

# A Network Cost Function for Clustered Ad Hoc Networks: Application to Group-Based Systems

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**Abstract**—Many wireless public safety and military networks are ad hoc networks, for which clustering is a well-known strategy to improve scalability. In addition these networks are structured according to a hierarchical organization, i.e. nodes belong to specific operational groups which implies that the traffic is mainly intra-group. So far, clustering algorithms are built using metrics such as node identifiers, node mobility, etc., and thus do not take into account the network hierarchical structure. The goal of this paper is therefore twofold: i) specify a way to benchmark clustering solutions from a system point of view, and ii) thanks to this benchmark determine the importance of using operational group information to build clusters. Therefore we define a novel network cost function based on additive metrics (e.g. delay) incorporating the traffic structure and the inter-clusters communications costs. Thanks to this function we show that the clustering solutions providing the best QoS to the end user depend on the group structure.

**Keywords**—Ad hoc network, Cluster, Network cost function, Operational group, End-to-end delay.

## I. INTRODUCTION

An ad hoc network is an infrastructure-less multi-hop wireless network where each node acts as a router, thus relaying packets on behalf of other nodes. Those networks are used when usual infrastructure based networks are not available or not suitable, such as in public safety or military networks. Such networks do not rely on the same MAC/NET solutions as usual communication technologies such as WiFi or 3GPP. In this context, specific communication solutions must be designed, especially concerning radio resource allocation. A preliminary step before allocating radio resources to nodes is to choose how the network will be organized. A first strategy is the *flat* network approach where radio resources are allocated thanks to peer to peer signalling exchanges between neighbor nodes. A second approach, considered here, consists in building sets of nodes called clusters in order to introduce hierarchy in the network and thus improve its scalability [1].

Numerous solutions for building clusters in mobile ad hoc networks have been defined [2] [3] [4], using parameters such as node identifiers, node degrees<sup>1</sup>, node mobility, node remaining battery energy, etc. A salient feature of public safety and military networks is that they are structured according to a hierarchical organization: each node belongs to specific

operational groups such as a squad, a section, etc., which implies that most traffic is intra-group. This is the context that is tackled in this paper. The authors of [5] have proposed a clustering algorithm using operational group (called group of interest) information but their work takes into account neither the dependence of traffic flows to operational groups nor the effect of radio links quality. Additionally, in [5] there is no explicit result about the benefit of building clusters using group information compared to not using it.

More generally, clustering solutions are compared using metrics such as number of clusters, number of nodes per cluster, etc. The limitation of this approach is that it does not provide any hint about the QoS that will be provided by the network to the user applications. Thus, there is a need to define a metric that could be used for that purpose.

The novelty of our work is the definition of a cost function  $J^{(0)}$  used to measure and compare the performance of clustering solutions in clustered ad hoc networks. More precisely this function measures the quality of a network partition using end-to-end path calculations with additive metrics (e.g. delay), and takes into account the fact that inter-cluster and intra-cluster communications have different costs. As a byproduct, this function is very useful for evaluating the benefit expected through the use of operational group information for obtaining the clustering solution.

The paper is organized as follows. The network model is described in Section II. The novel network cost function  $J^{(0)}$  is introduced in Section III. Application to group based systems and associated numerical results are detailed in Section IV. Section V is devoted to concluding remarks.

To improve readability, in this document *operational group* is contracted into *group*.

## II. NETWORK MODEL

Without loss of generality only connected<sup>2</sup> undirected graphs are considered in the remaining of this work. We consider a graph  $\mathcal{G}$  defined by its set of nodes  $\mathcal{V}$  and its set of edges  $\mathcal{E}$ . The number of nodes of  $\mathcal{G}$  is  $N := |\mathcal{V}|$  and the number of edges of  $\mathcal{G}$  is  $M := |\mathcal{E}|$ . The set of all partitions  $p$  of  $\mathcal{G}$  is called  $\mathcal{P}$ . In our model the parts of a partition are

<sup>1</sup>In a graph the degree of a node is the number of its 1-hop neighbors.

<sup>2</sup>In a connected graph there exists a path between any pair of vertices.

identified to the clusters. The weight of edge  $(i, j), \forall (i, j) \in \mathcal{E}$  is noted  $w_{i,j}$ . The weight  $w_{i,j}$  is a dimensionless quantity associated with the quality of link  $(i, j)$ . For example it can be a number of transmissions required to achieve a target packet error rate.

The set of groups  $\mathcal{O}$  is defined as  $\{\mathcal{O}_1, \dots, \mathcal{O}_{N_g}\}$  with  $N_g$  the number of groups. Let us note  $n_k^g$  the size of group  $\mathcal{O}_k$ . Each node belongs to only one group, i.e.  $\forall k \neq \ell, \mathcal{O}_k \cap \mathcal{O}_\ell = \emptyset$ , thus  $\sum_{k=1}^{N_g} n_k^g = N$ , and  $\bigcup_{k=1}^{N_g} \mathcal{O}_k = \mathcal{V}$ . A partition  $p$  of  $\mathcal{G}$  contains  $N_c$  clusters noted  $\mathcal{C}_k$  with  $k \in \{1, \dots, N_c\}$ . The size of cluster  $\mathcal{C}_k$  is noted  $n_k^c$ . Each node belongs to only one cluster, i.e.  $\forall k \neq \ell, \mathcal{C}_k \cap \mathcal{C}_\ell = \emptyset$ , thus  $\sum_{k=1}^{N_c} n_k^c = N$ , and  $\bigcup_{k=1}^{N_c} \mathcal{C}_k = \mathcal{V}$ . The diameter<sup>3</sup> of cluster  $\mathcal{C}_k$  is noted  $d_k$ .

### III. NOVEL NETWORK COST FUNCTION

#### A. Definition

To assess the quality of a partition  $p$  of  $\mathcal{G}$  we define a function  $J^{(0)}$  which represents the average cost of communications between all pairs of nodes in the network. It is defined as:

$$J^{(0)}(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{i,j} \cdot J_{i,j}^{(0)}(p), \quad (1)$$

where the factor  $1/N$  embodies the fact that all nodes  $i$  have equal probability to transmit,  $J_{i,j}^{(0)}(p)$  is the transmission cost between node  $i$  and node  $j$ , and  $\pi_{i,j}$  is the probability that node  $i$  chooses node  $j$  as a destination. By convention  $\pi_{i,i} = 0$ , and  $\sum_{j \in \mathcal{V}} \pi_{i,j} = 1, \forall i \in \mathcal{V}$ .

Finding the best clustering of the network will be equivalent to finding the set of best partitions  $\mathcal{P}'$  defined as:

$$\mathcal{P}' := \arg \min_{p \in \mathcal{P}} J^{(0)}(p). \quad (2)$$

Let us now explain how  $J_{i,j}^{(0)}(p)$  is elaborated. First, we assume that the routing process selects the shortest paths to establish communications in the network. The shortest path between node  $i$  and node  $j$  is defined here as the set  $\mathcal{S}_{i,j} = ((i, i_1), (i_1, i_2), \dots, (i_{L-1}, i_L), (i_L, j))$  for which the cumulated weights along this path,  $h_{i,j}$ , defined as:

$$h_{i,j} := \sum_{(i',j') \in \mathcal{S}_{i,j}} w_{i',j'}, \quad (3)$$

is minimum. Note that  $\mathcal{S}_{i,j}$  is independent of the partition  $p$ .

Fig. 1 provides an example of shortest path  $\mathcal{S}_{1,4}$  between source node 1 and destination node 4.

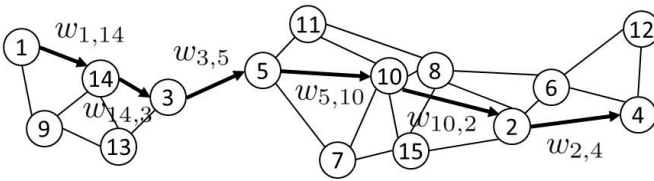


Fig. 1. Example of multi-hop shortest path.

In clustered networks, inter-cluster communications can be implemented using a MAC different from the one used for intra-cluster communications. This is justified by the fact that within a cluster a node called cluster-head (CH) can manage the radio resource management<sup>4</sup> (RRM) on behalf of the whole cluster, which allows the CH to optimize it locally. Conversely, inter-cluster RRM is done in a more distributed way (e.g. among the CHs of neighbor clusters) and is thus more difficult to optimize. Therefore, we reasonably assume that the costs of intra-cluster and inter-cluster communications are different. Consequently a path  $\mathcal{S}_{i,j}$  is split into the two subsets  $\hat{\mathcal{S}}_{i,j}(p)$  of intra-cluster links and  $\tilde{\mathcal{S}}_{i,j}(p)$  of inter-cluster links, leading respectively to the cumulated weights  $\hat{h}_{i,j}$  and  $\tilde{h}_{i,j}$ :

$$\hat{h}_{i,j}(p) := \sum_{(i',j') \in \hat{\mathcal{S}}_{i,j}(p)} w_{i',j'}, \quad (4)$$

$$\tilde{h}_{i,j}(p) := \sum_{(i',j') \in \tilde{\mathcal{S}}_{i,j}(p)} w_{i',j'}. \quad (5)$$

Following Eqs. (3)-(4)-(5), we have:

$$h_{i,j} = \hat{h}_{i,j}(p) + \tilde{h}_{i,j}(p). \quad (6)$$

For example in Fig. 2,  $\hat{h}_{1,4} = w_{1,14} + w_{14,3} + w_{5,10} + w_{2,4}$  and  $\tilde{h}_{1,4} = w_{3,5} + w_{10,2}$ .

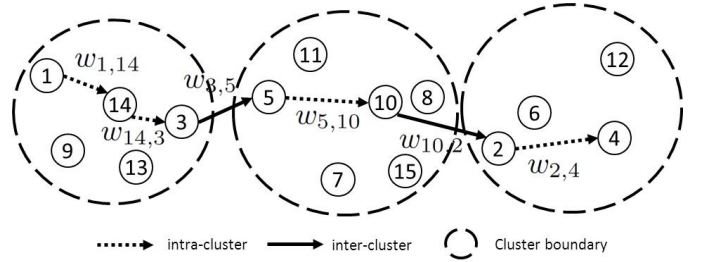


Fig. 2. Multi-hop shortest path with cluster boundaries.

In order to account for the difference between intra and inter-cluster communications, we define the cost from node  $i$  to node  $j$  as the weighted sum of  $\hat{h}_{i,j}(p)$  and  $\tilde{h}_{i,j}(p)$ :

$$J_{i,j}^{(0)}(p) := \hat{\gamma} \cdot \hat{h}_{i,j}(p) + \tilde{\gamma} \cdot \tilde{h}_{i,j}(p), \quad (7)$$

where  $\hat{\gamma} > 0$  is the cost associated with the intra-cluster communications, and  $\tilde{\gamma} \geq \hat{\gamma}$  the cost associated with the inter-cluster communications. Note that Eqs. (4)-(5)-(7) clearly show that  $J^{(0)}$  is applicable to additive metrics.

Noting  $\gamma := \tilde{\gamma}/\hat{\gamma}$  and summing Eq. (7) over all pairs of nodes we get:

$$J^{(0)}(p) = \frac{\hat{\gamma}}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{i,j} \cdot (\hat{h}_{i,j}(p) + \gamma \cdot \tilde{h}_{i,j}(p)). \quad (8)$$

One can remark that the factor  $\hat{\gamma}/N$  in Eq. (8) plays no role in the optimization problem of Eq. (2).

<sup>3</sup>The diameter of a graph is the length of its the longest shortest path.

<sup>4</sup>i.e. the allocation of transmit power, modulation and coding schemes, sub-channels, scheduling, admission control, etc.

Because  $h_{i,j}$  is independent of the way the network is clustered, it is useful to exhibit it in Eq. (8) leading to:

$$J^{(0)}(p) = A + (\gamma - 1) \cdot B(p), \quad (9)$$

with

$$A := \frac{\hat{\gamma}}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{i,j} \cdot h_{i,j},$$

$$B(p) := \frac{\hat{\gamma}}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{i,j} \cdot \tilde{h}_{i,j}(p).$$

So far, the expressions are a function of probabilities  $\pi_{i,j}$  which makes the model very general and applicable to several contexts. Particular values for  $\pi_{i,j}$  will be considered in Section IV in order to take into account the hierarchical organization of some ad hoc networks.

### B. Examples for practical $J^{(0)}$

In this subsection, through two examples we discuss how to use the generic cost function  $J^{(0)}$  with practical QoS metrics.

Firstly,  $J^{(0)}$  can be interpreted as an end-to-end delay. In that context the link weights  $w_{i,j}$  (see Eq. (3)) can be interpreted as the average number of transmissions required on node  $i$  to achieve a successful reception on node  $j$  (using an ARQ protocol) and  $\gamma$  can be viewed as the multiplicative coefficient of the delay induced by the difference of efficiency between inter-cluster and intra-cluster RRM. The value of  $J_{i,j}^{(0)}$  from Eq. (7) should be interpreted as the duration needed for node  $i$  to send successfully to node  $j$  one unit of traffic. Let us take the example of a TDMA based MAC in which the intra-cluster delay to send data  $\hat{\gamma}$  is equal to one 100 ms MAC frame while the inter-cluster delay  $\tilde{\gamma}$  is twice this value. Assuming a uniform traffic between all nodes, each destination node  $j$  receives an equal portion  $\pi_{i,j} = 1/14$  from node  $i$ . Considering the shortest path between nodes 1 and 4 of Fig. 2 and applying Eq. (7) with  $w_{i,j} = 1 \forall (i,j) \in \mathcal{E}$ , we get  $J_{1,4}^{(0)} = 800$  ms. The value of  $J^{(0)}$  in Eq. (9) is the average duration for a node to send successfully one unit of traffic to all nodes. Using the same example of Fig. 1 and Fig. 2 we get  $J^{(0)} = 376$  ms.

Secondly,  $J^{(0)}$  can be used to measure a network capacity consumption, which we want to minimize in order to maximize the throughput forwarded in the network. Using the same amount of radio resources, the higher the SNR of the link, the higher the amount of information that can be transmitted. Therefore to transmit the same amount of data, the use of good links leads to the consumption of less network radio resources. From that perspective the link weights  $w_{i,j}$  can be interpreted as the inverse of the spectral efficiency achievable on link  $(i,j)$ , related to the best modulation and coding scheme (MCS) usable on the link. To render  $w_{i,j}$  dimensionless, it must be divided by the spectral efficiency of a reference MCS. Compared to intra-cluster RRM that can be centralized on CH nodes, inter-cluster RRM is a distributed process. Consequently the channel state indications (CSI) used by inter-cluster have less accuracy than CSI used by intra-cluster RRM.

Therefore, intra-cluster RRM can use higher efficiency MCS than inter-cluster RRM, thus justifying  $\gamma > 1$ .

Some other examples can be found but are not evoked here due to space limitation.

### C. Theoretical results and additional constraints

Thanks to Eq. (9) it is easy to prove the following theorem. *Theorem 1:* The solutions of the problem in Eq. (2) are:

- 1) If  $\gamma = 1$ ,  $\mathcal{P}' = \mathcal{P}$ , and  $\forall p \in \mathcal{P}$ ,  $J^{(0)}(p) = A$ .
- 2) If  $\gamma > 1$ ,  $\mathcal{P}' = \{\mathcal{V}\}$ .

The first part of theorem 1 means that when  $\gamma = 1$ , the way clusters are built is not important. This case is not interesting in practice since there will always be a difference of cost between intra and inter-cluster communications. When  $\gamma > 1$ , the best and trivial solution is to build one cluster corresponding to the whole network. This solution is not acceptable because of the constraints related to the size of the clusters. If the cluster is too large the RRM is not simple anymore and the underlying assumption about a simple intra-cluster RRM does not hold anymore. Consequently, to ensure the validity of this assumption we add constraints on the clusters and define  $\mathcal{P}^c$  the subset of valid partitions as follows:

$$\mathcal{P}^c = \{p \in \mathcal{P} \text{ s.t. } p \text{ satisfies } C_1, C_2, C_3\},$$

with

$$C_1 : C_k \text{ is connected } \forall k \in \{1, 2, \dots, N_c\},$$

$$C_2 : n_{min} \leq n_k^c \leq n_{max} \quad \forall k \in \{1, 2, \dots, N_c\},$$

$$C_3 : d_k \leq d_{max} \quad \forall k \in \{1, 2, \dots, N_c\}.$$

First  $C_1$  ensures that each cluster is a connected subgraph of  $\mathcal{G}$ , allowing intra-cluster communication between all cluster members. Then  $C_2$  forces the number of cluster members to be neither too small nor too large, which makes sense from a RRM point of view. Finally  $C_3$  prevents nodes from the same cluster from being too far from the CH (in charge of RRM). The values of the parameters  $n_{min}$ ,  $n_{max}$  and  $d_{max}$  depend on the RRM process.

Now, our goal is to find the set of partitions  $\mathcal{P}^*$  solving the following problem:

$$\mathcal{P}^* = \arg \min_{p \in \mathcal{P}^c} J^{(0)}(p). \quad (10)$$

Again, thanks to Eq. (9), we have the following theorem:

*Theorem 2:* When  $\gamma > 1$ ,  $\mathcal{P}^*$  is independent of  $\gamma$ .

## IV. APPLICATION OF $J^{(0)}$ TO GROUP-BASED SYSTEMS

### A. Definition

We now consider that operational groups exist and that the traffic is structured according to these groups. To capture this fact, we consider that the probability that one node communicates with a node of the same group is equal to  $\alpha \in [0, 1]$  and thus the probability that one node communicates with a node in another group is equal to  $1 - \alpha$ . Since a node of group  $\mathcal{O}_k$  can communicate to  $n_k^g - 1$  nodes in the same group, the probability to reach one of these nodes is equal to

$\alpha/(n_k^g - 1)$ . The number of nodes of the other groups with which this node can communicate is equal to  $N - n_k^g$  with a corresponding probability of  $(1 - \alpha)/(N - n_k^g)$ . Thus, we have:

$$\pi_{i,j} := \begin{cases} \frac{\alpha}{n_k^g - 1} & \text{if } j \in \mathcal{O}_k, \\ \frac{1 - \alpha}{N - n_k^g} & \text{otherwise,} \end{cases} \quad (11)$$

with  $(i, j) \in \mathcal{V}^2$  and  $i \in \mathcal{O}_k$ .

### B. Assessment methodology

A first result we want to show is that groups have an impact on the clustering solution. To do this we find the set  $\mathcal{P}^u$  of optimal partitions when the traffic pattern does not depend on the groups, then we determine if these partitions are still good when the traffic pattern becomes dependent on the groups. A traffic independent of the groups is equivalent to  $\pi_{i,j} = \frac{1}{N-1}$ ,  $\forall (i, j) \in \mathcal{V}^2$ ,  $i \neq j$ , hence:

$$\mathcal{P}^u := \arg \min_{p \in \mathcal{P}^c} J^{(0)}(p) \Big|_{\pi_{i,j} = \frac{1}{N-1}, \forall (i,j) \in \mathcal{V}^2, i \neq j}.$$

To determine if the partitions  $p \in \mathcal{P}^u$  are still good when the traffic pattern becomes dependent on the groups according to Eq. (11), we calculate the following metric:

$$\delta_u(\alpha) := \frac{J_u^{(0)} - J_*^{(0)}}{\bar{J}_*^{(0)} - J_*^{(0)}}, \quad (12)$$

with  $J_u^{(0)}$  the highest value of  $J^{(0)}$  for all the partitions in  $\mathcal{P}^u$  when  $J^{(0)}$  is calculated with  $\pi_{i,j}$  defined as in Eq. (11),  $\bar{J}_*^{(0)} := \max_{p \in \mathcal{P}^c} J^{(0)}(p)$ , and  $J_*^{(0)} := \min_{p \in \mathcal{P}^c} J^{(0)}(p)$ . Let us call  $J^{(0)}$  interval the interval  $[J_*^{(0)}, \bar{J}_*^{(0)}]$ . The term  $\delta_u(\alpha)$  measures the performance loss obtained by not taking into account the dependence of traffic patterns to groups during cluster building. It takes its values in  $[0, 1]$  when  $J_u^{(0)}$  goes through the  $J^{(0)}$  interval: 0 is associated to the best partitions and 1 to the worst partitions.

Assuming that taking the group structure into account when building clusters is of interest, new algorithms have to be designed. In order to show that it is not an easy task (which is out of the scope of the paper), we propose hereafter a naive approach. The simplest way to follow would be to *force* all the members of a group to belong to the same cluster. However, due to the constraints it is not always possible. Therefore we propose the following procedure: i) for each group, find the node with the highest degree in the subgraph of  $\mathcal{G}$  induced by the members of this group and build a cluster including this node and its 1-hop neighbors; ii) for each nodes not yet member of any cluster, attach it to an existing cluster while making sure that  $C_3$  is satisfied, and build a new 1-node cluster when  $C_3$  is not satisfied. The obtained partition is denoted by  $p_g$ . Using the same idea as Eq. (12), the performance of the aforementioned naive algorithm is studied through the metric:

$$\delta_g(\alpha) := \frac{J^{(0)}(p_g) - J_*^{(0)}}{\bar{J}_*^{(0)} - J_*^{(0)}}.$$

Note that this procedure may lead to clusters with a number of nodes less than  $n_{min}$ , thus not satisfying  $C_2$ .

### C. Simulation setup

The random networks used to calculate the values of  $\delta_u(\alpha)$  and  $\delta_g(\alpha)$  are generated using the following procedure. First  $N_g$  nodes are placed randomly in a  $d \times d$  square area following a uniform distribution. Each of those nodes is the first node of each group  $\mathcal{O}_k$ ,  $\forall k \in \{1, 2, \dots, N_g\}$ , called  $V_k$ . Then all members of each group  $\mathcal{O}_k$  are placed relatively to  $V_k$  using polar coordinates  $(\rho, \theta)$ . The radius  $\rho$  is a random variable following the probability density function (p.d.f.):

$$f_\rho(d) = \begin{cases} \frac{2}{d_1 + d_2} & \text{if } 0 < d \leq d_1, \\ \frac{2(d_2 - d)}{(d_1 + d_2)(d_2 - d_1)} & \text{if } d_1 < d \leq d_2, \\ 0 & \text{otherwise.} \end{cases}$$

The angle  $\theta$  is a random variable following a uniform p.d.f. in  $[-\pi, +\pi)$ . When  $\theta$  leads to a node outside of the simulated area, another value is calculated. In this paper the following parameter values have been used:  $N = 30$ ,  $d_1 = 4000$ ,  $d_2 = 8000$ ,  $N_g = 6$ ,  $r = 5000$  and  $d = 20000$ . The number of edges of the 100 random networks used during our simulations lies in  $[146, 222]$  with an average value of 183 edges. Their network diameter lies in  $[5, 12]$  and is equal to 7.19 on average.

Thanks to Theorem 2 the quality of a partition does not depend on  $\gamma$ , so the only considered value is  $\gamma = 2$ . For the sake of simplicity, each  $w_{i,j}$  has been set constant and equal to 1,  $\forall (i, j) \in \mathcal{E}$ . Finally, the constraint parameters are:  $n_{min} = 4$ ,  $n_{max} = 8$  and  $d_{max} = 2$ . All the curves of section IV-D have been obtained by averaging over 100 networks. Note that all valid partitions have been found using a brute force approach. Among the 100 random networks, the minimum and maximum number of valid partitions were 672 and 49 647 650.

### D. Results

The cumulated distribution functions (c.d.f.) of  $\delta_u(\alpha)$  are plotted in Fig. 3.

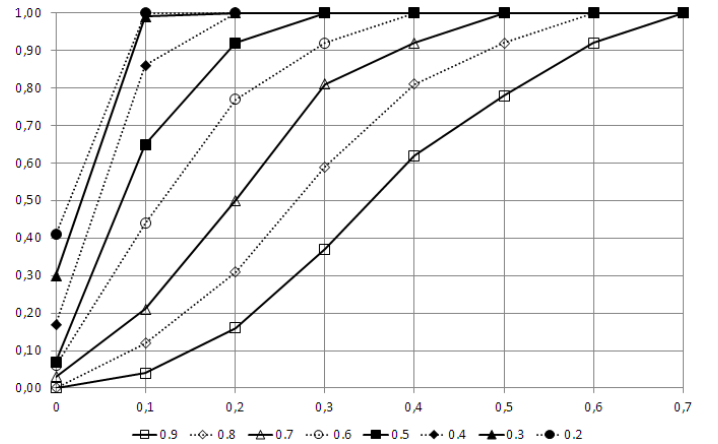


Fig. 3. c.d.f. of  $\delta_u(\alpha)$ .

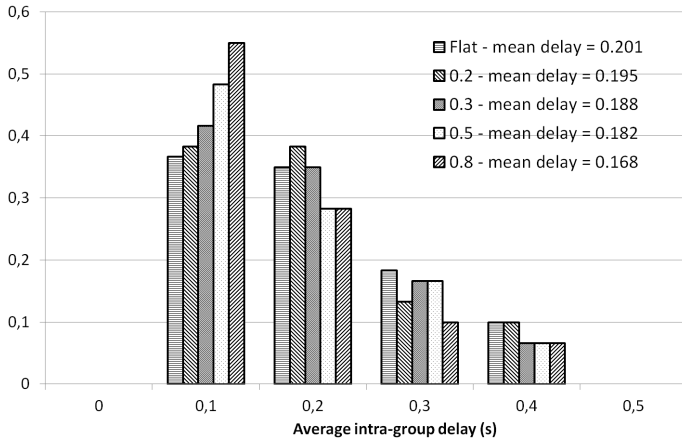


Fig. 4. End-to-end intra-group delay histogram.

The first information provided by these curves is that the partitions  $p \in \mathcal{P}^u$  that are the best when the traffic pattern does not depend on the groups, do not remain the best partitions when the traffic pattern depends on the group:  $\mathcal{P}^u \neq \mathcal{P}^*$ . For example when  $\alpha = 0.2$ , only 40% of the partitions  $p \in \mathcal{P}^u$  are also in  $\mathcal{P}^*$  and this proportion decreases to about 0% when  $\alpha = 0.8$  or 0.9. An additional result is that the partitions  $p \in \mathcal{P}^u$  are not only no longer the best but also become quite bad when the value of  $\alpha$  is high enough. For example when  $\alpha = 0.7$ , only 50% of these partitions have a  $J^{(0)}$  value in the first 20% of the  $J^{(0)}$  interval. Fig. 3 shows that when traffic flows are concentrated within groups, the group membership should be taken into account during cluster building. This result has been achieved thanks to the introduction of the benchmark network cost function  $J^{(0)}$ .

Fig. 4 illustrates the benefit of operational clustering, using the same example of the TDMA based MAC as in section III-B. This figure plots, for one particular network, the histogram of the average delay between nodes of the same group, calculated over all groups. Thanks to  $J^{(0)}$ , for each value of  $\alpha$  the best partitions  $p \in \mathcal{P}^*$  have been found and the associated delays have been determined. Fig. 4 shows that when  $\alpha$  increases the average delay decreases. Additionally, the lower  $\alpha$  is, the higher is the number of pairs of nodes in the same group with a minimum delay of 100 ms. This is a practical example of the benefit provided to the end-user when the group structure is taken into account.

Another result achieved thanks to  $J^{(0)}$  is that the naive clustering strategy consisting in trying to build one cluster per group is not the best way to build clusters. Out of the 100 networks clustered using this heuristic, only 56 satisfied the cluster size constraints, the remaining ones included clusters with only 1 or 2 nodes. Fig. 5 plots the c.d.f. of  $\delta_g(\alpha)$  for these 56 networks. This figure shows that the need for a clustering solution more clever than the naive one increases when  $\alpha$  decreases. For example when  $\alpha = 0.9$ , about 80% of these partitions have a  $J^{(0)}$  value in the first 10% of the  $J^{(0)}$  interval. These 80% are reduced to about 55% when  $\alpha = 0.7$ , and are

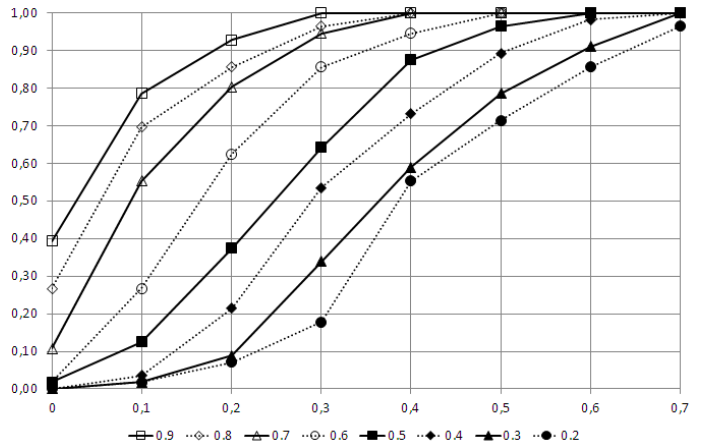


Fig. 5. c.d.f. of  $\delta_g(\alpha)$ .

close to 0% when  $\alpha < 0.5$ .

## V. CONCLUSION

The cost function  $J^{(0)}$  which can be used as a benchmark to compare different clustering solutions is a first result provided in this document. This function measures the quality of a network partition using end-to-end path calculations with additive metrics. It takes into account the fact that inter-cluster and intra-cluster communications have different costs and is flexible enough to cover both cases when the traffic distribution depends on the groups or not. A second result that we achieved is that making use of group information for clustering leads to better network performance. This result was illustrated with the practical application of  $J^{(0)}$  to intra-group delays. Thanks to our new cost function we have also shown that using a simple naive approach consisting of building one cluster per group does not lead to the best network performance. This justifies the need for more advanced clustering solutions using group information. Further work will investigate how  $J^{(0)}$  can be used to design distributed clustering algorithms.

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