Error probability approximation and codes selection in presence of multi-user interference for IR-UWB

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***Abstract*— An approximation for the Average Error Probability (AEP) of the Pulse Amplitude Modulation (PAM) Impulse Radio Ultra Wide Band (IR-UWB) systems in the presence of Multi-User Interference (MUI) is derived assuming that the spreading codes are fixed in time. The comparison of the proposed theoretical expression and the empirical result shows the accuracy of our approximation for both Direct-Sequence (DS) and Time-Hopping (TH) multiple access techniques. From this approximation, we deduce criterion that enable us to select the set of codes optimizing the performance in terms of the AEP.**

I. Introduction

The exact Average Error Probability (AEP) has been expressed in [8] for TH IR-UWB system. Their derivations are based on characteristic function. The obtained AEP is obviously accurate but the obtained expression is still too much complex and does not provide any highlight about the influence of design parameters (such as the multi-user codes). Actually if we would like to obtain a simple AEP closed-from expression, we need to carefully approximate the multi-user interference (MUI) distribution. In [1], it was shown that the Generalized Gaussian Distribution (GGD) is a relevant choice to describe the MUI distribution. Assuming that the MUI is Generalized-Gaussian distributed, we derive an accurate closed form expression for the AEP in function of the multiple access codes for both TH and DS multiple access techniques. From the obtained AEP approximation, we are able to exhibit the criterion that the multiple access codes have to minimize in order to optimize the AEP for both access techniques. Minimizing such a criterion leads the optimal multiple access to satisfy some constraints that are characterized in this paper. The obtained results can be easily extended to the Pulse Position Modulation (PPM) TH IR-UWB system.

This paper is organized as follows. In Section II, we introduce the transmitted signal model for both TH and DS PAM IR-UWB systems, the channel model and the rake receiver structure. In Section III, we assume the codes to be fixed, and we derive the AEP in closed-form for the two analyzed systems using the Generalized Gaussian (GG) approximation whose parameters are evaluated as well in terms of the multiple access codes. In Section IV, we present the criterion that the codes have to satisfy in order to improve the performance. Section V is devoted to numerical illustrations. By empirical simulations, we validate our GGD based approximation and we inspect also the impact of the codes choice on the performance. We show the AEP significantly decreases when codes are selected as suggested in Section IV. Conclusions are given in Section VI.

II. Signal model

We consider a PAM IR-UWB system with either TH or DS as a code division multiple access technique. Let N_u be the number of active users in the network. Each user transmits information asynchronously through a multipath channel. The transmitted signal from user n can be expressed similarly for both DS and TH access techniques as follows:

$$s_n(t) = \sum_{i=-\infty}^{\infty} d_n(i) \sum_{j=0}^{N_c-1} c_n(j) w(t - iT_s - jT_c - \theta_n) \quad (1)$$

where

- N_c is the number of chips per symbol,
- T_c is the duration of one chip,
- T_s is the symbol time,
- $w(t)$ is the normalized impulse of duration $T_w \ll T_c$,
- $d_n(i) \in \{-1, 1\}$ are the information symbols of user n, assumed to be independent and identically distributed,
- $\{c_n(j)\}_{j=0}^{N_c-1}$ is the multiple access code, with $c_n(j) \in \{-1, 1\}$ for DS scheme and $c_n(j) \in \{0, 1\}$ is the Developed Time Hopping (DTH) code [3], associated with user n,
- θ_n denotes the time asynchronism, assumed to be uniformly distributed random variable within $[0, T_s]$,

The receiver input signal is the sum of the attenuated and delayed transmitted signals from the different users. Its expression is given by

$$r(t) = \sum_{n=1}^{N_u} \sqrt{P_n} \left(\sum_{k=1}^{N_p} A_n^k s_n(t - \tau_n^k) \right) + n(t) \quad (2)$$

where A_n^k and τ_n^k are the amplitude and the delay of the k^{th} path between the user n and the receiver, N_p is the number of paths, assumed to be the same for all the users, P_n is the received power, and $n(t)$ is an additive zero-mean white Gaussian noise.

The multipath channel model we employ is that proposed generally for UWB systems. The amplitude A_n^k is usually assumed to be dependent on the delay τ_n^k as $A_n^k = a_n^k f(\tau_n^k)$, where a_n^k are independent zero-mean random variables (rv) which account for the amplitude statistics and $f(\cdot)$ is a function which indicates the variation of the amplitude according to the delay. For sake of simplicity, we also consider that the channel impulse response is normalized, i.e., $\sum_{n=0}^{N_p}(A_n^k)^2 = 1$. The information about random received powers is provided by the set $\{P_n\}_{n=1,\cdots,N_u}$. The rv τ_n^k are assumed to be independent between users but are usually correlated for a given user. The distribution of the variables τ_n^k and a_n^k are provided in the IEEE 802.15.3a standard [9]. When only one cluster is considered (which is not restrictive as mentioned in [10]), the delay τ_n^k follows a Poisson distribution. The attenuation $a_n^k = p_n^k \cdot \beta_n^k$, where $p_n^k \in \{\pm 1\}$ are equiprobable and β_n^k are log-normal rv. The function $f(\cdot)$ is defined by $f(\tau_n^k) = e^{-\tau_n^k/\gamma}$, where γ is the path power-decay time.

Without loss of generality, the user of interest is assumed to be the user 1. We consider the rake receiver of user 1, commonly used for multipath channel systems, with $L_r \leq N_p$ fingers. We also assume that the receiver is synchronized, i.e., $\theta_1 = 0$. Thus, the rake receiver output is given by

$$z = \sum_{\ell \in \mathcal{L}} A_1^\ell \int_0^{T_s} r(t+\tau_1^\ell) \cdot v_1(t) \mathrm{d}t \quad (3)$$

where $v_1(t) = \sum_{j=0}^{N_c-1} c_1(j) w(t - jT_c)$ is the template signal associated with user 1 and $\mathcal{L}$ is the selected subset paths with Card($\mathcal{L}$)=L_r. Using Eqs. (2)-(3), the rake receiver output is equal to

$$z = \sum_{\ell \in \mathcal{L}} A_1^\ell \sum_{n=1}^{N_u} \sqrt{P_n} \sum_{k=1}^{N_p} A_n^k y_{k,\ell,n}(\theta_n) + \eta \quad (4)$$

where $\eta = \sum_{\ell \in \mathcal{L}} A_1^\ell \int_0^{T_s} n(t+\tau_1^\ell) v_1(t) \mathrm{d}t$ is the filtered Gaussian noise, and

$$\begin{aligned} y_{k,\ell,n}(\theta_n) &= d_n(-Q_n^{k,\ell}) \left[\mathcal{C}_{1,n}^+(q_n^{k,\ell}) r(\epsilon_n^{k,\ell})\right. \\ &+ \left.\mathcal{C}_{1,n}^+(q_n^{k,\ell}+1)\, r(\epsilon_n^{k,\ell} - T_c)\right] \\ &+ d_n(-Q_n^{k,\ell}-1) \left[\mathcal{C}_{1,n}^-(q_n^{k,\ell}) r(\epsilon_n^{k,\ell})\right. \\ &+ \left.\mathcal{C}_{1,n}^-(q_n^{k,\ell}+1) r(\epsilon_n^{k,\ell} - T_c)\right], \end{aligned} \quad (5)$$

with $Q_n^{k,\ell} = \lfloor(\theta_n + \tau_n^k - \tau_1^\ell)/T_s\rfloor$, $q_n^{k,\ell} = \lfloor(\theta_n + \tau_n^k - \tau_1^\ell - Q_n^{k,\ell} T_s)/T_c\rfloor$, and $\epsilon_n^{k,\ell} = \theta_n + \tau_n^k - \tau_1^\ell - Q_n^{k,\ell} T_s - q_n^{k,\ell}$ which lies in $[0, T_c]$. We also put

$$\mathcal{C}_{m,n}^-(q) = \sum_{k=0}^{q-1} c_m(k) c_n(k-q) \quad (6)$$

$$\mathcal{C}_{m,n}^+(q) = \sum_{k=q}^{N_c-1} c_m(k) c_n(k-q) \quad (7)$$

and $r(s) = \int_{-\infty}^{+\infty} w(t) w(t-s) \mathrm{d}t$. Notice that the rake receiver output given by Eq. (4) can be decomposed as [3]:

$$z = z_U + z_I + z_M + \eta \quad (8)$$

where,

- z_U is the *U*seful part of user 1 signal, and is given by $z_U = \sqrt{P_1} \sum_{\ell \in \mathcal{L}} (A_1^\ell)^2 y_{\ell,\ell,1}(0)$
- z_I is the *I*nter-symbol interference from user 1, and is given by $z_I = \sqrt{P_1} \sum_{\ell \in \mathcal{L}} A_1^\ell \sum_{k \neq \ell = 1}^{N_p} A_1^k y_{k,\ell,1}(0)$
- z_M is the *M*ulti-user interference, and is given by $z_M = \sum_{\ell \in \mathcal{L}} A_1^\ell \sum_{n=2}^{N_u} \sqrt{P_n} \sum_{k=1}^{N_p} A_n^k y_{k,\ell,n}(\theta_n)$

Unlike z_I and z_M, the useful part z_U and the filtered noise η do not depend on the multiple access codes. Fortunately, if the channel is short enough compared to symbol period, the term z_I can be neglected by inserting a guard time [7]. However, the MUI can only be mitigated by a judicious choice of the multiple access codes. In the sequel, for sake of simplicity, on the one hand, we assume that $z_I = 0$, and on the other hand, we consider the set of received powers fixed, i.e., the obtained closed-form expression will depend on the realization of $\{P_n\}_{n=1,\cdots,N_u}$.

III. AEP APPROXIMATION BASED ON GENERALIZED GAUSSIAN DISTRIBUTION

The first works dealing with the performance of UWB systems assumed that the MUI was a Gaussian distributed random variable. Later, it has been proved in [5], [6] that the Gaussian approximation is not valid in many cases. Recently, it has been proposed to use the GGD to describe the MUI distribution in TH IR-UWB system [1] in AWGN context. By simulation, we remarked that the MUI can still be well modeled by GGD when IEEE 802.15.3a standard based multipath channels (described in Section II) are implemented. We remind that the GGD writes as follows [4]:

$$p(x) = \frac{\sqrt{\Gamma_c(3/\alpha)}}{2\sigma\sqrt{\Gamma_c(1/\alpha)}\Gamma_c(1+1/\alpha)} e^{-\left|\frac{\sqrt{\Gamma_c(3/\alpha)}}{\sigma\sqrt{\Gamma_c(1/\alpha)}} x\right|^\alpha} \quad (9)$$

where $\sigma^2 = \mathbb{E}[x^2]$ is the variance, $\alpha > 0$ is the so-called shape parameter, and $x \mapsto \Gamma_c(x)$ is the Gamma function. Remark that when $\alpha = 2$, $p(x)$ corresponds to a Gaussian distribution. In the sequel, given the set of received powers, we derive the AEP when the MUI is assumed to be GG distributed.

A. AEP approximation based on GGD

First of all, notice that the useful signal z_U in Eq. (8) is equal to $z_U = d_1(0)\sqrt{P_1} N_s$ where N_s is the repetition factor ($N_s = N_c$ for DS-UWB system; $N_s = N_f$ for TH-UWB system where N_f is the number of frame). Then, since the GGD is symmetric and the PAM modulation is equilikely, the AEP is given by:

$$\overline{P}_e = \mathrm{Prob}(\nu > \sqrt{P_1} N_s) = \int_{\sqrt{P_1} N_s}^{+\infty} p_\nu(x) dx \quad (10)$$

where $\nu = z_M + \eta$ is the term disturbing the decision and where $p_\nu(x)$ is its distribution.

As z_M is assumed to be GG distributed and as η is Gaussian distributed, i.e., GG distributed, we know that ν is also well approximated by a GGD. Indeed, in [2], one has been mentioned that the sum of two GG distributed variables can be approximated by a GG distributed variable as well.

Consequently, the distribution of ν is described by Eq. (9) whose the shape parameter and variance are α and σ^2 respectively. Note that the expressions of α and σ^2 in terms of multiple access codes will be calculated in Section III-B. By replacing $p_\nu(x)$ in Eq. (10) with its expression in Eq. (9) and by doing tedious but straightforward algebraic manipulations, we get

$$\overline{P}_e = \frac{1}{2\alpha\Gamma_c(1+1/\alpha)}\Gamma_i\left[\frac{1}{\alpha},\left(\frac{N_s\sqrt{\Gamma_c(3/\alpha)}}{\sigma\sqrt{\Gamma_c(1/\alpha)}}\right)^{\alpha}\right] \quad (11)$$

where $\Gamma_i[.,.]$ is the so-called incomplete Gamma function defined by $\Gamma_i[a,x] = \int_x^{+\infty} t^{a-1}\exp(-t)\mathrm{d}t$.

B. GGD parameters vs the multiple access codes

In the sequel, we denote by α_M and σ_M^2 the shape parameter and variance of signal z_M respectively. Terms $\alpha_\eta = 2$ and σ_η^2 stand for the shape parameter and variance of noise η respectively. Let us now focus on the derivation of σ^2 and α. As z_M and η are zero mean, we have

$$\sigma^2 = \sigma_M^2 + \sigma_\eta^2. \quad (12)$$

As mentioned [1], the shape parameter α of a GGD is related to the 4^{th} and 2^{th} order moment as follows

$$\alpha = F^{(-1)}\left(\frac{D^4}{\sigma^4}\right) \quad (13)$$

where $D^4 = \mathbb{E}[\nu^4]$, and where $F^{(-1)}(.)$ is the reciprocal function of $x \mapsto F(x) = \Gamma_c(5/x)\Gamma_c(1/x)/\Gamma_c^2(3/x)$. Let $D_M^4 = \mathbb{E}[z_M^4]$ and notice that $\mathbb{E}[\eta^4] = 3\sigma_\eta^4$. Like the 2^{th} order moment, the 4^{th} order moment of ν can be expressed in function of those of z_M and η as

$$D^4 = D_M^4 + 3\sigma_\eta^4 + 6\sigma_M^2\sigma_\eta^2. \quad (14)$$

In order to determine perfectly the statistics of ν in terms of the multiple access codes, we only need to derive σ_M^2 and D_M^4 in terms of the multiple access codes. In [1], the average of σ_M^2 and D_M^4 over all the TH multiple access codes were evaluated. In our work, we remind that the multiple access codes are fixed since we would to select them according to the minimization of the obtained AEP approximation. Notice that the expectation for deriving the 2^{th} and 4^{th} order moments is achieved over the channel amplitude a_n^k, the symbol d_n, the asynchronism θ_n and the delay τ_n according to this order.

1) Closed-form expression of $\sigma_M^2 := \mathbb{E}_{a,d,\theta,\tau}[z_M^2]$: in [3] dedicated to TH-UWB system, we have

$$\sigma_M^2 = \frac{\gamma_1}{T_s}\sum_{n=2}^{N_u} P_n\Psi_n \sum_{q=0}^{N_c-1}\left[\mathcal{C}_{1,n}^{-2}(q) + \mathcal{C}_{1,n}^{+2}(q)\right] \quad (15)$$

where $\gamma_1 = \int r^2(t)\mathrm{d}t$, $\Psi_n = \sum_{\ell\in\mathcal{L}}\mathbb{E}_\tau[I_1^\ell]\sum_{k=1}^{N_p}\mathbb{E}_\tau[I_n^k]$ with $I_n^k = \mathbb{E}_a[(A_n^k)^2]$.

Thanks to Eq. (1), one can see that the MUI can be similarly represented for TH-UWB and DS-UWB when employing PAM modulation. Consequently the variance given by Eq. (15) remains valid for DS-UWB system.

2) Closed-form expression of $D_M^4 := \mathbb{E}_{a,d,\theta,\tau}[z_M^4]$: the MUI can be decomposed as $z_M = \sum_{n=2}^{N_u} z_{M,n}$ where $z_{M,n} = \sqrt{P_n}\sum_{k=1}^{N_p} A_n^k y_{k,\ell,n}(\theta_n)$ is the interference associated with user n. Since the symbols d_n and d_n^3 are zero-mean, the expectation of z_M^4 over the amplitude a and the symbol d is given by

$$\mathbb{E}_{a,d}[z_M^4] = \sum_{n=2}^{N_u}\mathbb{E}_{a,d}[z_{M,n}^4] + 6\sum_{\substack{n,m=2\\n\neq m}}^{N_u}\mathbb{E}_{a,d}[z_{M,n}^2]\mathbb{E}_{a,d}[z_{M,m}^2]$$

As the time-support of $r(\cdot)$ is much less than T_c, we have $r^p(\epsilon)r^q(\epsilon - T_c) = 0$, $\forall p,q$. Consequently, we get

$$\begin{aligned}\mathbb{E}_{a,d}[z_{M,n}^4] =& P_n^2\sum_{\substack{k=1\\\ell\in\mathcal{L}}}^{N_p} J_1^\ell J_n^k\Big[\left(\mathcal{C}_{1,n}^{+4}(q_n^{k,\ell}) + \mathcal{C}_{1,n}^{-4}(q_n^{k,\ell})\right)r^4(\epsilon_n^{k,\ell})\\ &+\left(\mathcal{C}_{1,n}^{+4}(q_n^{k,\ell}+1) + \mathcal{C}_{1,n}^{-4}(q_n^{k,\ell}+1)\right)r^4(\epsilon_n^{k,\ell}-T_c)\\ &+6\mathcal{C}_{1,n}^{+2}(q_n^{k,\ell})\mathcal{C}_{1,n}^{-2}(q_n^{k,\ell})r^4(\epsilon_n^{k,\ell})\\ &+6\mathcal{C}_{1,n}^{+2}(q_n^{k,\ell}+1)\mathcal{C}_{1,n}^{-2}(q_n^{k,\ell}+1)r^4(\epsilon_n^{k,\ell}-T_c)\Big]\end{aligned}$$

with $J_n^k = \mathbb{E}_a[(A_n^k)^4]$.

The expectation of $\mathbb{E}_{a,d}[z_{M,n}^4]$ over the uniform variable θ_n is obtained by $\mathbb{E}_{a,d,\theta}[z_{M,n}^4] = \frac{1}{T_s}\int_{T_s}\mathbb{E}_{a,d}[z_{M,n}^4]d\theta$. By writing the integral over $[0,T_s]$ as a sum of integrals over the subinterval $[0,T_c]$, and by taking into account the periodicity of the multiple access codes, we find

$$\begin{aligned}\mathbb{E}_{a,d,\theta}[z_{M,n}^4] =& \frac{\gamma_2}{T_s}P_n^2\sum_{\substack{k=1\\\ell\in\mathcal{L}}}^{N_p} J_1^\ell J_n^k\\ &\times\sum_{q=0}^{N_c-1}\left[\mathcal{C}_{1,n}^{+4}(q) + \mathcal{C}_{1,n}^{-4}(q) + 6\mathcal{C}_{1,n}^{+2}(q)\mathcal{C}_{1,n}^{-2}(q)\right]\end{aligned}$$

with $\gamma_2 = \int r_{ww}^4(t)\mathrm{d}t$.

Finally, using previous equalities, averaging $\mathbb{E}_{a,d,\theta}[z_{M,n}^4]$ over the delays $\tau_n^k - \tau_1^\ell$, and reminding that $\sigma_M^2 = \sum_{n=2}^{N_u}\mathbb{E}_{a,d,\theta,\tau}[z_{M,n}^2]$ leads to the following expression for the 4^{th} order moment

$$\begin{aligned}D_M^4 =& \frac{\gamma_2}{T_s}\sum_{n=2}^{N_u}P_n^2\Phi_n\Big[\frac{3(N_u-2)}{N_u-1}\sigma_M^4\\ &+\sum_{q=0}^{N_c-1}\mathcal{C}_{1,n}^{+4}(q) + \mathcal{C}_{1,n}^{-4}(q) + 6\mathcal{C}_{1,n}^{+2}(q)\mathcal{C}_{1,n}^{-2}(q)\Big]\end{aligned} \quad (16)$$

where $\Phi_n = \sum_{\ell\in\mathcal{L}}\mathbb{E}_\tau[J_1^\ell]\sum_{k=1}^{N_p}\mathbb{E}_\tau[J_n^k]$ and where σ_M^2 is given by Eq. (15).

IV. Multiple access codes minimizing the AEP

Given N_s and the receiver powers, the AEP (see Eq. (11)) depends only on α and σ^2. For a given σ^2, one can remark that $\overline{P}_e$ decreases when α increases at high SINR. Therefore, in order to minimize the performance, i.e., the AEP, we have to select the codes that minimize σ^2 and then maximize α. Thanks to Eq. (13) and the monotonic decreasing property of $F^{(-1)}(.)$, maximizing α is equivalent to minimizing D^4

when σ^2 is fixed. Notice that the high SINR assumption is not restrictive since we would like to improve the error floor occuring in the Rake receiver. Thanks to Eqs. (12)-(14) and thanks to the independence of σ_η^2 with respect to the multiple access codes codes, minimizing σ^2 and D^4 with respect to the multiple access boils down to minimizing σ_M^2 and D_M^4 with respect to the multiple access codes. Unlike the previous sections of this paper, we need hereafter distinguish the multiple access codes selection for TH-UWB and DS-UWB systems since the codes belong to $\{0,1\}$ and $\{-1,1\}$ in TH and DS case respectively.

A. Optimal Developed Time Hopping codes

Before going further, we introduce the following proposition for which we omit the proof due to the lack of space. Proof can be done similarly to the proof of Theorem 1 in [3].

Proposition 1: Let us consider a pair of DTH code (c_m, c_n) that satisfies $\sum_{q=0}^{N_c-1} \mathcal{C}_{m,n}^{+2}(q) + \mathcal{C}_{m,n}^{-2}(q) = N_s^2$, then

$$\sup_q \mathcal{C}_{m,n}^{+}(q) = 1 \textit{ and } \sup_q \mathcal{C}_{m,n}^{-}(q) = 1 \tag{17}$$

We are then able to state Theorem 1 which characterize the Developed Time Hopping codes minimizing the AEP.

Theorem 1: The AEP of user of interest 1 is minimum, if and only if, the set of pair of DTH codes $\{(c_1, c_n),\ n = 2, \ldots, N_u\}$, satisfies

$$\sum_{q=0}^{N_c-1} \left(\mathcal{C}_{1,n}^{+}(q) + \mathcal{C}_{1,n}^{-}(q)\right)^2 = N_s^2 \tag{18}$$

Notice that, in [3], the authors suggest to select the DTH codes minimizing the variance, i.e., satisfying $\sum_{q=0}^{N_c-1} (\mathcal{C}_{1,n}^{+2}(q) + \mathcal{C}_{1,n}^{-2}(q)) = N_s^2$ which corresponds to a larger set of codes than the set of codes verifying Eq. (18).

Proof: We aim to identify the pair of codes that minimizes both σ_M^2 and D_M^4 given by Eqs. (15) and (16) respectively. In [3], it has been already proven that a pair of DTH codes minimizes σ_M^2 if and only if $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+2}(q) + \mathcal{C}_{1,n}^{-2}(q) = N_s^2$. Let us consider a pair of codes that satisfies this last condition and let us prove that D_M^4 is minimal if and only if $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+}(q)\mathcal{C}_{1,n}^{-}(q) = 0$. Using Eq. (17) and noting that $\mathcal{C}_{1,n}^{-}(q),\ \mathcal{C}_{1,n}^{+}(q) \geq 0$, we can deduce that $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+4}(q) + \mathcal{C}_{1,n}^{-4}(q)$ is minimal as well and is equal to N_s^2. Hence, D_M^4 is minimal if and only if $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+2}(q)\mathcal{C}_{1,n}^{-2}(q) = 0$, or equivalently, $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+}(q)\mathcal{C}_{1,n}^{-}(q) = 0$ due to Eq. (17) for the DTH codes minimizing the variance and due to the positivity of $\mathcal{C}_{1,n}^{-}(q)$ and $\mathcal{C}_{1,n}^{+}(q)$. This concludes the proof. ■

B. Optimal Direct Sequence codes

Before exhibiting the Optimal Direct Sequence codes in Theorem 2, we introduce two preliminary propositions.

Proposition 2: Let (c_m, c_n) be two Direct Sequence codes of length N_c. We have

$$\sum_{q=0}^{N_c-1} \mathcal{C}_{m,n}^{+2}(q) + \mathcal{C}_{m,n}^{-2}(q) \geq N_c \tag{19}$$

Proof: Given Eqs. (6)-(7), we show that $\mathcal{C}_{m,n}^{-}(q)$ and $\mathcal{C}_{m,n}^{+}(q)$ consist of q and $N_c - q$ terms respectively. Each term belongs to $\{\pm 1\}$. When N_c is odd, q and $N_c - q$ does not have the same parity. Consequently, when q is even, $\mathcal{C}_{m,n}^{-}(q)$ is lower-bounded by 0 and $\mathcal{C}_{m,n}^{+}(q)$ is lower-bounded by 1 which implies that $\mathcal{C}_{m,n}^{+2}(q) + \mathcal{C}_{m,n}^{-2}(q)$ is lower-bounded by 1. When q is odd, we just have to permute the role of $\mathcal{C}_{m,n}^{-}(q)$ and $\mathcal{C}_{m,n}^{+}(q)$. Then we deduce immediately that $\sum_{q=0}^{N_c-1} \mathcal{C}_{m,n}^{+2}(q) + \mathcal{C}_{m,n}^{-2}(q)$ is lower bounded by N_c the number of terms in the sum. When N_c is even, similar proof can be done. ■

Proposition 3: Let (c_1, c_n) be a pair of Direct Sequence code satisfying $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+2}(q) + \mathcal{C}_{1,n}^{-2}(q) = N_c$.

- *In the case of even N_c: $|\mathcal{C}_{1,n}^{+}(q)| = |\mathcal{C}_{1,n}^{-}(q)| = 0$ if q is even; and $|\mathcal{C}_{1,n}^{+}(q)| = |\mathcal{C}_{1,n}^{-}(q)| = 1$ if q is odd.*
- *In the case of odd N_c: $|\mathcal{C}_{1,n}^{+}(q)| = 1$ and $|\mathcal{C}_{1,n}^{-}(q)| = 0$ if q is odd; and $|\mathcal{C}_{1,n}^{+}(q)| = 0$ and $|\mathcal{C}_{1,n}^{-}(q)| = 1$ if q is even.*

Due to the lack of space, the proof of Proposition 3 is omitted. Nevertheless the proof can be done in similar way of those of Proposition 2.

Theorem 2: The AEP of user of interest 1 is minimum, if and only if, the set of pair of DS codes $\{(c_1, c_n),\ n = 2, \ldots, N_u\}$, satisfies

$$\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+2}(q) + \mathcal{C}_{1,n}^{-2}(q) = N_c \tag{20}$$

Unlike TH scheme, we also see that minimizing jointly the variance and the shape parameter of MUI distribution is equivalent to minimizing the variance only, in DS scheme context.

Proof: Due to lack of space, we only prove that $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+2}(q) + \mathcal{C}_{1,n}^{-2}(q) = N_s$ yields that σ_M^2 and D_M^4 are minimal. The proof of the reverse implication can be easily done by using Propositions 2 and 3. Let $\{(c_1, c_n)\}$ be a set of pair of DS code satisfying Eq. (20). Thanks to Eq. (19), we have that the variance σ_M^2 (given by Eq. (15)) is minimal. Let us now focus on D_M^4 given by Eq. (16). By means of Proposition 3, one can easily check that $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+4}(q) + \mathcal{C}_{1,n}^{-4}(q)$ is minimal and is equal to N_c. The second term of D_M^4, given by $\sum_{q=0}^{N_c-1} \mathcal{C}_{1,n}^{+2}(q)\mathcal{C}_{1,n}^{-2}(q)$, is equal to $N_c/2$ if N_c is even and 0 otherwise, and thus identical for any code minimizing the variance. Therefore, Eq. (20) leads to the joint minimization of σ_M^2 a,d D_M^4. ■

V. Numerical results

We consider an AWGN channel and a normalized Gaussian impulse

$$w(t) = A_w \sqrt{\frac{2}{\pi}} \frac{\cos(2\pi f_c t)}{\lambda} e^{-\frac{t^2}{2\lambda^2}}$$

with A_w is a normalized factor such that $\int_{-\infty}^{+\infty} w^2(t)\mathrm{d}t = 1$, $f_c = 6.85$ GHz and $\lambda = 9.107 \times 10^{-2}$ ns.

In Figures (1) and (2) we compared the theoretical AEP approximation given by Eq. (11) (displayed in dotted lines) to the empirical Bit Error Rate (displayed in solid lines) with

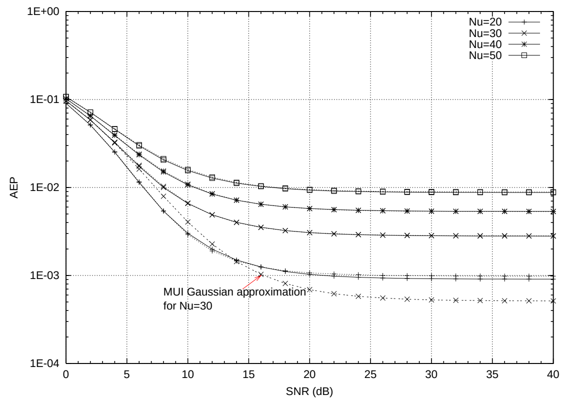

Fig. 1. Theoretical AEP and BER for PAM TH-UWB system with $N_c = 16$, $N_s = 4$, $T_c = 3$ ns and random codes.

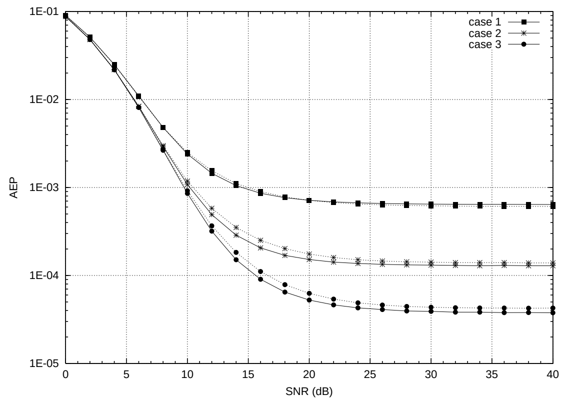

Fig. 3. Performance wrt the codes properties for PAM TH-UWB system with $N_c = 24$, $N_s = 4$, $T_c = 3$ ns and $N_u = 30$.

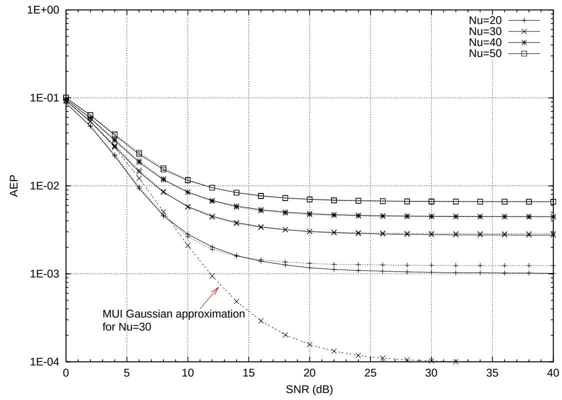

Fig. 2. Theoretical AEP and BER for PAM DS-UWB system with $N_c = N_s = 6$, $T_c = 8$ ns and random codes.

different values of N_u for TH-UWB and DS-UWB systems respectively. The symbol time is equal to $T_s = 48$ ns for both systems. For TH system, the number of chips N_c is equal to 16 and the repetition factor N_s is equal to 4. For DS system, we have $N_s = N_c = 6$. Both figures show the accuracy of our approximation when the codes are chosen at random for the different N_u values. The error probability with the Gaussian approximation ($\alpha = 2$ in Eq. (11)) is also plotted in both figures for $N_u = 30$. The Gaussian approximation clearly underestimates the error probability for TH-UWB and DS-UWB systems as already observed in [5].

Let us now consider a PAM TH-UWB system with $N_u = 30$ active users, the symbol time $T_s = 72$ ns, the number of chips is $N_c = 24$ and the repetition factor $N_s = 4$. In Figure 3, we inspect the impact of the multiple access codes on the performance. We are interesting to three cases: case 1 corresponds to random codes, case 2 corresponds to the codes minimizing the MUI variance σ_M^2 as done in [3], and case 3 corresponds to the codes verifying Eq. (18). By comparing these different cases, we notice that the last case leads to the best performance. The selection of the codes that minimize only the variance does not guarantee a minimal error probability. These codes nevertheless improve the performance with respect to the random codes.

VI. Conclusions

An accurate error probability approximation for both PAM TH and DS IR-UWB systems has been derived assuming the MUI distribution is well modeled by GGD for any set of fixed multiple access codes. We then were able to select the multiple access codes minimizing the error probability. Numerical results show significant gains while selecting the appropriate codes in TH context.

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