A Practical Scheme to Achieve Optimal Diversity-Multiplexing Tradeoff for High Diversity Gains for Half-Duplex Relay Channels

Nassar Ksairi
SUPELEC, Plateau de Moulon
91192 Gif-sur-Yvette Cedex
nassar.ksairi@supelec.fr

Pascal Bianchi, Philippe Ciblat
Telecom Paris Tech, 46 rue Barrault,
75634 Paris Cedex 13
Email: {bianchi, ciblat}@telecom-paristech.fr

Walid Hachem
CNRS / Telecom Paris Tech (UMR 5141),
46 rue Barrault, 75634 Paris Cedex 13
Email: hachem@telecom-paristech.fr

***Abstract*—In this paper, we study the Diversity-Multiplexing Tradeoff (DMT) of the Decode or Quantize and Forward (DoQF) protocol recently proposed in [1] for the half duplex single relay channel. Our results show that this static relaying protocol achieves the 2 by 1 MISO bound for multiplexing gains $r < 0.25$. The DMT of the classical non orthogonal Decode and Forward (DF) protocol is also derived. We show that the DoQF protocol outperforms in terms of DMT the DF protocol on a range of low multiplexing gains, and performs as well as the latter on the rest of the range of multiplexing gains.**

I. Introduction

It is well known now that communication in wireless networks can considerably benefit from the idle nodes that are likely to be present in the proximity of the transmitter. This can be achieved by letting these nodes relay the transmitted signal towards the destination. This cooperation technique, which creates a virtual Multiple-Input Multiple-Output (MIMO) system can provide two type of gains: diversity gain and multiplexing gain. And as in MIMO systems, there is a fundamental tradeoff between these two gains. This tradeoff can be captured by the *Diversity-Multiplexing Tradeoff* (DMT), the performance measure introduced by Zheng and Tse [2] for MIMO Rayleigh channels. We now recall the definition of the DMT as it was provided in [2]. Denote by ρ the signal to Noise Ratio (SNR) and let R the transmission rate be a function of ρ. Denote by P_o the outage probability associated with the scheme. A relaying scheme is said to achieve *multiplexing gain* r and *diversity gain* $d(r)$ if the transmission rate and the outage probability satisfy

$$\lim_{\rho\to\infty} \frac{R(\rho)}{\log\rho} = r \qquad \lim_{\rho\to\infty} \frac{\log P_o(\rho)}{\log\rho} = -d(r)\,. \tag{1}$$

From now on, $d(r)$ as defined above will be referred to as the DMT of the relaying scheme. It is well known that the DMT of any relaying scheme with N relay nodes is upper bounded by the DMT of a $(N+1)\times 1$ MISO system which is given by $d_{\text{MISO}}(r) = (N+1)(1-r)^+$. The recent work [3] suggests the existence of relaying schemes which permit to achieve this upperbound. Authors of [3] claim in particular that the "Quantize and Map (QM)" scheme can be used to achieve the MISO upperbound on the entire range of multiplexing gains. However, the design of DMT-optimal schemes which involve simple coding-decoding strategies at the relay and the destination, and which leads to practical transmit-receive architectures, is still a challenging issue.

The DMT has been used in the literature to evaluate the performance of different relaying schemes. In the class of amplify-and-forward relaying schemes, some of the known protocols are the Non orthogonal Amplify and Forward (NAF) scheme proposed by Nabar *et al.* in [4], and the Slotted Amplify and Forward (SAF) proposed by Yang *et al.* in [5]. As for the family of decode and forward schemes, the relay listens to the source during some time (the first slot of transmission) and tries to decode the source message. Only if it succeeds, the source message is retransmitted during the remaining time (the second slot). In the context of this family of relaying schemes, Azarian *et al.* proposed in [6] a dynamic protocol called Dynamic Decode and Forward (DDF). which achieves the MISO upperbound on the range of low multiplexing gains $r < 0.5$. The price to be payed for this performance gain is that the time the relay waits before decoding is random (dynamic). In general, static relaying protocols which fix the relay waiting time are simpler to implement. One of these static schemes is the *non orthogonal* [1] Decode and Forward (DF) protocol [7]. To the best of our knowledge, the DMT of this protocol has never been computed, but is known [6] to be dominated by the DMT of the DDF. One of the main weaknesses of decode-and-forward schemes is due to the fact that the relay remains inactive during the second slot of transmission if it fails to decode the source message during the first slot. In order to improve the performance of these schemes, [1] proposes a novel protocol: the Decode or Quantize and Forward (DoQF). In this proposed scheme, the relay does not remain silent in the case of failure in decoding the source message, but instead quantizes the received signal vector using a well chosen distortion value, and forwards a coded version of the quantized vector towards the destination. In this paper, we use the DMT measure to evaluate the performance of DoQF and

[1] By "non orthogonal" we mean that the relay and the destination transmit simultaneously during the second slot. This scheme will be referred to in this paper simply as the DF scheme

we show that it achieves the 2×1 MISO bound for high diversity orders, (low multiplexing gains), more precisely for multiplexing gains satisfying $r < 0.25$.

Paper Contributions

1) We Derive the DMT of the DoQF scheme and we prove its optimality for $r \leq 0.25$.

2) We Derive the DMT of the DF scheme. To the best of our knowledge, the DMT of the *non orthogonal* DF has never been derived in the half-duplex case. Only the DMT of the *orthogonal* DF exists in the literature [8].

The main assets of DoQF are: *i)* DoQF is DMT-optimal for $r < 0.25$ and it outperforms the DMT of DF, *ii)* it has the best *outage gain* performance in a large class of relaying protocols, and *iii)* it involves a practical receiver structure that can be implemented in practice.

II. Notations and Assumptions

We consider a half-duplex single relay channel where the source (node 0) needs to send information at a rate of R nats per channel use towards the destination (node 2) with the aid of the relay (node 1). To this end, the source has at its disposal a frame of length T and a dictionary of $\lfloor e^{RT} \rfloor$ Gaussian independent vectors with independent $\mathcal{CN}(0,1)$ elements each. The radio channels between the different nodes of the network are assumed to be independent Rayleigh channels and we denote by $H_{ij} \sim \mathcal{CN}(0,1)$ the complex random variable representing the radio channel between node i and node j. The power gain of this channel will be denoted by $G_{ij} = |H_{ij}|^2$. We partition the word X_0 transmitted by the source as $X_0 = [X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}]^{\mathrm{T}}$ where the length of X_{00} and X_{01} is $t_0 T$ and $t_1 T$ respectively with $t_1 = 1 - t_0$. Here $t_0 < 1$ is a fixed parameter. The relay listens to the source message for a duration of $t_0 T$ channel uses. At the end of this period of time that we refer to as slot 0, The signal of size $t_0 T$ received by the relay writes

$$Y_{10} = \sqrt{\rho} H_{01} X_{00} + V_{10} \ , \tag{2}$$

where ρ represents the Signal to Noise Ratio (SNR) and V_{10} is the unit variance AWGN at the relay. The main difference , as we will see, between the DoQF protocol and the DF protocol is in the way the relay behaves when it is not able to decode the message X_{00} embedded in the received signal Y_{10}.

From now on, R the transmission rate is assumed in accordance with (1) to be a function of the SNR and to satisfy $R = r \log \rho$. We also write as usual $f(\rho) \doteq \rho^d$ if $\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log(\rho)} = d$. Finally, $(x)^+ = \max(0, x)$.

III. DMT of the Half-Duplex Single Relay DF

In order to have a reference of comparison for the performance of DoQF, we derive in this section the DMT of the non orthogonal DF protocol. We begin with a brief description of this protocol. Next, we derive the outage probability associated with the protocol in order to compute finally its DMT using (1). As stated in Section II, the source transmits a code word $X_0 = [X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}]^{\mathrm{T}}$ of length T. The relay listens to the source message during slot 0 for a duration of $t_0 T$ channel uses. At the end of slot 0, the relay attempts to decode X_{00}. If it succeeds, the relay transmits the corresponding codeword X_{11} during the remainder of the frame (slot 1) to the destination. By referring to (2), we can check that the relay is able to decode the source message if the event

$$\mathcal{E} = \{w : t_0 \log(1 + \rho G_{01}(w)) > R\} \tag{3}$$

is realized. We denote by X_{11} the code word transmitted by the relay in this case during slot 1. The destination receives the signal

$$[Y_{20}^{\mathrm{T}}, Y_{21}^{\mathrm{T}}]^{\mathrm{T}} = \mathbf{H}_{\mathcal{E}} [X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}, X_{11}^{\mathrm{T}}]^{\mathrm{T}} + [V_{20}^{\mathrm{T}}, V_{21}^{\mathrm{T}}]^{\mathrm{T}} \ , \tag{4}$$

where $\mathbf{H}_{\mathcal{E}} = \begin{bmatrix} \sqrt{\rho} H_{02} \mathbf{I}_{t_0 T} & 0 & 0 \\ 0 & \sqrt{\rho} H_{02} \mathbf{I}_{t_1 T} & \sqrt{\rho} H_{12} \mathbf{I}_{t_1 T} \end{bmatrix}$ and V_{20} (resp. V_{21}) is the unit variance AWGN at the destination during slot 0 (resp. slot 1). We denote by $P_{o,1}$ the outage probability of the destination conditioned to $\mathcal{E}$.

$$P_{o,1} = \Pr[t_0 \log(1 + \rho G_{02}) + t_1 \log(1 + \rho G_{02} + \rho G_{12}) \leq R] \ . \tag{5}$$

Let $d_1(r) = -\lim_{\rho\to\infty} \frac{\log P_{o,1}(\rho)}{\log \rho}$ designate the DMT of $P_{o,1}$. In the case the relay does not succeed in decoding the source message *i.e,* when the complementary event $\overline{\mathcal{E}}$ is realized, the destination receives the following signal

$$[Y_{20}^{\mathrm{T}}, Y_{21}^{\mathrm{T}}]^{\mathrm{T}} = \mathbf{H}_{\overline{\mathcal{E}}} [X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}]^{\mathrm{T}} + [V_{20}^{\mathrm{T}}, V_{21}^{\mathrm{T}}]^{\mathrm{T}} \ , \tag{6}$$

where $\mathbf{H}_{\overline{\mathcal{E}}} = \begin{bmatrix} \sqrt{\rho} H_{02} \mathbf{I}_{t_0 T} & 0 \\ 0 & \sqrt{\rho} H_{02} \mathbf{I}_{t_1 T} \end{bmatrix}$. We denote by $P_{o,2}$ the outage probability of the destination conditioned to the event $\overline{\mathcal{E}}$, and by $d_2(r)$ the DMT associated with $P_{o,2}$.

$$P_{o,2} = \Pr[\log(1 + \rho G_{02}) \leq R] \ . \tag{7}$$

The outage probability P_o associated with the DF protocol is

$$P_o = P_{o,1} \Pr[\mathcal{E}] + P_{o,2} \Pr[\overline{\mathcal{E}}] = P_{o,1}(1 - P_{o,r}) + P_{o,2} P_{o,r} \ ,$$

where $P_{o,r} = \Pr[\overline{\mathcal{E}}]$ is the probability that the relay does not succeed in decoding the source message. Define $d_r(r)$ as the DMT of $P_{o,r}$. It is clear that the outage probability P_o, and hence its DMT, is function of the parameter t_0. Therefore, we should first compute the DMT of DF for fixed values of t_o, say $d(t_0, r) = -\lim_{\rho\to\infty} \frac{\log P_o(\rho)}{\log \rho} = \min\{d_1(r),\ d_2(r) + d_r(r)\}$. The final DMT of the protocol, which we denote by $d_{\mathrm{DF}}^*(r)$, can be obtained by maximizing $d(t_0, r)$ w.r.t t_0

$$d_{\mathrm{DF}}^*(r) = \sup_{t_0} d(t_0, r) \ . \tag{8}$$

The following theorem provides the closed-form expression of $d_{\mathrm{DF}}^*(r)$ and of $t_0^*(r)$, the argument of the supremum in (8).

Theorem 1: The diversity-multiplexing tradeoff achieved by the half-duplex single relay DF protocol is

$$d_{\mathrm{DF}}^*(r) = \begin{cases} 2 - \frac{2}{3-\sqrt{5}} r & \text{for } 0 \leq r \leq \frac{\sqrt{5}-1}{\sqrt{5}+1} \\ (2-r)(1-r) & \text{for } \frac{\sqrt{5}-1}{\sqrt{5}+1} < r \leq 1 \ . \end{cases} \tag{9}$$

Moreover, the optimal value of t_0, as function of r, that permits to achieve this DMT is given by

$$t_0^*(r) = \begin{cases} \frac{2}{\sqrt{5}+1} & \text{for } 0 \leq r \leq \frac{\sqrt{5}-1}{\sqrt{5}+1} \\ \frac{1}{2-r} & \text{for } \frac{\sqrt{5}-1}{\sqrt{5}+1} < r \leq 1 \ . \end{cases} \tag{10}$$

The DMT of DF given by (9) is plotted in Figure 1, where we can see that the DF protocol does not achieve the MISO upperbound for any value of the multiplexing gain r. We will see in the next section that a better performance can be achieved by the DoQF scheme. Note also that the value of t_0^* of the *non orthogonal* DF plotted in Figure 2 is a function of the multiplexing gain r and is different from 0.5 for all the values of r. This novel result is to be compared with the classical choice of $t_0 = 0.5$ which was proved in the literature to achieve the DMT of the *orthogonal* DF protocol. The proof of Theorem 1 is omitted from this paper.

IV. DMT of the Half-Duplex Single Relay DoQF

We begin with description of the protocol as it was introduced in [1]. Next, the outage probability associated with the protocol is derived in order to compute its DMT. As in the DF protocol, the relay in DoQF listens during slot 0 to the source signal (2) and is able to decode the source message if the event $\mathcal{E}$ defined by (3) is realized. The outage probability of the destination conditioned to the event $\mathcal{E}$ is, as in DF, equal to $P_{o,1}$ given by (5). The difference between DoQF and DF arises when the relay fails to decode the source message. In this case, the relay quantizes the received signal during slot 0 and transmits a coded version of the quantized vector towards the destination during slot 1 using the following steps.

a) Quantization: Denote by $\tilde{Y}_{10}$ the quantized version of the received vector Y_{10}. Vector $\tilde{Y}_{10}$ is constructed as follows. Clearly, all t_0T components of vector Y_{10} are independent and $\mathcal{CN}(0, \rho G_{01}+1)$ distributed. Denote by $\Delta^2 = \Delta^2(\rho)$ the desired squared-error distortion:

$$\mathbb{E}|\tilde{Y}_{10}(i) - Y_{10}(i)|^2 \leq \Delta^2(\rho) \,.$$

The Rate Distortion Theorem for Gaussian sources [9] tells us that, provided that the following two conditions are satisfied

$$Q > \log\left(\frac{\rho G_{01}+1}{\Delta^2}\right) \text{ [a]}, \;\; \rho G_{01}+1 > \Delta^2 \text{ [b]} \qquad (11)$$

for some $Q > 0$, then there exists a $(\lfloor e^{Qt_0T} \rfloor, t_0T)$-rate distortion code which is achievable for the distortion Δ^2. In practice, such a code can be constructed by properly selecting the quantized vector $\tilde{Y}_{10}$ among a quantizer-codebook formed by $\lfloor e^{Qt_0T} \rfloor$ independent random vectors with distribution $\mathcal{CN}(0, (\rho G_{01}+1-\Delta^2)\mathbf{I}_{t_0T})$. Note that condition (11)-[b] is necessary for the construction of the above code because it ensures that $\rho G_{01}+1-\Delta^2$, the variance of each component of the code words is positive. Vector $\tilde{Y}_{10}$ is selected among this codebook in such a way that sequences Y_{10} and $\tilde{Y}_{10}$ are jointly typical w.r.t. the joint distribution $p_{(Y,\tilde{Y})}$ given by $Y = \tilde{Y} + \Delta Z$ where $\tilde{Y}$ and Z are independent r.v with respective distributions $\mathcal{CN}(0, \rho G_{01}+1-\Delta^2)$ and $\mathcal{CN}(0,1)$. Parameter Q can be interpreted as the number of nats used to quantize one component of the received vector Y_{10}. This parameter must be chosen such that condition (11)-[a] is satisfied. This is why we choose

$$Q = Q(\rho) = \log\left(\frac{\rho G_{01}+1}{\Delta^2(\rho)}\right) \,, \qquad (12)$$

which guaranties that condition (11)-[a] is satisfied with the smallest number of nats per channel use that must be forwarded to the destination. In order to complete the definition of the quantizer, we still need to define the way Δ^2 depends on the SNR ρ. In this paper, we assume that

$$\Delta^2(\rho) = \rho^\delta \,,$$

where parameter δ will be fixed later. If δ is negative, then *fine quantization* will be achieved at high SNR *i.e,* $\lim_{\rho\to\infty}\Delta^2(\rho) = 0$. However, we do not force δ *a priori* to be negative but instead we optimize it in order to maximize the DMT of the protocol.

Remark: Condition (11)-[b] states that the quantization step is possible in case the event

$$\mathcal{S} = \left\{ w : \rho G_{01}+1 > \Delta^2 \right\}. \qquad (13)$$

is realized. In case the complementary event $\overline{\mathcal{S}}$ is realized, the relay does not quantize the source message and remains silent during slot 1. The latter case happens with negligible probability provided that δ is chosen properly.

b) Forwarding the Relay Message: During the second slot of length t_1T, the relay must forward the index of the quantized vector among the possible $\lfloor e^{Qt_0T} \rfloor$ ones. To that end, the relay uses a Gaussian codebook with rate Qt_0/t_1. If we denote by X_{11} the corresponding codeword, the signal transmitted by the relay can be written as $\sqrt{\phi(\rho)}X_{11}$, where $\phi(\rho)$ is the power of the relay. Function $\phi(\rho)$ should be selected in such a way that the power constraint is satisfied.

Remark: In order to make explicit this power constraint, let us derive the mean energy spent by both the source and the relay to transmit a block of RT nats. The source transmits the signal $[\sqrt{\rho}X_{00}, \sqrt{\rho}X_{01}]$ spending the energy $E_0 = \rho T$. If the event $\mathcal{E}$ is realized *i.e,* if the relay decodes the source message, then the relay transmits the signal $\sqrt{\rho}X_{11}$ and spends $\rho t_1 T$ Joules. In the other case, the relay transmits $\sqrt{\phi(\rho)}X_{11}$ spending $\phi(\rho)t_1T$ Joules. Recalling that $\Pr[\mathcal{E}] = 1 - P_{o,r}$, the mean energy spent by the relay is then $E_1 = \rho t_1 T(1 - P_{o,r}) + \phi(\rho)t_1TP_{o,r}$. The power constraint should ensure that E_0+E_1 the total energy spent by the network does not exceed the energy the source would have spent in the non cooperative case *i.e,* $E_0 + E_1 \leq \text{constant} \times T\rho$, which leads to

$$\rho T + \rho t_1(1-P_{o,r}) + \phi(\rho)t_1TP_{o,r} \leq \text{constant} \times T\rho \,. \qquad (14)$$

Now since we are evaluating the performance of the DoQF protocol from a DMT perspective, this power constraint should be examined in the asymptotic regime when ρ tends to infinity. For that sake, one can easily verify using the definition of $P_{o,r}$ and assuming $R = r\log\rho$ that $P_{o,r} \doteq \rho^{-(1-r/t_0)^+}$. After some simple manipulations on (14), the power constraint in the asymptotic regime can be written as $\phi(\rho)\rho^{-(1-r/t_0)^+} \dot{\leq} \rho$ leading to the following condition $\phi(\rho) \dot{\leq} \rho^{1+(1-r/t_0)^+}$. In the sequel, we choose $\phi(\rho) = \rho^{1+(1-r/t_0)^+}$.

c) Processing at Destination: In case the event $\mathcal{S}$ defined by (13) is realized, condition (11) - [b] will be satisfied and the relay will quantize the source message. In this case, the destination proceeds as follows. It tries first to recover X_{11} the relay message received during slot 1 and uses it to help decode the source message. The signal received by the destination during the second slot can be written as

$$Y_{21} = \sqrt{\phi(\rho)} H_{12} X_{11} + \sqrt{\rho} H_{02} X_{01} + V_{21} \, . \qquad (15)$$

Note that (15) can be seen as a MAC channel. In order to recover X_{11} (and consequently $\tilde{Y}_{10}$) from (15), the destination interprets the source contribution as noise. It succeeds in recovering $\tilde{Y}_{10}$ in the case the event

$$\mathcal{F} = \left\{ w : t_1 \log \left(1 + \frac{\phi(\rho) G_{12}(w)}{\rho G_{02}(\rho) + 1} \right) > Q(\rho) t_0 \right\} \qquad (16)$$

is realized. We distinguish between three possible cases.

Events $\mathcal{S}$ and $\mathcal{F}$ are realized: In this case, the contribution of X_{11} in (15) can be canceled, and the resulting signal can be written as $Y_{21}' = \sqrt{\rho} H_{02} X_{01} + V_{21}$. In order to decode the source message in this case, the overall received signal can be reconstructed as $Y_2 = [Y_{20}^{\mathrm{T}}, \frac{1}{\sqrt{1+\Delta^2}} \tilde{Y}_{10}^{\mathrm{T}}, (Y_{21}')^{\mathrm{T}}]^{\mathrm{T}}$:

$$Y_2 = \sqrt{\rho} \mathbf{H}_{\mathcal{F}} [X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}]^{\mathrm{T}} + \check{V}_{10}, \qquad (17)$$

where $\mathbf{H}_{\mathcal{F}} = \begin{bmatrix} H_{02} \mathbf{I}_{t_0 T} & 0 \\ \frac{1}{1+\Delta^2} H_{01} \mathbf{I}_{t_0 T} & 0 \\ 0 & H_{02} \mathbf{I}_{t_1 T} \end{bmatrix}$ and $\check{V}_{10}$ is a unit variance AWGN. Conditionally to events $\overline{\mathcal{E}}$, $\mathcal{F}$ and $\mathcal{S}$, the outage probability can be expressed [1] as

$$P_{o,2} = \Pr \Big[t_1 \log(1 + \rho G_{02}) + t_0 \log \left(1 + \rho G_{02} + \frac{\rho}{1 + \Delta^2(\rho)} G_{01} \right) < R | \overline{\mathcal{E}}, \mathcal{F}, \mathcal{S} \Big] \, . \qquad (18)$$

Denote by $d_2(r)$ the DMT associated with the probability $P_{o,2} \times \Pr[\overline{\mathcal{E}}, \mathcal{F}, \mathcal{S}]$ *i.e*, $d_2(r) = -\lim_{\rho \to \infty} \frac{\log(P_{o,2} \times \Pr[\overline{\mathcal{E}}, \mathcal{F}, \mathcal{S}])}{\log \rho}$. Note from (16) and (18) that $d_2(r)$ is a function of parameters t_0 and δ.

Events $\mathcal{S}$ and $\overline{\mathcal{F}}$ are realized: The destination will only be able to use Y_{20}, the signal received during slot 0, to recover the source message. Note that, since $Y_{20} = \sqrt{\rho} H_{02} X_{00} + V_{20}$, the outage probability conditionally to events $\overline{\mathcal{E}}$, $\overline{\mathcal{F}}$ and $\mathcal{S}$ is

$$P_{o,3} = \Pr[t_0 \log(1 + \rho G_{02}) < R | \overline{\mathcal{E}}, \overline{\mathcal{F}}, \mathcal{S}] \, . \qquad (19)$$

Denote by $d_3(r)$ the DMT associated with the probability $P_{o,3} \times \Pr[\overline{\mathcal{E}}, \overline{\mathcal{F}}, \mathcal{S}]$: $d_3(r) = -\lim_{\rho \to \infty} \frac{\log(P_{o,3} \times \Pr[\overline{\mathcal{E}}, \overline{\mathcal{F}}, \mathcal{S}])}{\log \rho}$.

Event $\overline{\mathcal{S}}$ is realized: In this case, condition (11) - [b] is not satisfied and the relay does not quantize the source message. This is the case of a non cooperative transmission and we can easily verify that the outage probability conditionally to events $\overline{\mathcal{E}}$ and $\overline{\mathcal{S}}$ can be given by $P_{o,4} = \Pr\Big[\log(1 + \rho G_{02}) < R \Big]$. Denote by $d_4(r)$ the DMT associated with the probability $P_{o,4} \times \Pr[\overline{\mathcal{E}}, \overline{\mathcal{S}}]$. Finally, recall the definition of $P_{o,1}$ given by (5) as the outage probability of the destination conditioned to the event $\mathcal{E}$, and let $d_1(r)$ denote the DMT of this probability. The outage probability of the DoQF protocol is

$$P_o = P_{o,1} \Pr[\mathcal{E}] + P_{o,2} \Pr[\overline{\mathcal{E}}, \mathcal{F}, \mathcal{S}] + P_{o,3} \Pr[\overline{\mathcal{E}}, \overline{\mathcal{F}}, \mathcal{S}] + P_{o,4} \Pr[\overline{\mathcal{E}}, \overline{\mathcal{S}}] \, . \qquad (20)$$

And the DMT associated with P_o is equal to

$$d(t_0, \delta, r) = \min\{d_1(r), d_2(r), d_3(r), d_4(r)\} \, . \qquad (21)$$

Note that this DMT depends on t_0 and δ since $d_1(r)$, $d_2(r)$ and $d_3(r)$ are functions of these two parameters. The final DMT of DoQF, denoted by $d_{\mathrm{DoQF}}^*(r)$, is defined as follows

$$d_{\mathrm{DoQF}}^*(r) = \sup_{t_0, \delta} d(t_0, \delta, r). \qquad (22)$$

Theorem 2: The DMT $d_{\mathrm{DoQF}}^*(r)$ associated with the DoQF protocol is given by Figure 1. In particular, for $r \leq 0.25$

$$d_{\mathrm{DoQF}}^*(r) = 2(1 - r)^+ \, ,$$

so that the DMT of DoQF coincides with the MISO bound for $r \leq 0.25$. For $r > 0.25$, the DMT is given by

$$d_{\mathrm{DoQF}}^*(r) = \sup_{t_0, \, \delta > 0} \min\{d_1(r), d_2(r), d_3(r), d_4(r)\}$$

where $d_1(r)$ is defined by (30) and $d_2(r)$ by (23), and

$$d_3(r) = 2\left(1 - \frac{r}{t_0}\right)^+ + \left(\left(2 + \frac{t_0}{t_1}\right) \left(1 - \frac{r}{t_0}\right)^+ - (1 - \delta) \frac{t_0}{t_1} \right)^+ ,$$

$$d_4(r) = (1 - r)^+ + \max\left\{ \left(1 - \frac{r}{t_0}\right)^+ , (1 - \delta)^+ \right\} .$$

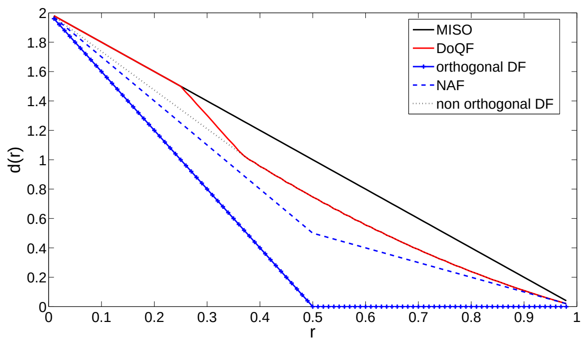

Fig. 1. DMT of the DF and DoQF protocols

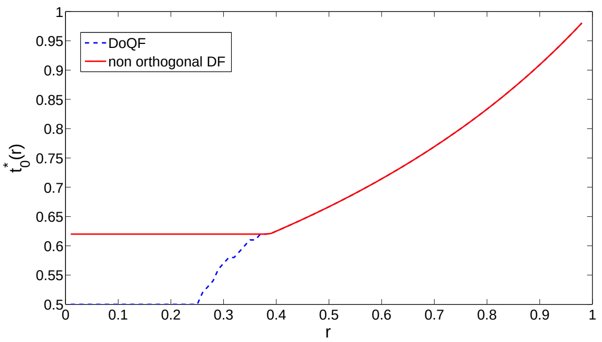

Fig. 2. $t_0^*(r)$ for DF and DoQF

Comments on Theorem 2: Theorem 2 states that the MISO upperbound can be reached by DoQF for $r < 0.25$. Denote

$$d_2(r) = \begin{cases} (1-r/t_0)^+ + \max\{(1-r)^+, (1-r)^+/t_1 - \delta t_0/t_1 - (1-r/t_0)^+ t_0/t_1\}, & \text{for } t_0 \leq 0.5 \ \& \ 1-r \geq t_0 - t_1(1-r/t_0)^+ \\ (1-r/t_0)^+ + \max\{(1-r)^+, (1-\delta)t_0/t_1 - (1+t_0/t_1)(1-r/t_0)^+\}, & \text{for } t_0 \leq 0.5 \ \& \ 1-r < t_0 - t_1(1-r/t_0)^+ \\ (1-r)^+ + \max\{(1-r/t_0)^+, 1-r-\delta\}, & \text{for } t_0 > 0.5 \ \& \ 1-r \geq t_0 - t_1(1-r/t_0)^+ \\ (1-r)^+ + \max\{(1-r/t_0)^+, 1-\delta-(1-r/t_0)^+ t_1/t_0 - (1-r)^+ t_1/t_0\}, & \text{for } t_0 > 0.5 \ \& \ 1-r < t_0 - t_1(1-r/t_0)^+ \end{cases} \quad (23)$$

by $t_0^*(r)$ and $\delta^*(r)$ the argument of the supremum in (22) *i.e,* the value of t_0 and δ that permits to achieve the DMT of the protocol. $t_0^*(r)$ is plotted in Figure 2 and it is worth mentioning here that for $r < 0.25$, the MISO bound is reached with $t_0^*(r) = 0.5$ and $\delta^*(r) = 0$. For higher multiplexing gains, the DMT can be obtained by solving (22) using proper numerical methods (for example, exhaustive search in a 2D grid of values of t_0 and δ). Figure 1 shows that the DMT of the protocol deviates from the MISO bound for $r > 0.25$ when the recovery of the quantized vector from the MAC channel of (15) becomes a burden for the destination. Nonetheless, the DMT of DoQF remains better than that of DF on a range of medium multiplexing gains. But for higher values of r, quantization at the relay can no more improve the DMT of DoQF which becomes equal to the DMT of DF. We note also from Figure 1 that the DMT of DoQF is better than the DMT of the Non orthogonal Amplify and Forward (NAF) on the entire range of multiplexing gains. A sketch of the proof of Theorem 2 is provided in the Appendix.

APPENDIX

Recalling the definition of $d(t_0, \delta, r)$ given in (21), we need first to compute $d_1(r)$, $d_2(r)$, $d_3(r)$ and $d_4(r)$ defined in Section IV. However, for lack of space, only the derivation of $d_1(r)$, the DMT of $P_{o,1}$ (5), is provided in this paper. In the following, we assume that $R = r \log \rho$ in accordance with (1), and we define the *exponential order* associated with channel H_{ij} as $\alpha_{ij} = -\frac{\log G_{ij}}{\log \rho}$. We can easily verify that α_{ij} is a *Gumbel* distributed random variable with the probability density function $f_{\alpha_{ij}}(\alpha) = \log \rho \, e^{\alpha} e^{-e^{-\alpha \log \rho}}$.

DMT associated with $P_{o,1}$: We can derive the DMT of $P_{o,1}$ using the definition of the exponential orders given earlier as follows. We insert $R = r \log \rho$, $G_{02} = \rho^{-\alpha_{02}}$ and $G_{12} = \rho^{-\alpha_{12}}$ in (5) to obtain $P_{o,1} \doteq \Pr(t_0(1-\alpha_{02})^+ + (1-t_0)(1-\min(\alpha_{02}, \alpha_{12}))^+ < r)$, or

$$P_{o,1} \doteq \int_{\mathcal{O}} f_{\alpha_{02}}(\alpha_{02}) f_{\alpha_{12}}(\alpha_{12}) d\alpha_{02} d\alpha_{12} \,, \quad (24)$$

with $\mathcal{O} = \{(\alpha_{02}, \alpha_{12}) \in \mathbb{R}^2 | t_0(1-\alpha_{02})^+ + (1-t_0)(1-\min(\alpha_{02}, \alpha_{12}))^+ < r\}$. Inserting in (24) the expression of $f_{\alpha_{ij}}(\alpha)$ given above, $P_{o,1}$ can be written now as

$$P_1 \doteq \int_{\mathcal{O}} (\log \rho)^2 \rho^{-(\alpha_{02}+\alpha_{12})} e^{-\rho^{-\alpha_{02}}} e^{-\rho^{-\alpha_{12}}} d\alpha_{02} d\alpha_{12} \,.$$

It can be shown (refer to [2]) that the term $(\log \rho)^2$ can be dropped from the latter equation without loosing its exactness. Moreover, integration in the same equation can be restricted to positive values of α_{02} and α_{12}. Define $\mathcal{O}^+ = \mathcal{O} \cap \mathbb{R}_+^2$. The probability $P_{o,1}$ can now be written as $P_{o,1} \doteq \int_{\mathcal{O}^+} \rho^{-(\alpha_{02}+\alpha_{12})} d\alpha_{02} d\alpha_{12}$, and consequently the DMT associated with $P_{o,1}$ as defined by (1) can be written as

$$d_1(r) = \inf_{(\alpha_{02}, \alpha_{12}) \in \mathcal{O}^+} (\alpha_{02} + \alpha_{12}). \quad (25)$$

This infimum can be computed by partitioning $\mathcal{O}^+$ into four subsets according to whether α_{02}, α_{12} are smaller or larger than 1. Recalling the definition of $\mathcal{O}^+$ we can verify that

$$\inf_{(\alpha_{0,2}, \alpha_{1,2}) \in \mathcal{O}^+, \alpha_{0,2} > 1, \alpha_{1,2} > 1} (\alpha_{0,2} + \alpha_{1,2}) = 2 \quad (26)$$

$$\inf_{(\alpha_{0,2}, \alpha_{1,2}) \in \mathcal{O}^+, 1 \geq \alpha_{0,2}, \alpha_{1,2} > 1} (\alpha_{0,2} + \alpha_{1,2}) = 1 + (1-r)^+ \quad (27)$$

$$\inf_{(\alpha_{0,2}, \alpha_{1,2}) \in \mathcal{O}^+, \alpha_{0,2} > 1, 1 \geq \alpha_{1,2}} (\alpha_{0,2} + \alpha_{1,2}) = 1 + (1 - r/(1-t_0))^+ \quad (28)$$

The case $\alpha_{02} \leq 1$, $\alpha_{12} \leq 1$ requires some attention. This is why we use in this case (as was done in [6]) a graphical representation of $\mathcal{O}^+$ in order to compute $\inf_{(\alpha_{0,2}, \alpha_{1,2}) \in \mathcal{O}^+} (\alpha_{0,2} + \alpha_{1,2})$. The details of this derivation are omitted from this paper but will be given in its extended version. Here we only give the result of this derivation:

$$\inf_{\substack{(\alpha_{0,2}, \alpha_{1,2}) \in \mathcal{O}^+ \\ 1 \geq \alpha_{0,2}, 1 \geq \alpha_{1,2}}} (\alpha_{0,2} + \alpha_{1,2}) = \begin{cases} 2(1-r)^+ & \text{for } t_0 \leq 0.5 \\ (1-r)^+/t_0 & \text{for } t_0 > 0.5 \end{cases} \quad (29)$$

Finally, $d_1(r)$ in (25) can be obtained as the minimum of the four infima given by equations (26) $\sim$ (29):

$$d_1(r) = \begin{cases} 2(1-r)^+ & \text{for } t_0 \leq 0.5 \\ 1 + (1 - r/(1-t_0))^+ & \text{for } t_0 > 0.5 \text{ and } r < 1 - t_0 \\ (1-r)^+/t_0 & \text{for } t_0 > 0.5 \text{ and } r \geq 1 - t_0 \end{cases} \quad (30)$$

REFERENCES

[1] P. Bianchi, P. Ciblat and W. Hachem, "Outage Performance of a Novel Relaying Protocol: Decode or Quantize and Forward," *ISITA08*, 2008.

[2] L. Zheng and D. N. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 6, June 2005.

[3] S. Pawar, S. Avestimehr and D. N. Tse, "Diversity multiplexing tradeoff of the half-duplex relay channel," *Proceedings of Allerton Conference on Communication, Control, and computing,*, Sep. 2008.

[4] R. U. Nabar, H. Bolcskei and F. W. Kneubuehler, "Fading Relay Channels: Performance Limits and Space-Time Signal Design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, Aug. 2004.

[5] S. Yang and J.-C. Belfiore, "Towards The Optimal Amplify-and-Forward Cooperative Diversity Scheme," *IEEE Trans. Inform. Theory*, vol. 53, no. 9, Sep. 2007.

[6] K. Azarian, H. El Gamal and P. Schniter, "On the achievable Diversity-Multiplexing Tradeoff in half-Duplex Cooperative Channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 12, Dec. 2005.

[7] M. Katz and S. Shamai, "Relaying protocols for two colocated users," *IEEE Trans. Inform. Theory*, vol. 52, no. 6, June, 2006.

[8] J. N. Laneman, D. N. Tse and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, Dec. 2004.

[9] T. Cover and J. Thomas,"Elements of Information Theory," *John Wiley, 2nd edition*, 2006.