Performance computation of cross-layer Hybrid ARQ schemes at IP layer in the presence of corrupted acknowledgments

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***Abstract*—Hybrid ARQ (HARQ) schemes operate at the PHY and Data Link (DL) layers and their performance have been naturally studied at the DL level. However, all the modern systems are running under the IP protocol. Therefore, in order to get realistic performance of the whole system considering the multiple layer stacks, performance analysis at the IP layer is crucial. Very little work has been done so far in this direction, except in [1] which proposes a cross-layer optimization strategy between MAC and IP layers for ARQ, and in [7], [8] which consider the HARQ case. In this paper, we study the effect of erroneous feedbacks at the DL layer on the performance at the IP layer considering both conventional HARQ schemes as well as the cross-layer strategy mentioned above. We derive in closed-form expressions the performance in terms of packet error rate (PER), efficiency, and delay with respect to the error probability of the feedback channel.**

I. Introduction

HARQ protocols combine ARQ retransmission schemes along with channel coding capabilities. They enable to provide reliable link transmission in wireless systems in varying channels. They have been used from the beginning in cellular standards (GSM GPRS/EDGE) and are still included in the most recent ones (WiMAX, 3GPP LTE). The motivation of this work is twofold. First, we consider the performance study at the IP layer. Although the design of conventional HARQ system is done at the DL layer (*a.k.a.* MAC layer), most of the systems run an IP protocol and it is thus of great importance to study the performance at the IP layer. Indeed, in the spirit of the cross-layer concept, the optimum performance at the MAC layer does not guaranty that the performance will be optimal at the IP layer. Moreover, recent cross-layer strategy between the MAC layer and the IP layer has been proposed in [1] in order to enhance the performance at the IP layer and thus for which the performance at the MAC does not make sense. Second, retransmission schemes are implemented using an acknowledgment process sent by the receiver which tells the transmitter whether the packets are correctly received (ACK) or not (NACK). A lot of work has been done investigating HARQ performance under the assumption of error-free (referred in the sequel as *perfect*) feedback. One can cite for instance [2], [3], [4], [5], [6] which analytically study HARQ performance at the MAC level, [7], [8], [9] at the network level and [10] at the application level. However, since the acknowledgments are transmitted over the air in wireless systems, they are prone to transmission errors. Only a small amount of analytical studies about the impact of non-perfect feedback on HARQ has been done and only at the MAC layer. One can cite studies of the efficiency for Type-I HARQ in the case of an infinite number retransmissions [11], [12]. In the case of a finite retransmissions, analytical expressions of the efficiency were derived in [13] for a Type-I HARQ and in [14] for the Type-II HARQ with Chase Combining. All the reference previously-mentioned tackle the Stop and Wait (SW) protocol. Other analysis have been done based on Markov Chain for the Selective Repeat (SR) protocol [15], [16], [17] and for the Go-Back-N (GBN) protocol [18]. Thus, our goal in this paper is to derive in closed-form the Packet Error Rate (PER), efficiency and delay at the IP layer when the feedback (fb) channel is corrupted. Moreover we will consider both conventional and cross-layer strategy. Note that our results are an extension of [7], [8] to the case of corrupted feedback.

The paper is organized as follows. We first describe the system model in Section II including the corrupted feedback channel and the cross-layer strategy. Then, we derive the performance metric in closed-form in Section III starting with the cross-layer strategy, the conventional one being deduced as a particular case. In Section IV, the results are numerically illustrated. Concluding remarks are drawn in Section V.

II. System model

A. Layer and HARQ model

We consider here the first three layers of the ISO model respectively referred in the sequel to as PHY (physical layer - layer 1), DL (data link layer - layer 2) and NET (network layer - layer 3). We assume that the NET protocol is the Internet Protocol (IP) but our work can be applied to any other protocol. We also assume that the incoming IP datagrams are fragmented at the DL layer into N fragments (FR) of equal length. Each fragment is then transmitted following an HARQ process, *i.e.* transformed into MAC packet(s) according to the considered HARQ scheme and then encoded, modulated and sent through the wireless channel. At the receiver side, the PHY packet is demodulated, decoded, and sent to the DL. The HARQ process checks if the received MAC packet allows to successfully decode the fragment (with no error) and sends back to the transmitter an acknowledgment: an ACK when the fragment is successfully received or a NACK otherwise. If the transmitter receives an ACK, it starts the HARQ process with the next fragment. Otherwise, it sends another MAC packet according to the considered HARQ scheme. Once the N fragments are correctly received, the receiver reassembles the fragments to make an IP datagram that is sent to the NET layer. The number of trials per fragment is usually limited to bound the maximum transmission delay, and we put $M_{\max}$ the maximum number of transmission (also referred in the sequel to as *credit*) allowed per fragment. Note that the assumptions considered in this work follow those of [7], [8]: *i*) we consider a block fading channels for which realizations of the channel are independent between 2 MAC packet transmissions, *ii*) we assume that the HARQ has equal MAC packet length which fits a large amount of HARQ schemes.

B. Cross-layer strategy

The conventional retransmission schemes depicted in the previous section are usually applied at the DL layer where the HARQ scheme manages the fragments one after the other independently. For truncated schemes, once the $M_{\max}$-th transmission fails, the fragment is dropped and the retransmission process is started again with the next fragment. Recently, based on the fact that if one fragment is missing at the receiver side, the corresponding IP packet at the NET layer is dropped, the authors in [1] proposed to enhance the ARQ scheme by granting a global transmission credit, noted C, to the set of fragments belonging to the same IP packet before being reassembled. Thus, rather than allowing each of the N fragments to be transmitted $M_{\max}$ times, the new scheme allocates C transmissions to the set of the N fragments. Results in [1] show that this cross-layer strategy outperforms the conventional one in terms of PER. In the later we will refer the conventional strategy to as Fragment-Based Strategy (FBS) and the cross-layer one to as XL-Based Strategy (XBS). In [1] the strategy was applied to the ARQ scheme and in this paper, we apply it to the HARQ in general.

C. Feedback channel model

We assume that the acknowledgments are CRC encoded and that the CRC is strong enough to neglect the probability of miss detection. Thus, each time the acknowledgment is corrupted (contains at least one error), the CRC detects it. When an error is detected, we systematically consider the acknowledgment as a NACK. With this strategy, a corrupted NACK will be considered as a NACK (nothing change) and a corrupted ACK as a NACK as well (the acknowledgment is erroneous). Indeed, confusing an ACK by a NACK is much less damageable than the contrary. We define then the probability that such an event occurs $p_{\mathrm{fb}} := \Pr\{\text{an acknowledgment is corrupted}\}$ which corresponds to the event $\mathrm{ACK} \rightarrow \mathrm{NACK}$.

D. Metrics definitions

The metrics considered in this paper are: mean packet error rate, efficiency, and mean packet delay. For the FBS, all the metrics can be considered at the FR level and at the IP level, whereas for the XBS, only the IP level makes sense. For the notations, we put the subscript 'FR' for the FR level and 'IP' for the IP level. We also put the superscript 'F' for the conventional strategy FBS and 'X' for the cross-layer strategy XBS. Thus, to one metric "Z", we will consider three different values: $\mathrm{Z}_{\mathrm{FR}}^{\mathrm{F}}$ (or eventually denoted by Z_{FR} since $\mathrm{Z}_{\mathrm{FR}}^{\mathrm{X}}$ does not make sense), $\mathrm{Z}_{\mathrm{IP}}^{\mathrm{F}}$, and $\mathrm{Z}_{\mathrm{IP}}^{\mathrm{X}}$.

The **packet error rate**, noted P, is defined as the probability that a packet fails to be transmitted. The **efficiency**, noted η, is defined as the average number of correctly received bits per transmitted bit. The packet **delay**, noted n, is defined as the average number of MAC packets needed to transmit one fragment (or IP datagram) correctly from the transmitter to the receiver. Notice the tilde (˜) is added to the metrics evaluated under the corrupted feedback assumption whereas the metrics without tilde are associated with the usual perfect feedback assumption.

III. Performance derivation at IP layer

A. The cross-layer strategy case

The packet error probability depends on the quality of the feedback channel. Indeed, the number of possible transmissions of the ith fragment is driven by the number of transmission done the $(i-1)$ previous fragments. If the previous fragments have lost a lot of transmissions due to corrupted ACK, the fragment i will strongly suffer since less transmissions are now allowed for it. We denote by $\tilde{p}_n(i)$ the probability of decoding n fragments in i transmissions and receiving n ACKs (at the transmitter side).

Let us focus on the packet detection probability. A packet will be well detected iff the N fragments are correctly received in i transmissions with $i \in \{N, \cdots, C\}$. The N fragments need i transmissions to be received if, for each $k \in \{N-1, \cdots, i-1\}$,

i) the first $N-1$ fragments are correctly received (and thus "sent" which mean that the corresponding ACKs have effectively been received at the transmitter side) and a ACK is eventually received for the $(N-1)$th fragment in k transmissions.

ii) Then the last Nth fragment is sent and correctly received in $(i-k)$ transmissions (regardless of the ACK/NACK received at the transmitter side).

The event i) has a probability $\tilde{p}_{N-1}(k)$ while the event ii) has a probability $p_1(i-k)$. Therefore, we obtain

$$\tilde{P}_{\mathrm{IP}}^{\mathrm{X}} = 1 - \sum_{i=N}^{C} \sum_{k=N-1}^{i-1} \tilde{p}_{N-1}(k) p_1(i-k). \tag{1}$$

Similarly to [7], we find

$$\tilde{n}_{\rm IP}^{\rm X} = \frac{1}{1-\tilde{P}_{\rm IP}^{\rm X}} \sum_{i=N}^{C} i \frac{\tilde{p}_N(i)}{(1-p_{\rm fb})^{\mathbf{1}\{i=C\}}}. \quad (2)$$

where $\mathbf{1}\{A\}$ is the indicator of the event A. The term $(1-p_{\rm fb})^{\mathbf{1}\{i=C\}}$ has been inserted in order to take into account that the ACK (for the last fragment), when the transmission credit C has been reached (*i.e.*, when no more transmission is possible), needs not be necessary received.

We recall that $\tilde{n}_{\rm IP}^{\rm X}$ is the average number of MAC packet needed to transmit one IP packet (*i.e.*, N successive fragments) correctly from the transmitter to the receiver under imperfect feedback channel. The term

$$\tilde{\eta}_{\rm IP}^{\rm X} = \frac{\rho N(1-\tilde{P}_{\rm IP}^{\rm X})}{C\tilde{P}_{\rm IP}^{\rm X} + (1-\tilde{P}_{\rm IP}^{\rm X})\tilde{n}_{\rm IP}^{\rm X}}, \quad (3)$$

where ρ is the ratio between the fragment packet length and the PHY packet length.

For XBS case, each metric is directly determined by the knowledge of $p_1(i)$ and of $\tilde{p}_n(i)$. We remind that the closed-form expression for $p_1(i)$ is available in [7], [8]. The closed-form expression for $\tilde{p}_n(i)$ is given in the next proposition and corresponds to one of the main contributions of this paper.

Proposition 1: We have $\forall n \geq 1$

$$\tilde{p}_n(i) = (1-p_{\rm fb})^n \sum_{\underline{q}\in Q_{i,n}} \prod_{j=1}^{n} \sum_{k_j=1}^{q_j} p_1(k_j) {p_{\rm fb}}^{q_j-k_j}, \quad (4)$$

with

$$Q_{i,n} = \left\{ \underline{q} \in \mathbb{N}_*^n / \sum_{j=1}^{n} q_j = i \right\}.$$

Proof: For $N \leq i \leq C$, we obtain by enumerating

$$\tilde{p}_N(i) = \sum_{\underline{q}\in Q_{i,N}} \prod_{n=1}^{N} \Pr\{\text{FR } \#n \text{ received in } q_n, \text{ and ACK received}\}.$$

We decompose the ACK of the nth fragment in q_n transmissions whose k_n transmissions until the correct decoding of the fragment (at the receiver side), and then $(q_n - k_n)$ transmissions for correctly receiving the ACK at the transmitter side. This leads to

$$\Pr\{\text{FR } \#n \text{ received in } q_n \text{ and ACK received}\} = \sum_{k_n=1}^{q_n} p_1(k_n) {p_{\rm fb}}^{q_n-k_n}(1-p_{\rm fb}).$$

■

As the set $Q_{i,n}$ is very huge for practical values of i and n, the term $\tilde{p}_n(i)$ is difficult to evaluate numerically in practice. To overcome this drawback, one can remark that this term can be calculated recursively as proposed in the next proposition.

Proposition 2: We have

$$\tilde{p}_1(i) = (1-p_{\rm fb}) \sum_{m=1}^{i} p_1(m) {p_{\rm fb}}^{i-m}, \quad (5)$$

$$\tilde{p}_n(i) = \sum_{j=1}^{i-n+1} \tilde{p}_{n-1}(i-j)\tilde{p}_1(j), \quad \forall n \geq 2. \quad (6)$$

Proof: To prove this proposition, $Q_{i,n}$ has to be partitioned as follows: n fragments are received in i transmissions iff the $(n-1)$ previous fragments are received in $(i-j)$ trials, for $j \in \{1,\dots,i-n+1\}$. The initialisation of the procedure is given by the probability (5) of receiving one fragment when C transmissions are allowed. ■

B. The standard strategy case

In the case of standard strategy FBS, the packet error probability is not modified when the feedback channel is imperfect. The fact that the ACK is modified into a NACK during the reverse link does not changed the good decoding at the receiver side. Consequently, $\tilde{P}_{\rm IP}^{\rm F} = P_{\rm IP}^{\rm F}$ as already-mentioned in [14] when only MAC level is considered. We remind that closed-form expressions for $P_{\rm IP}^{\rm F}$ are available in [7].

Nevertheless the fragments sent after the reception of a wrong NACK lead to a loss in efficiency and delay at MAC and IP level since, for instance, the transmitter will send useless redundant fragments whereas it should have sent new data fragments if the ACK was correctly received. One can notice that the average number of sent MAC packets when the fragment is not correctly received is identical to the case of perfect channel feedback. As a consequence, by following the same reasoning as that in [7], [8], one can easily deduce that, under the assumption of i.i.d. fragments,

$$\tilde{n}_{\rm IP}^{\rm F} = N\tilde{n}_{\rm FR} \quad (7)$$

and

$$\tilde{\eta}_{\rm IP}^{\rm F} = \frac{\rho(1-P_{\rm FR})^N}{M_{\max}P_{\rm FR} + (1-P_{\rm FR})\tilde{n}_{\rm FR}}. \quad (8)$$

where $\tilde{n}_{\rm FR}$ is the average number of MAC packet needed to transmit one fragments correctly from the transmitter to the receiver (so, we work at the MAC level) when the feedback is imperfect and where $P_{\rm FR}$ is the packet error probability of each fragment (at the MAC level). Thus, only this delay $\tilde{n}_{\rm FR}$ has to be evaluated in closed-form. It can be obtained by putting $N=1$ in Eq. (2) and by replacing $\tilde{P}_{\rm IP}^{\rm X}$ with $P_{\rm FR}$. Therefore, we have

$$\tilde{n}_{\rm FR} = \frac{1}{1-P_{\rm FR}} \sum_{i=1}^{M_{\max}} i \frac{\tilde{p}_1(i)}{(1-p_{\rm fb})^{\mathbf{1}\{i=M_{\max}\}}} \quad (9)$$

with $\tilde{p}_1(i)$ given by Eq. (5) where C has to be replaced with $M_{\max}$.

C. Comparison with the literature

We would like now compare your results with the literature. First of all, by setting $p_{\text{fb}} = 0$ in every previous equations (*i.e.*, by considering perfect feedback), we find out all the closed-form expressions given in [8], [9].

We remind that our derivations are the first one at the IP level when the feedback is imperfect. So to do the comparison with the state-of-the-art involving imperfect feedback, we have to focus on the MAC level, and so to put $N = 1$ in our proposed expressions.

1) The efficiency is identical to that provided in [14] (at the MAC level). The delay given in Eq. (9) is different from that given in [14, Eq. (6)]. In [14], the delay is viewed as the inverse of the efficiency which is true only when the maximum transmission credit is infinite or when the delay is defined as the average number of packets to be sent between two consecutive successful decodings; Here, the delay is the number of packets to be sent to receive correctly one fragment. The obtained delay is also different from that of [13] up to the term $i = M_{\max}$;
2) In the Type-I HARQ case, we have $p_1(i) = (1 - \pi_0){\pi_0}^{i-1}$ with $\pi_0 = \Pr\{\text{MAC packet KO}\}$. Then, the expressions can be dramatically simplified as follows

$$\begin{aligned}\tilde{n}_{\text{FR}} &= \frac{1-\pi_0}{(1-P_{\text{FR}})(p_{\text{fb}}-\pi_0)} \\ &\times \left(M_{\max}\left({p_{\text{fb}}}^{M_{\max}} - {\pi_0}^{M_{\max}}\right)\right. \\ &+ (1-p_{\text{fb}}) \\ &\times \left.\left(p_{\text{fb}} f_{M_{\max}}(p_{\text{fb}}) - \pi_0 f_{M_{\max}}(\pi_0)\right)\right), \quad (10)\end{aligned}$$

with $f_n(x) := \sum_{k=1}^{n-1} kx^{k-1}$. When infinite transmission credit is considered, we have $\lim_{M_{\max}\to\infty} \tilde{n}_{\text{FR}} = 1/(1-\pi_0) + p_{\text{fb}}/(1-p_{\text{fb}})$ which is in perfect agreement with that given in [11], [12].

As a conclusion, our new closed-form expressions for the delay and the efficiency at the IP level only depend on the "adapted" delay at the MAC level to the imperfect channel case. Our expressions (at the MAC and IP levels) are general since they hold for any HARQ and transmission credit.

IV. Numerical illustrations

We will illustrate our results with two kinds of HARQ:

i) a ARQ scheme (corresponding to a Type-I HARQ without coding),
ii) and a Type-II HARQ scheme (actually, a Chase combining noted CC-HARQ).

Both strategies FBS and XBS have been considered. Simulations are done under AWGN channel. Each MAC packet contains 128 information bits. A convolutional code of rate $1/2$ with generator polynomial (35,23) is used when HARQ is considered. The PHY layer is a QPSK modulation.

In Fig. 1, we plot the theoretical and empirical efficiency versus SNR when the feedback channel error probability is fixed to $p_{\text{fb}} = 10^{-1}$. Our theoretical expressions are in perfect agreement with the simulations which confirms the accuracy of our derivations.

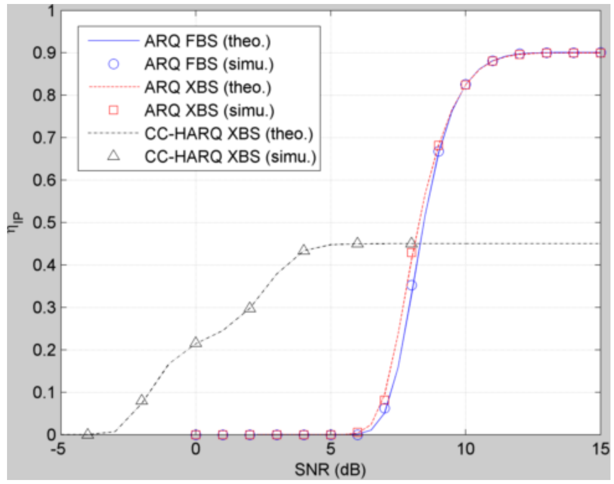

Fig. 1. Theoretical and empirical efficiency (IP) versus SNR ($N = 3$, $M_{\max} = 3$, $C = 9$).

In Fig. 2, we display the delay for ARQ and CC-HARQ under perfect and imperfect feedback channels. When the feedback channel is imperfect, the ACK/NACKs are inserted into packets of 32 bits (1bit for ACK/NACK, 15 bits may contain other information such as the packet number, 16 bits for the CRC in order to detect error on the previous 16 "information" feedback bits). Then these 32bits may be encoded through the convolutional code used in the direct link. Finally the reverse link is assumed to be AWGN with the same SNR as the direct link. We observe that the delay is dramatically influenced by the feedback channel quality, and the feedback channel has to be well protected to ensure quite the same performance as in perfect feedback scheme.

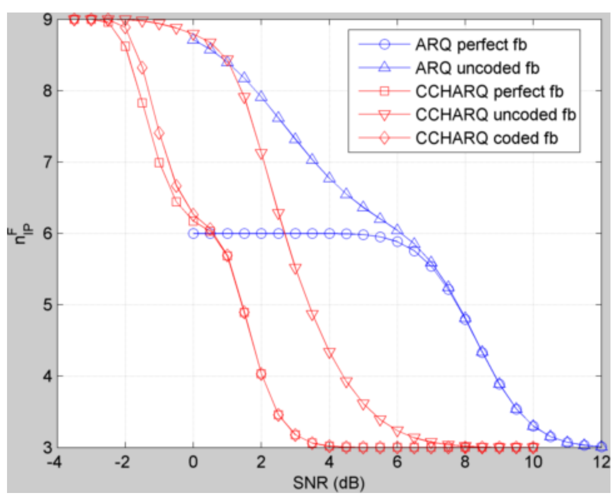

Fig. 2. Delay (IP) for ARQ and HARQ under various feedback schemes (FBS, $N = 3$, $M_{\max} = 3$).

In Fig. 3, we consider the efficiency versus SNR for CC-HARQ (in FBS and XBS contexts) when the feedback is perfect or imperfect (with or without coding), and modeled as in Fig. 2. Once again, we observe that the feedback channel has to be well protected (*i.e.* with a FEC) to keep the performance close to the perfect case. We also remark that the XBS is more sensitive to imperfect feedback than the FBS.

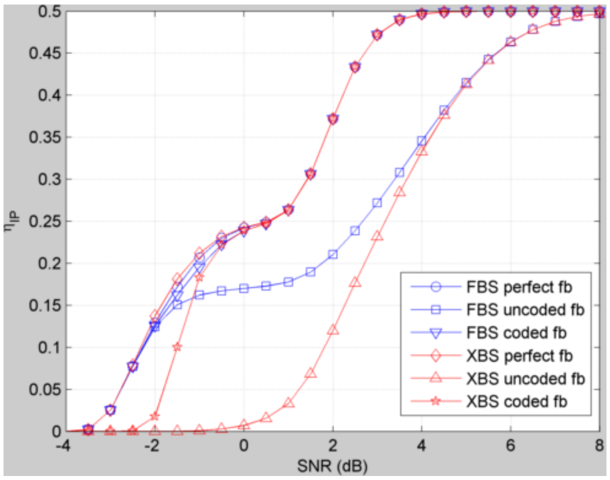

Fig. 3. Efficiency (IP) for HARQ under various feedback schemes ($N = 3$, $M_{\max} = 3$, $C = 9$).

In Fig. 4, we plot the PER versus SNR for CC-HARQ (in FBS and XBS contexts) when the feedback is perfect, is assumed to offer fixed p_{fb} (imperfect), is uncoded (imperfect case) or is coded (imperfect case). We show, as expected, that the PER for FBS is not sensitive to imperfect feedback channel. In contrast, the PER for XBS may be far away from the ideal case. As the main advantage of XBS is to improve the PER [1], the feedback channel has to be well designed for XBS in order to still have a practical interest.

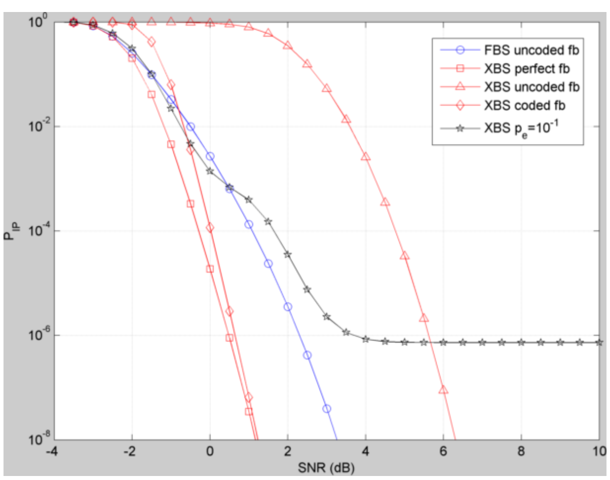

Fig. 4. PER (IP) for HARQ under various feedback schemes ($N = 3$, $M_{\max} = 3$, $C = 9$).

V. Conclusion

We have theoretically and numerically analyzed the impact of a corrupted feedback channel of any HARQ scheme at the IP level. The standard fragmented approach (between MAC and IP levels) as well as a cross-layer approach have been investigated. We have remarked that the feedback channel has to be well-protected to ensure similar performance to the perfect case, especially for the cross-layer approach.

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