. For a given $\phi = \phi_g$, the optimal $P_s$ is obtained by the following theorem.

*Theorem 2*. For a given $\phi = \phi_g$, the optimal $P_s$ denoted by $P_s^*$ is obtained by

- If $\sum_{i=1}^{M} G(\phi_g,\theta_i)h(d_i)(X_i - N\sigma_w^2) \leq 0$ then $P_s^* = 0$. In practice, this is equivalent to the case where the transmitter is estimated to be "off".
- If $\sum_{i=1}^{M} G(\phi_g,\theta_i)h(d_i)(X_i - N\sigma_w^2) > 0$ then $P_s^*$ is the unique solution of $\frac{\partial LL}{\partial P_s} = 0$, with $\frac{\partial LL}{\partial P_s} = \sum_{i=1}^{M} -\Bigg[\frac{NG(\phi_g,\theta_i)h(d_i)}{2\big(P_s G(\phi_g,\theta_i)h(d_i)+\sigma_w^2\big)} + \frac{G(\phi_g,\theta_i)h(d_i)X_i}{2\big(P_s G(\phi_g,\theta_i)h(d_i)+\sigma_w^2\big)^2}\Bigg]$, where $X_i = \sum_{n=1}^{N} x_i^2[n]$, which can be solved e.g. by Newton method.

**Proof.** In order to prove Theorem 2, first we calculate $\frac{\partial LL(P_s,\phi_g)}{\partial P_s}$, and we obtain

$$\frac{\partial LL(P_s,\phi_g)}{\partial P_s} = \sum_{i=1}^{M}\Bigg[-\frac{NG(\phi,\theta_i)h(d_i)}{2\big(P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2\big)} + \frac{G(\phi,\theta_i)h(d_i)X_i}{2\big(P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2\big)^2}\Bigg]. \tag{5}$$

It is clear the the negative term in (5), i.e. $-\frac{NG(\phi,\theta_i)h(d_i)}{2\left(P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2\right)}$ is increasing in $P_s$, while the positive term, i.e. $\frac{G(\phi,\theta_i)h(d_i)X_i}{2\left(P_s G(\phi,\theta_i)h(d_i)+\sigma_w^2\right)^2}$ is decreasing in $P_s$. Further, it is clear that the rate of increase of the negative term is slower than the rate of decrease of the positive term. This shows that the negative term of (5) can intersect the positive term only once. For $P_s = 0$, $\frac{\partial LL(P_s,\phi_g)}{\partial P_s}$ has two possibilities as follows.

- If $\frac{\partial LL(P_s,\phi_g)}{\partial P_s}\Big|_{P_s=0} \leq 0$ and thus $\sum_{i=1}^{M} G(\phi_g,\theta_i)h(d_i)(X_i - N\sigma_w^2) \leq 0$, with increasing $P_s$, the positive term reduces while the negative term increases, and hence $\frac{\partial LL(P_s,\phi_g)}{\partial P_s}$ remains not positive. Therefore, the optimal $P_s$ in this case is $P_s^* = 0$.
- If $\frac{\partial LL(P_s,\phi_g)}{\partial P_s}\Big|_{P_s=0} > 0$ and thus $\sum_{i=1}^{M} G(\phi_g,\theta_i)h(d_i)(X_i - N\sigma_w^2) > 0$, then the positive and negative terms will intersect each other only once at $P_s^* > 0$, and after that $\frac{\partial LL(P_s,\phi_g)}{\partial P_s}$ becomes negative. Therefore, the optimal $P_s$ in this case is the unique root of $\frac{\partial LL}{\partial P_s}$. ■

Then, we insert $P_s^*$ in (3), and thus the optimal $\phi$ and consequently optimal $P_s$ can be estimated by solving the following line-search problem,

$$\max_{\phi}\ LL(P_s^*,\phi) \text{ s.t. } 0^\circ \leq \phi \leq 360^\circ, \tag{6}$$

where $P_s^*(\phi)$ is obtained from Theorem 2. The joint estimation of $P_s$ and $\phi$ using (6) is depicted in a more clear way in Algorithm 1.

**Algorithm 1** Joint $P_s$ and $\phi$ estimation algorithm.

**Input:** $\phi = 0$, $\delta\phi$ as the search step size,
1: Step 1: Find $P_s^*$ for $\phi$ from Theorem 2, and store $\phi$, $P_s^*(\phi)$, and $LL(P_s^*,\phi)$.
2: Step 2: $\phi = \phi + \delta\phi$.
3: **while** $\phi \leq 360^\circ$ **do** go to step 1.
4: **end while**
5: Find $(\phi, P_s^*(\phi))$ which has the maximum $LL(P_s^*,\phi)$ in storage.
6: **if** $P_s^* = 0$ **then**
7: announce the transmitter is "off".
8: **else**
9: Estimate $P_s$ and $\phi$ by $(\phi, P_s^*(\phi))$.
10: **end if**

*Remark 1*: Looking at the estimator, we can see that the sensors only need to communicate the accumulated energy (i.e. $X_i$) of the received samples to the FC. This reduces the communications overhead significantly.

After finding the algorithm in (6), the Cramer-Rao-Bound (CRB) can be derived as a benchmark for estimation accuracy evaluation. After some algebraic calculations we obtain the $\text{CRB}(P_s,\phi) = \text{CRB}(P_s) + \text{CRB}(\phi)$ as follows,

$$\text{CRB}(P_s,\phi) = \frac{2}{N(\mathcal{A}-\mathcal{B})}\Bigg[\sum_{i=1}^{M}\left(\frac{P_s h(d_i)G^{'}(\phi,\theta_i)}{P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2}\right)^2 + \sum_{i=1}^{M}\left(\frac{G(\phi,\theta_i)h(d_i)}{P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2}\right)^2\Bigg], \qquad (7)$$

with $G^{'}(\phi,\theta_i) = \frac{\partial G(\phi,\theta_i)}{\partial\phi}$, and

$$\mathcal{A} = \sum_{i=1}^{M}\left(\frac{P_s h(d_i)G^{'}(\phi,\theta_i)}{P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2}\right)^2 \times \sum_{i=1}^{M}\left(\frac{G(\phi,\theta_i)h(d_i)}{P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2}\right)^2,$$

and

$$\mathcal{B} = \left(\sum_{i=1}^{M}\frac{P_s h^2(d_i)G(\phi,\theta_i)G^{'}(\phi,\theta_i)}{\left(P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2\right)^2}\right)^2.$$

Further, the individual CRB for $P_s$ and $\phi$ are given by

$$\text{CRB}(P_s) = \frac{2}{N(\mathcal{A}-\mathcal{B})}\left[\sum_{i=1}^{M}\left(\frac{P_s h(d_i)G^{'}(\phi,\theta_i)}{P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2}\right)^2\right], \qquad (8)$$

and

$$\text{CRB}(\phi) = \frac{2}{N(\mathcal{A}-\mathcal{B})}\left[\sum_{i=1}^{M}\left(\frac{G(\phi,\theta_i)h(d_i)}{P_sG(\phi,\theta_i)h(d_i)+\sigma_w^2}\right)^2\right]. \qquad (9)$$

We can see that as the number of sensors $N$ increases, CRB$(P_s,\phi)$ decreases. Opposite effect can be observed for $\sigma_w^2$, i.e. CRB increases with $\sigma_w^2$. However, the effect of the number of sensors $M$, $P_s$ and $d_i$ on CRB is not straightforward.

## 4. SIMULATION RESULTS

In this section, We evaluate the performance of the estimation algorithm in Section 3. We assume a source with a symmetric antenna pattern defined as

$$G(\phi,\theta_i) = \begin{cases} 100\exp(-|\phi-\theta_i|) & \text{if } 0^\circ \le |\phi-\theta_i| \le 180^\circ; \\ 0 & \text{else.} \end{cases} \qquad (10)$$

According to Proposition 1 we place the sensors such that $\theta_i = (i-1)\frac{360^\circ}{M}$, and without loss of generality we assume the sensors are equally distanced from the source, and thus $\forall i : d_i = d$. In all the simulations, we assume DoT to be $\phi = 60^\circ$, $P_s = 0$ dBW, transmit frequency denoted by $f$ to be 18 GHz, $\gamma = 2$, and $\sigma_w^2 = -136$ dBW which approximately represents the noise power of a 5 MHz bandwidth and noise temperature of $T = 360^\circ$ K receiver.

Fig. 3 depicts the normalized mean square error (NMSE) of the estimated parameters $P_s$ and $\phi$ with the number of samples $N$. In this figure, three sensors are considered for cooperative estimation setup, which are located at the distance of $d = 1000$ m to the source. The simulation result is averaged over 1000 runs. It is clear that as $N$ increases, NMSE for both parameters decreases. We can further see that the convergence rate for $\phi$ is much faster than the one of $P_s$. This can be explained by looking at the CRB$(P_s)$ and CRB$(\phi)$ in (8) and (9), respectively. It is clear that the presence of $P_s$ in the numerator of (8) makes the convergence rate slower than the one of $\phi$.

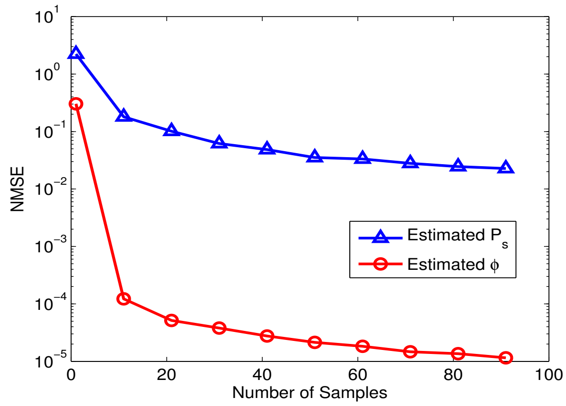


**Fig. 3**. NMSE of $P_s$ and $\phi$ versus number of samples, with $P_s = 0$ dBW, $\sigma_w^2 = -136$ dBW, $f = 18$ GHz, $\gamma = 2$, $M = 3$, $\theta_i = (i-1)\frac{360}{3}$ for $i = 1,2,3$, and $d = 1000$ m.

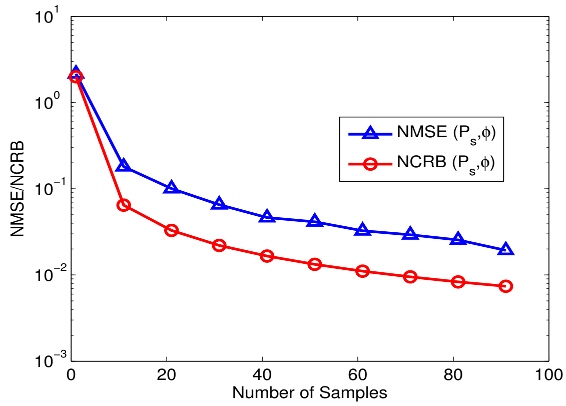


**Fig. 4**. NMSE and NCRB of $(P_s,\phi)$ versus the number of samples, with $P_s = 0$ dBW, $\sigma_w^2 = -136$ dBW, $f = 18$ GHz, $\gamma = 2$, $M = 3$, $\theta_i = (i-1)\frac{360}{3}$ for $i = 1,2,3$, and $d = 1000$ m.

In Fig. 4, the CRB performance is evaluated versus the number of samples for the same scenario as in Fig. 3. Here, we particularly depict the normalized CRB (NCRB) of $(P_s,\phi)$ (i.e. NCRB$(P_s)$+NCRB$(\phi)$), and compared with the total NMSE (i.e. NMSE$(P_s,\phi)$ = NMSE$(P_s)$ + NMSE$(\phi)$). We can see that the estimator performs quite close to the CRB for few number of samples.

## 5. CONCLUSION

Joint estimation of power and DoT of a directive source was considered in this paper. After deriving the sufficient identifiability condition, the required estimation algorithm as well as the CRB were developed. Particulalry, we developed the algorithm for a case where the source signal is unknown and random, and further the sensors are not synchronized in sampling, and nor synchronized with the source. The accuracy of the developed algorithm was shown to be close to CRB. Future works include power and DoT estimation for multiple directive sources.

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