

Early-Drop based Hybrid ARQ in a Cross-layer context

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Abstract—We aim to study a specific cross-layer optimized Hybrid Automatic Retransmission reQuest (HARQ) introduced in [1] and called *early-drop HARQ*. We remind that in [1], instead of giving a transmission credit per MAC packet (as usually done), it is proposed to share the transmission credit among all the MAC packets belonging to the same IP packet. The early-drop version stops the retransmission as soon as the number of remaining MAC packets is higher than the number of possible transmission attempts. As the early-drop has never been analyzed neither through simulations nor analytical derivations, the purpose of this paper is to fill this gap. As the packet error rate and the delay are not modified, we only focus on the efficiency which is expressed in closed-form.

I. INTRODUCTION

Hybrid ARQ (HARQ) techniques are promising solutions for high data rates mobile systems, such as LTE [2] or Wimax. Indeed, these techniques enable us to obtain a relevant trade-off between a low packet error rate (via the forward error correction (FEC)), and a reasonable (and controllable) efficiency and delay at any SNR via the repetition mechanism.

The literature on HARQ mechanisms are usually twofold: on the one hand, it is of interest to find powerful error correcting codes (see [3]–[5] for the physical layer, [6] for the application layer); on the other hand, it is of interest to analyze the HARQ mechanisms through closed-form expressions for useful metrics such as the packet error rate, the delay, and the efficiency (see [7]–[10] which only focus on the MAC layer).

In order to improve the HARQ mechanism, it is proposed in [1] to operate a cross-layer optimization between the MAC and the IP layers. The idea was the following one: let us consider that an IP packet is fragmented into N MAC packets on which a HARQ mechanism is applied. Usually, each MAC packet has its own transmission credit. Such an approach can be called Fragment Based Strategy (FBS). So, in [1], the transmission credit is shared among the N MAC packets belonging to the same IP packet. This approach is called IP packet Based Strategy (IBS). It has been observed that the PER is improved at the expense of a higher delay. The efficiency, as seen in [11], is in contrast very similar.

In [11]–[13], a unified framework for analyzing analytically any HARQ at any layer with and without cross-layer optimization has been developed. Closed-form expressions for most current HARQ metrics (namely, packet error rate, efficiency and delay) have been provided for FBS and IBS.

A way for improving the efficiency of the IBS, called *early drop*, has been roughly mentioned in [1]. The idea is to stop the IP packet transmission as soon as the number of remaining MAC packets is higher than the remaining number of transmission attempts. Nevertheless, this new way has been investigated neither through extensive simulations nor analytical derivations. Therefore, we propose, in this paper, to express in closed-form the efficiency of the early-drop based HARQ. Notice that the other metrics (packet error rate, delay) are not modified.

The paper is organized as follows: in Section II, we describe more precisely the early drop HARQ mechanism and introduce some notations. In Section III, we provide the new closed-form expressions for the efficiency in general case and some particular cases (ARQ, etc). We also prove that the early drop "trick" improves the efficiency whatever the kind of HARQ. In Section IV, numerical illustrations are provided. Finally Section V is devoted to concluding remarks.

II. EARLY-DROP HARQ MECHANISM

At the transmitter side, the MAC layer has to transmit several IP packets of length L_{IP} . Each IP packet is split into N MAC packets of length $L_{MAC} = L_{IP}/N$. From each MAC packet, some sub-blocks are generated in order to be transmitted by the PHY layer. Due to the lack of space and for the sake of simplicity, we hereafter only introduce the so-called IR-HARQ (where IR stands for Incremental Redundancy). Notice that our later derivations actually hold for any kind of HARQ.

Each MAC packet for which a header and a CRC have been added is encoded by a Forward Error Correcting Code of rate R_0 (known as *mother code*). The encoded MAC packet is then split into t_0 PHY packets, usually through a puncturing of the mother code. The PHY packets (denoted PPACKET) associated with the same MAC packet are then numbered as $\{\text{PPACKET}(i)\}_{i=1}^{t_0}$. The transmitter transmits sequentially PPACKET(1) up to PPACKET(t_0) upon error detection. If the MAC packet is still not received after the transmission of the last PHY packet PPACKET(t_0), the first PHY packet PPACKET(1) is transmitted again and so on. These PHY packets are sent through a propagation channel (that may be Gaussian one, Rayleigh one, Frequency-Selective one, etc). The length of the i -th PHY packet is δ_i for $i \in \{1, \dots, t_0\}$,

and we denote by $w_k = \sum_{i=1}^k \delta_{i \bmod t_0}$ the number of channel used to transmit a single MAC packet in k transmissions.

At the receiver side, the incoming PHY packet is decoded and sent to the MAC layer which decides to send back an ACKnowledgment (ACK) or a Negative ACKnowledgment (NACK) to the transmitter accordingly. To make a decision on the MAC packet, the receiver has the following sequential process: checking the CRC for PPACKET(1); if PPACKET(1) is not correctly received, it sends a NACK and it receives afterwards PPACKET(2). Then checking the CRC of the both previous PHY packets concatenation (associated with a FEC of lower rate), and so on until the reception of PPACKET(t_0) which is concatenated with the ($t_0 - 1$) previous PHY packets and then decoded as the mother code of rate R_0 , followed by the CRC checking. Then, if the MAC packet is not received after PPACKET(t_0) decoding and the transmission credit is not reached, the received packet memory is flushed and the process starts again.

We recall that, in a conventional manner (denoted FBS), the PHY packet transmission credit is the same for each MAC packet. The IBS approach, introduced in [1], leads to an ARQ scheme enhancement by providing a global transmission credit, denoted by C , to the set of MAC packets belonging to the same IP packet. Notice that a straightforward extension to HARQ scheme of this principle has been given in [11].

The early drop (ED) strategy can only be applied to the IBS context. This technique allows a transmitter to discard the IP packet at the j -th MAC packet if the remaining transmission credit is not high enough for transmitting the ($N - j$) last MAC packets.

III. NEW CLOSED-FORM EXPRESSION FOR EFFICIENCY

A. General case

As the early drop approach only modifies the packet processing when the IP packet will not be correctly received, the Packet Error Rate and the delay (defined as the average number of packet transmissions when an IP packet is successfully received) are identical to those given in [1], [11], [12]. In contrast, the efficiency is changed.

As said in [13], the efficiency of any HARQ mechanism at the IP layer can be written

$$\eta = \frac{L_{IP}(1 - \Pi)}{\Pi \tilde{n} + (1 - \Pi) \hat{n}}, \quad (1)$$

where

- Π stands for the IP packet error rate. The closed-form expression for Π given in [11], [13] remains valid for the ED approach.
- \hat{n} is the average number of bits sent given that IP packet has correctly been received. The closed-form expression given in [12] for \hat{n} remains valid for the ED approach.
- \tilde{n} is the average number of bits sent given that the current IP packet reception fails. The closed-form expression given in [11] for \tilde{n} is modified by the ED approach since the ED does not manage the transmission credit in the same way than other approach when the IP packet fails.

Therefore our main goal is now to find a closed-form expression for \tilde{n} .

We denote with \tilde{n}^{ed} the average number of bits sent given that the current IP packet failed when ED is employed. We have to enumerate each MAC packets combination corresponding to a failure of the IP packet. The set of these combinations describes the event \mathcal{E} that can be decomposed as follows: $\mathcal{E} = \cup_{\ell=1}^N D(\ell)$ where

- $D(1) = \{\text{MAC packet \#1 consumes } C - N + 2 \text{ credits}\}$,
- $D(\ell) = \{\text{MAC packet \#1 OK and MAC packet \#2 OK and ... and less than } N - \ell \text{ credits left during MAC packet \#\ell \text{ transmission}\}$ for $\ell \in \{2, \dots, N - 1\}$,
- $D(N) = \{\text{MAC packet \#1 OK and ... and MAC packet \#(N - 1) OK and MAC packet \#N KO with the remaining credit}\}$.

Before going further, we have to introduce the following notations. Let $p_1(k)$ be the probability of receiving one MAC packet in exactly k PHY packets transmissions. Let $q(k)$ be the probability of receiving a MAC packet with errors after k PHY packet transmissions. Let us now explicit the probabilities of each event $D(\ell)$.

- $\ell = 1$: whenever it is received or not, the MAC packet #1 consumes at least $C - N + 2$ trials which leads to

$$\Pr \{D(1)\} = q(C - N + 1) \quad (2)$$

and the number of bits sent for this event is equal to $d(1) = w_{C-N+2}$.

- $\ell \in \{2, \dots, N - 1\}$: let us assume that the MAC packet # k (with $k \leq \ell - 1$) has been successfully received and has used i_k transmissions. Then, the consumed transmission credit is equal to $m_{\underline{i}}(k) = \sum_{j=1}^k i_j$ for $k \in \{1, \dots, \ell - 1\}$. The IP packet will not be received if the MAC packet # ℓ consumes at least $C - m_{\underline{i}}(\ell - 1) - (N - \ell) + 1$ credits, whenever it is received or not. Such an event is denoted by $D_{\underline{i}}(\ell)$. Therefore, we have

$$\Pr \{D(\ell)\} = \sum_{\underline{i} \in \mathcal{T}_\ell} \Pr \{D_{\underline{i}}(\ell)\}$$

where $\mathcal{T}_\ell = \{\underline{i} \in \mathbb{N}_*^{\ell-1} \mid m_{\underline{i}}(\ell - 1) = \sum_{k=1}^{\ell-1} i_k < C - N + \ell\}$ and

$$\Pr \{D_{\underline{i}}(\ell)\} = q(C - m_{\underline{i}}(\ell - 1) - N + \ell) \prod_{k=1}^{\ell-1} p_1(i_k) \quad (3)$$

and the number of bits sent for the event $D_{\underline{i}}(\ell)$ is equal to $d_{\underline{i}}(\ell) = r_{\underline{i}}(\ell - 1) + w_{C - m_{\underline{i}}(\ell - 1) - (N - \ell) + 1}$ with $r_{\underline{i}}(\ell) = \sum_{k=1}^{\ell} w_{i_k}$.

- $\ell = N$: similar derivations lead to

$$\Pr \{D(N)\} = \sum_{\underline{i} \in \mathcal{T}_N} \Pr \{D_{\underline{i}}(N)\} \quad (4)$$

where $D_{\underline{i}}(N)$ is defined as in Eq. (3) by putting $\ell = N$. The number of transmitted bits for the event $D_{\underline{i}}(N)$ however is $d_{\underline{i}}(N) = r_{\underline{i}}(N - 1) + w_{C - m_{\underline{i}}(N - 1)}$.

The delay \check{n}^{ed} corresponds to the sum of the number of bits $d_{\underline{i}}(\ell)$ weighted by the probability of the event $D_{\underline{i}}(\ell)$ divided by the IP packet error rate. Therefore, we finally have

$$\check{n}^{ed} = \frac{1}{\Pi} \left(d(1) \Pr \{D(1)\} + \sum_{\ell=2}^N \sum_{\underline{i} \in \mathcal{T}_\ell} d_{\underline{i}}(\ell) \Pr \{D_{\underline{i}}(\ell)\} \right). \quad (5)$$

The term $1/\Pi$ occurs since \check{n} is calculated given that the IP packet has not been correctly received.

We are now able to obtain the following result. The proof is reported in Appendix A.

Result 1. *Let η be the efficiency of an IBS based HARQ at the IP level. Let η^{ed} be the efficiency of the same system but when early drop is carried out. We have then*

$$\eta^{ed} \geq \eta.$$

B. Special cases

a) *Equal Packet Length:* We now assume that all PHY packets have the same length. This assumption is often done in the literature. For example, when IR-HARQ is considered, if the mother code has a rate $R_0 = 1/t_0$ and the punctured code rates have the shape $\{1/t\}_{t=1, \dots, t_0}$, then the equal PHY packet length assumption is satisfied.

When $\sum_{k=1}^{\ell-1} i_k = s$, we have

$$w_{i_k} = i_k L_{\text{MAC}}, \quad m_{\underline{i}}(\ell-1) = s, \quad r_{\underline{i}}(\ell-1) = s L_{\text{MAC}}$$

since $\delta_k = L_{\text{MAC}}, \forall k$.

As for general packet lengths, we have to distinguish the three cases:

- $\underline{\ell} = 1$:

$$d(1) \Pr \{D(1)\} = L_{\text{MAC}}(C - N + 2) \Pr \{D(1)\}.$$

- $\underline{\ell} \in \{2, \dots, N-1\}$: one can easily check that

$$\mathcal{T}_\ell = \bigcup_{s=\ell-1}^{C-N+\ell-1} Q_{s,\ell}$$

where $Q_{s,\ell}$ is the subset of \mathcal{T}_ℓ such that $\sum_{k=1}^{\ell-1} i_k = s$. As a consequence,

$$\sum_{\underline{i} \in \mathcal{T}_\ell} d_{\underline{i}}(\ell) \Pr \{D_{\underline{i}}(\ell)\} = \sum_{s=\ell-1}^{C-N+\ell-1} \sum_{\underline{i} \in Q_{s,\ell}} d_{\underline{i}}(\ell) \Pr \{D_{\underline{i}}(\ell)\}.$$

When $\underline{i} \in Q_{s,\ell}$, we have

$$d_{\underline{i}}(\ell) = (C - N + \ell + 1) L_{\text{MAC}}.$$

Therefore

$$\sum_{\underline{i} \in \mathcal{T}_\ell} d_{\underline{i}}(\ell) \Pr \{D_{\underline{i}}(\ell)\} = L_{\text{MAC}}(C - N + \ell + 1) \Pr \{D(\ell)\}.$$

- $\underline{\ell} = N$: similar derivations can be done. We have thus

$$\sum_{\underline{i} \in \mathcal{T}_N} d_{\underline{i}}(N) \Pr \{D_{\underline{i}}(N)\} = L_{\text{MAC}} C \Pr \{D(N)\}.$$

Finally, we obtain

$$\begin{aligned} \check{n}^{ed} &= L_{\text{MAC}}(C - N + 1) \\ &+ \frac{L_{\text{MAC}}}{\Pi} \left[\sum_{\ell=1}^N \ell \Pr \{D(\ell)\} - \Pr \{D(N)\} \right]. \quad (6) \end{aligned}$$

b) *Type-I HARQ:* for such a kind of HARQ, the PHY packets are all identical and actually equal to the MAC packet (eventually encoded by a fixed rate FEC). Moreover the PHY packets are treated one by one at the receiver side. Therefore, we can work with the expression of \check{n}^{ed} given in Eq. (6). Due to the simple relation between the PHY packet and the MAC packet, we will be able to exhibit simple closed-form expression for the terms $\Pr \{D(\ell)\}$. Notice that the ARQ scheme is a Type-I HARQ one for which the MAC packet corresponds to data without FEC. As the PHY packets are identical and handled independently in the Type-I HARQ context, their error rates are the same and denoted by π_0 . Before going further, we have to remind [14] that

$$\Pi = I_{\pi_0}(C - N + 1, N)$$

where $I_x(a, b) := B_x(a, b)/B(a, b)$ is the regularized Beta function, $B_x(a, b)$ is the incomplete Beta function as defined in [15, Eq. (8.391)] and $B(a, b) = B_1(a, b)$ the Beta function. From now, we need to evaluate $\Pr \{D(\ell)\}$. One can prove that

$$\Pr \{D(\ell)\} = |\mathcal{T}_\ell| (1 - \pi_0)^{\ell-1} \pi_0^{C-N+1}$$

where $|\mathcal{T}_\ell|$ is the cardinality of the set \mathcal{T}_ℓ . By convention, we put $|\mathcal{T}_1| = 1$. One can check that

$$|\mathcal{T}_\ell| = \sum_{s=\ell-1}^{C-N+\ell-1} \binom{s-1}{\ell-2} = \binom{C-N+\ell-1}{\ell-1}.$$

Therefore it remains to calculate

$$\sum_{\ell=1}^N \ell \Pr \{D(\ell)\} = \sum_{\ell=1}^N \ell \binom{C-N+\ell-1}{\ell-1} (1-\pi_0)^{\ell-1} \pi_0^{C-N+1}.$$

In Appendix B, it is proven that

$$\begin{aligned} \sum_{\ell=1}^N \ell \Pr \{D(\ell)\} &= \frac{\pi_0 + K(1 - \pi_0)}{\pi_0} I_{\pi_0}(K, N) \\ &- \frac{\pi_0^{K-1}(1 - \pi_0)^N}{B(K, N)}. \end{aligned}$$

with $K = C - N + 1$. Finally we have

$$\begin{aligned} \check{n}^{ed} &= \left(\frac{\pi_0 + K}{\pi_0} - \frac{\pi_0^{K-1}(1 - \pi_0)^N}{B_{\pi_0}(K, N)} \right) \\ &- \binom{C-1}{N-1} (1 - \pi_0)^{N-1} \pi_0^K L_{\text{MAC}}. \quad (7) \end{aligned}$$

We remind that, in the non early drop case, we have [11]

$$\check{n}^{ned} = C L_{\text{MAC}}. \quad (8)$$

Clearly, the early drop analysis is much more complex than the non-early drop case. Nevertheless we succeed to express \check{n} in closed-form through Eq. (7).

IV. NUMERICAL RESULTS

In this section, we will inspect the relevance of the early drop approach for different kinds of HARQ. Before going further, we would like to validate our analytical expressions. Therefore, in Figure 1, we compute the closed-form expressions and we also evaluate the efficiency through extensive Monte-Carlo simulations. For several types of HARQ (ARQ or HARQ with Chase Combining) and of channel (Gaussian or Rayleigh), we observe a nice agreement between our expressions and the estimated points.

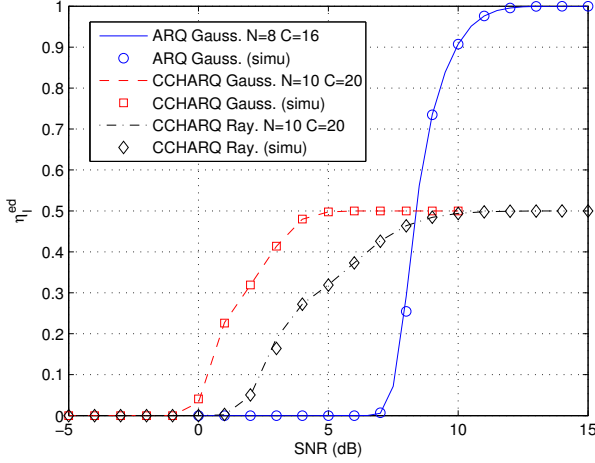


Figure 1. Theoretical and empirical efficiencies versus SNR for different types of HARQ when early drop approach is applied.

For the sake of clarity we will now consider, for instance, an ARQ scheme with QPSK over an AWGN channel. Numerous other simulations have been run for different configurations ; all of them lead to similar comments. In Figure 2, we remark the little gain in efficiency. Thus, the early drop approach does not provide a significant gain but just an incremental one.

Let us explain this phenomenon. In Figure 3, we plot the two terms involved in the denominator of Eq. (1) versus SNR when early drop is employed or not. We observe that the main difference between the terms $\Pi\tilde{n}$ occurs at low and medium SNR. As soon as the SNR is large enough, the early drop does not provide improvement since $\Pi\tilde{n}$ (circles) and $\Pi\tilde{n}^{ed}$ (squares) have quite the same value. At low SNR, the gain in $\Pi\tilde{n}$ is important. But as the efficiency is very low (around 0) for both approaches, the improvement is significant in relative value but keeps very small and thus useless in absolute value. However, at medium SNR, the efficiency is slightly improved.

In Table I, we inspect the relative efficiency gains defined as

$$G = \frac{\eta^{ed} - \eta}{\eta},$$

averaged over different SNR uniformly distributed between 5 and 10 dB for different channels and HARQ mechanisms. The gain in percent is quite small but remains of interest.

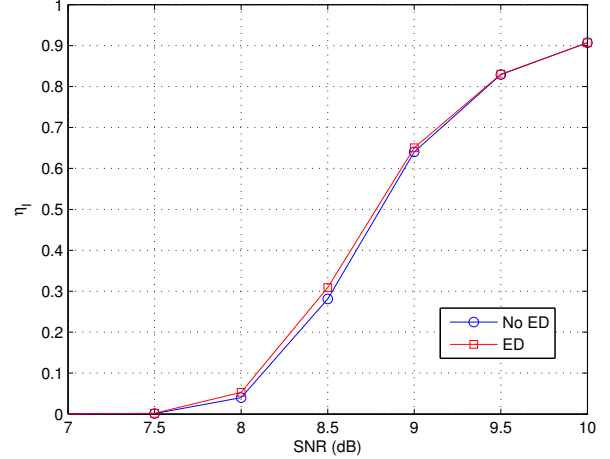


Figure 2. Efficiency versus SNR (ARQ scheme, AWGN channel, QPSK constellation, $N = 14$, and $C = 21$).

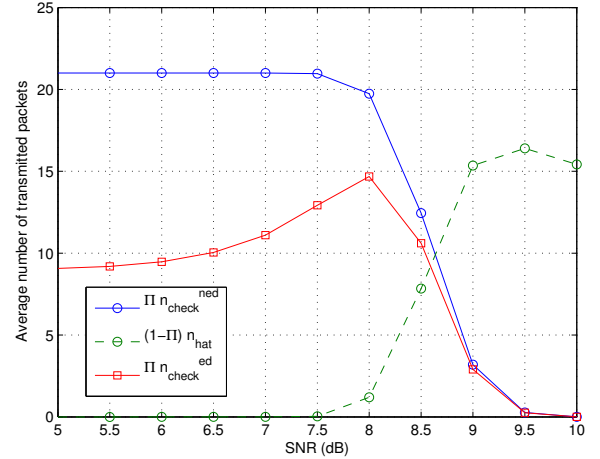


Figure 3. $\Pi\tilde{n}$ and $(1 - \Pi)\hat{n}$ versus SNR (ARQ scheme, AWGN channel, QPSK constellation, $N = 14$, and $C = 21$).

	G (%) over AWGN	G (%) over Rayleigh
ARQ	12.4	17.1
CC-HARQ	4.0	3.8
IR-HARQ	2.6	1.8

Table I
AVERAGE RELATIVE EFFICIENCY GAIN FOR DIFFERENT HARQ TYPES
($N = 8$, $C = 16$).

V. CONCLUSION

In this paper, we have deeply analyzed the so-called early-drop technique well adapted to cross-layer designed HARQ schemes. A closed-form expression for the efficiency has been derived, and it has been shown that the efficiency is only slightly improved when early drop is used, without degrading the other performance metrics.

A. Proof of Result 1

In non-early drop context, a more precise description than $D_{\underline{i}}(\ell)$ is needed since we have to know how the MAC packets $\#\ell'$ (with $\ell' > \ell$) are handled. Therefore we can decompose $D_{\underline{i}}(\ell)$ as follows: $D_{\underline{i}}(\ell) = \cup_{\underline{i}'} D_{\underline{i}, \underline{i}'}(\ell)$ where $D_{\underline{i}, \underline{i}'}(\ell)$ represents a certain way of handling the remaining $N - \ell - 1$ MAC packets given that the ℓ first MAC packets are handled as in $D_{\underline{i}}(\ell)$. Then, we have to replace in Eq. (5)

$$\sum_{\underline{i} \in \mathcal{T}_\ell} d_{\underline{i}}(\ell) \Pr \{D_{\underline{i}}(\ell)\}$$

with

$$\sum_{\underline{i} \in \mathcal{T}_\ell} \sum_{\underline{i}'} d_{\underline{i}, \underline{i}'}(\ell) \Pr \{D_{\underline{i}, \underline{i}'}(\ell)\}$$

where $d_{\underline{i}, \underline{i}'}(\ell)$ is the cost in packets of the event $D_{\underline{i}, \underline{i}'}(\ell)$. As in early drop context the transmission stops as soon as $D_{\underline{i}}(\ell)$ occurs, we have

$$d_{\underline{i}, \underline{i}'}(\ell) \geq d_{\underline{i}}^{ed}(\ell)$$

which implies that

$$\tilde{n} \geq \tilde{n}^{ed}$$

and concludes the proof.

B. Simplification for Type-I HARQ

The purpose is to find $\sum_{\ell=1}^N \ell \Pr \{D(\ell)\} = \pi_0^{C-N+1} f(1 - \pi_0)$, where f is the (analytical) function defined for $x \in [0, 1]$ by

$$f(x) = \sum_{\ell=1}^N \ell \binom{C - N + \ell - 1}{\ell - 1} x^{\ell-1}.$$

Our approach will be to find a closed-form expression for a primitive F of f ,

$$F(x) = \sum_{\ell=1}^N \binom{C - N + \ell - 1}{\ell - 1} x^\ell$$

and then to calculate its derivative. Due to the page limitation, we only give a sketch of the proof, and some simplifications are left to the reader. First, by using the symmetry property $\binom{n}{k} = \binom{n}{n-k}$, $\forall n \geq k$ and an index reorganization, we obtain that

$$F(x) = \frac{G(x)}{x^{C-N}} \text{ with } G(x) = \sum_{\ell=C-N+1}^C \binom{\ell-1}{C-N} x^\ell.$$

Then, by using the Pascal identity for binomial coefficients [15, Eq. (3.1.4)], it can be shown that G satisfies the following differential equation

$$y - \frac{x(1-x)}{K} y' = \mu x^{C+1} \quad (9)$$

with $\mu = \binom{C}{K}$ and $K = C - N + 1$. Standard resolution methods can be used to solve Eq. (9), and we have

$$G(x) = \left(\frac{x}{1-x} \right)^K (\lambda - K\mu B_x(N, K)).$$

Due to [15, Eq. (3.1.4)], we have $\mu K = 1/B(N, K)$. Thus

$$F(x) = \frac{x}{(1-x)^K} (\lambda - I_x(N, K)).$$

In order to characterize λ , let us consider Π . One can check that

$$\Pi = \sum_{\ell=1}^N \Pr \{D(\ell)\} = \frac{\pi_0^K}{1 - \pi_0} F(1 - \pi_0) = \lambda - I_{1-\pi_0}(N, K).$$

As $\lim_{\pi_0 \rightarrow 1} \Pi = 1$ and $\lim_{\pi_0 \rightarrow 1} I_{1-\pi_0}(N, K) = 0$, we have $\lambda = 1$. Thus

$$F(x) = \frac{x}{(1-x)^K} (1 - I_x(N, K)).$$

Moreover, as $1 - I_x(b, a) = I_{1-x}(a, b)$, we finally have

$$F(x) = \frac{x}{(1-x)^K} I_{1-x}(K, N). \quad (10)$$

To find the final result, just take the derivative of $F(\cdot)$ given in Eq. (10).

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