Outage Probability Optimization of Certain Wireless Relaying Protocols

Walid Hachem
CNRS / ENST (UMR 5141)
46 rue Barrault
75634 Paris Cedex 13
France
walid.hachem@enst.fr

Pascal Bianchi
Supélec
Plateau de Moulon
91192 Gif-Sur-Yvette Cedex
France
pascal.bianchi@supelec.fr

Philippe Ciblat
ENST
46 rue Barrault,
75634 Paris Cedex 13
France
philippe.ciblat@enst.fr

***Abstract*— In the context of wireless relay networks operating on slow fading channels, the outage probability optimization is of central importance. It is often hard to give a closed form expression of the outage probability $P_{\rm o}$ for all possible values of the Signal to Noise Ratio (SNR). On the other hand, it is possible to analyze the behaviour of $P_{\rm o}$ in the asymptotic regime where the SNR ρ converges to infinity. In this regime, $\rho^{N+1} P_{\rm o}$ usually converges to a constant ξ where N is the number of relays. This paper presents a general method for deriving and minimizing ξ with respect to the power distribution between the source and the relays, and with respect to the durations of the slots specified by the relaying protocol. Convexity of ξ with respect to the design parameters is shown. The method applies to a general class of radio channels that includes the Rayleigh and the Rice channels as particular cases. Decode-and-Forward as well as Amplify-and-Forward protocols are considered in the half duplex mode. While the proposed approach is designed for the high SNR regime, simulations show that outage probability is reduced in a similar proportion at moderate SNR.**[1]

I. Introduction

In digital wireless communications over slow fading channels unknown at the transmitter side, spatial diversity by means of relaying is a new and efficient solution against the effect of channel fades ([1], [2], [3], [4], [5]). Indeed it is unlikely that all the links (source-destination, source-relay, relay-destination) are simultaneously subject to deep fades. Hence, the performance improves with respect to a communication without relaying technique.

In the context of communications over slow fading channels, the relevant performance measure from the information theoretic point of view is the so called outage probability, which is the probability that Shannon's mutual information lies beneath a given threshold. In a N–relay network with single antenna terminals, the outage probability $P_{\rm o}$ usually satisfies $\lim_{\rho\to\infty}\left(\rho^{N+1}P_{\rm o}\right) = \xi$ where ρ is the Signal to Noise Ratio (SNR) and ξ is a constant. This equation indicates in particular that the diversity order of our N–relay network is $N+1$. In the sequel, we call the constant ξ "outage gain" factor.

This paper is devoted to the outage probability minimization for an N relay network. Each relay is half duplex. Decode and Forward (DF) [2] and Amplify-and-Forward (AF) [3] relaying strategies are analyzed. The parameters involved in this minimization are the slots relative durations and the powers given to the source and to the relays. The minimization relies on a statistical knowledge of the source-relays, source-destination and relays-destination channels and can be performed by some resource allocation unit.

Our outage minimization relies on the minimization of the constant ξ introduced above with respect to powers and slot durations. We believe that the outage gain factor minimization, as done in this paper, is a relevant approach for the issue of outage probability minimization in the cooperative framework. Even though this minimization concerns (strictly speaking) the high SNR regime, simulations show that minimizing ξ reduces the outage probability in a significant proportion at moderate SNRs also.

We show in particular that the outage gain factor is a convex function of the powers and the slot durations for the considered protocols. We do not make any assumption on the channels probability distributions except for the fact that the probability densities of the channels power gains do not vanish at zero. This assumption is satisfied in particular by the so-called Rayleigh and Rice channels.

Concerning the literature, one can remark that outage probability derivation and its minimization (at least in some cases) has attracted attention in the context of multiple antenna point to point communications ([6], [7], [8] just to name these). In contrast, in the context of cooperative networks, only a few contributions dealing with this problem can be found ([9], [10], [11]). Furthermore outage gain factor derivations in the cooperative framework have been done only in a few papers. We can cite on that subject the early papers [2] and [3] devoted to cooperative networks, where outage gain factor derivations are made for certain classes of DF and AF protocols, mostly in the context of Rayleigh channels. In this contribution, we propose simpler derivations and generalize the outage gain factor results to most channel distributions encountered in practice, and to the situation where slot durations are not necessarily equal. Furthermore, convexity and optimization issues are new to the best of our knowledge.

In Section II, the outage gain factor is studied for a class

[1]This work has been partially funded by SYSTEMATIC/URC and ANR grant "SESAME".

of DF protocols. The AF case is considered in Section III. In both Sections II and III, we begin with the single relay case, then we extend the results to the N-relay case. Section IV is devoted to simulations. Mathematical proofs are provided in the long version of this paper [12].

A. General notations and channel assumptions

We denote by N the number of relays in the network. Node 0 will coincide with the source, nodes 1 to N are the relays and node $N+1$ is the destination. As the transmitted data frame is divided into slots, we shall denote by X_{in} the random vector that represents the message transmitted by node i during slot n. The signal received by node i during slot n will be denoted Y_{in}. Moreover, during slot n, node i is corrupted by an Additive White Gaussian Noise (AWGN) vector V_{in} with unit variance elements.

We denote by H_{ij} the complex random variable (r.v.) representing the scalar radio channel that conveys data from node i to node j. The power gain of this channel will be $G_{ij} = |H_{ij}|^2$. All r.v. G_{ij} are assumed to have densities $f_{G_{ij}}(x)$ which are right continuous at zero. We denote by c_{ij} the limit $c_{ij} = f_{G_{ij}}(0^+)$ and we assume that all these limits are positive. In particular, in the Rayleigh case, $H_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$, and we have $c_{ij} = \sigma_{ij}^{-2}$. More generally, in the Ricean case $H_{ij} \sim \mathcal{CN}(a_{ij}, \sigma_{ij}^2)$, and one can show that $c_{ij} = (1/\sigma_{ij}^2)\exp(-|a_{ij}|^2/\sigma_{ij}^2)$. All channels H_{ij} are assumed independent and available at the receivers only. In fact, we will see that the only information required by the resource allocation unit reduces to the constants $\{c_{0,i}\}_{i=1,\ldots,N+1}$ and $\{c_{i,N+1}\}_{i=1,\ldots,N}$. This information can be sent from the different receivers to the resource allocation unit at a negligible cost.

II. The DF Protocol

A. Outage Probability in the Single Relay case

In this section, we study the following DF protocol already considered in [4], [13]: the source (node 0) needs to send information at a rate of R nats per channel use towards the destination (node 2). To this end, the source has as its disposal a frame of length T and a dictionary of $\lfloor e^{RT} \rfloor$ Gaussian independent vectors with independent $\mathcal{CN}(0,1)$ elements each. Call X_0 the $T \times 1$ vector (dictionary element) transmitted by the source. The relay (node 1) listens to the source message for a duration of $t_0 T$ channel uses where t_0 is a fixed parameter. At the end of this period of time that we refer to as slot 0, the relay attempts to decode the source message. In case of success, the relay searches in a dictionary independent of the source dictionary the word corresponding to the source's message and it transmits it during slot 1 to the destination. Let us partition the word X_0 transmitted by the source as $X_0 = [X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}]^{\mathrm{T}}$ where the lengthes of X_{00} and X_{01} are $t_0 T$ and $t_1 T$ respectively with $t_1 = 1 - t_0$. The signal of size $t_0 T$ received by the relay during slot 0 writes

$$Y_{1,0} = \sqrt{\alpha_0 \rho} H_{01} X_{0,0} + V_{1,0}$$

The parameter ρ will represent the total power spent by the source and the relay to transmit the message as we shall see in a moment. The gain $\sqrt{\alpha_0}$ is an amplitude gain applied by the source. Recall that the random vector $V_{1,0}$ represents the unit variance AWGN received by the relay. Assuming that the relay has a perfect knowledge of the channel H_{01}, it will be able to decode the source message if the event $\mathcal{E} = \{\omega : t_0 \log(1 + \alpha_0 \rho G_{01}(\omega)) > R\}$ is realized. In case $\mathcal{E}$ is realized, the relay will transmit during slot 1 the signal $\sqrt{\alpha_1 \rho} X_{11}$ of length $t_1 T$ where $\sqrt{\alpha_1}$ is the amplitude gain of the relay. In that case, the destination receives the signal $Y_2 = [Y_{20}^{\mathrm{T}}, Y_{21}^{\mathrm{T}}]^{\mathrm{T}}$ given by the equation

$$Y_2 = \sqrt{\rho} \mathbf{H}_{\mathcal{E}} \begin{bmatrix} X_{00} \\ X_{01} \\ X_{11} \end{bmatrix} + V_2$$

where

$$\mathbf{H}_{\mathcal{E}} = \begin{bmatrix} \sqrt{\alpha_0} H_{02} \mathbf{I}_{t_0 T} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sqrt{\alpha_0} H_{02} \mathbf{I}_{t_1 T} & \sqrt{\alpha_1} H_{12} \mathbf{I}_{t_1 T} \end{bmatrix},$$

and V_2 is the unit variance AWGN received by the destination. Notice that probability distribution of the vector $[X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}} X_{11}^{\mathrm{T}}]^{\mathrm{T}}$ is $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{(1+t_1)T})$. Conditionally to the event $\mathcal{E}$, the outage probability $P_{\mathrm{o},1}$ for the destination is therefore

$$\begin{aligned} P_{\mathrm{o},1} &= \mathbb{P}\left[\log\det(\rho \mathbf{H}_{\mathcal{E}_1} \mathbf{H}_{\mathcal{E}_1}^* + \mathbf{I}) \le RT \parallel \mathcal{E}\right] \\ &= \mathbb{P}\left[t_0 \log(1 + \alpha_0 \rho G_{02}) + \right. \\ &\qquad \left. t_1 \log(1 + \alpha_0 \rho G_{02} + \alpha_1 \rho G_{12}) \le R\right] . \end{aligned}$$

In case the relay does not succeed in decoding the source message, which corresponds to the complementary event $\overline{\mathcal{E}}$, the destination simply receives

$$Y_2 = \sqrt{\rho} \begin{bmatrix} \sqrt{\alpha_0} H_{02} \mathbf{I}_{t_0 T} & 0 \\ 0 & \sqrt{\alpha_0} H_{02} \mathbf{I}_{t_1 T} \end{bmatrix} \begin{bmatrix} X_{0,0} \\ X_{0,1} \end{bmatrix} + V_2$$

Therefore, conditionally to $\overline{\mathcal{E}}$, the outage probability $P_{\mathrm{o},2}$ is $P_{\mathrm{o},2} = \mathbb{P}[\log(1 + \alpha_0 \rho G_{02}) \le R]$. In conclusion, the outage probability P_{o} associated with this protocol is $P_{\mathrm{o}} = P_{\mathrm{o},1}\mathbb{P}[\mathcal{E}] + P_{\mathrm{o},2}\mathbb{P}[\overline{\mathcal{E}}] = P_{\mathrm{o},1}(1 - P_{\mathrm{or}}) + P_{\mathrm{o},2} P_{\mathrm{or}}$ where $P_{\mathrm{or}} = \mathbb{P}\left[\overline{\mathcal{E}}\right] = \mathbb{P}\left[t_0 \log(1 + \alpha_0 \rho G_{01}) \le R\right]$ is the relay's outage probability. We need to show that $\rho^2 P_{\mathrm{o}}$ converges as $\rho \to \infty$ (resulting in a diversity order of 2), to derive the outage gain factor $\xi_{\mathrm{DF}} = \lim_{\rho \to \infty} \rho^2 P_{\mathrm{o}}$, and to minimize ξ_{DF} with respect to α_0, α_1 and t_1, subject to $t_1 \in (0,1)$ and to a power constraint. To make this constraint explicit, let us derive the total energy spent by the network to transmit a RT nat symbol. Whatever is the behaviour of the relay, the source transmits the signal $(\sqrt{\alpha_0 \rho} X_{00}, \sqrt{\alpha_0 \rho} X_{01})$. Therefore, the energy E_0 spent by the source is $E_0 = \alpha_0 \rho T$. The energy E_1 spent by the relay is $E_1 = \alpha_1 \rho t_1 T \mathbb{P}[\mathcal{E}] = \alpha_1 \rho t_1 T (1 - P_{\mathrm{or}})$. One can easily show that $P_{\mathrm{or}} = \mathcal{O}(1/\rho)$, hence the total energy E used to transmit one symbol satisfies $E = E_0 + E_1 = \rho T \left(\alpha_0 + \alpha_1 t_1 (1 - P_{\mathrm{or}})\right) \approx \rho T \left(\alpha_0 + \alpha_1 t_1\right)$ for large ρ. Our power constraint for large SNR is therefore $\alpha_0 + \alpha_1 t_1 \le 1$. This constraint becomes tight as $\rho \to \infty$ and is conservative for moderate values of ρ. Note that it is not convex in α_0, α_1, t_1

because the function $g(\alpha_1, t_1) = \alpha_1 t_1$ is not convex. It will be convenient to replace it with a convex constraint by making the change of variables $\beta_0 = \alpha_0$ and $\beta_1 = \alpha_1 t_1$. With these new variables, the power constraint becomes

$$\beta_0 + \beta_1 \le 1 \ . \tag{1}$$

The result is provided by the following proposition:

Proposition 1: With respect to the parameters t_1, β_0 and β_1, the outage gain factor $\xi_{\mathrm{DF}}(t_1, \beta_0, \beta_1)$ for the single relay DF protocol described above is given by

$$\begin{aligned}\xi_{\mathrm{DF}}(t_1, \beta_0, \beta_1) = \\ \frac{c_{01}c_{02}}{\beta_0^2}(\exp(R) - 1)\left(\exp\left(\frac{R}{1-t_1}\right) - 1\right) \\ + \frac{c_{02}c_{12}t_1}{\beta_0\beta_1}\left(\frac{1}{4t_1 - 2}\exp(2R) - \frac{t_1}{2t_1 - 1}\exp\left(\frac{R}{t_1}\right) + \frac{1}{2}\right) \ .\end{aligned} \tag{2}$$

Moreover, the function $\xi_{\mathrm{DF}}(t_1, \beta_0, \beta_1)$ is convex in the domain $(t_1, \beta_0, \beta_1) \in (0, 1) \times (0, \infty)^2$.

Outage probability minimization reduces to minimizing the right hand of (2) given the constraint (1). This reduces to minimizing ξ_{DF} on the line segment of $\mathbb{R}_+^2$ defined by $\beta_0 + \beta_1 = 1$. The function $\xi_{\mathrm{DF}}(t_1, \beta_0, 1 - \beta_0)$ defined on the open square $(0, 1)^2$ is convex as it coincides with the restriction of $\xi_{\mathrm{DF}}(t_1, \beta_0, \beta_1)$ to that line segment. Furthermore, it is clear that $\xi_{\mathrm{DF}}(t_1, \beta_0, 1 - \beta_0)$ goes to infinity on the frontier of $(0, 1)^2$. Therefore, the minimum is in the interior of $(0, 1)^2$, and can be obtained easily by a numerical method.

B. Outage Probability in the N–Relay case

In this paragraph we turn to the study of a DF protocol in the N–relay case. The protocol we shall consider is illustrated by Figure 1. We have $N+1$ slots numbered from 0 to N, slot

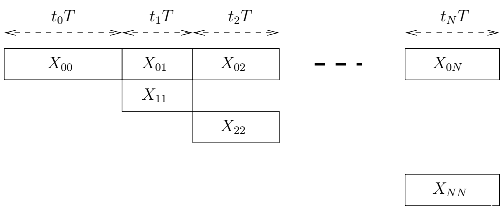

Fig. 1. DF Protocol for N relays

n having the duration $t_n T$. The source transmits during all the frame. Relay n transmits during slot n if it succeeds in decoding the signals sent in slots 0 to $n-1$ by the source and by those active relays among relays 1 to $n-1$. Source and relays dictionaries are independent.

Our purpose is to derive the expression of the outage gain factor ξ_{DF} given by $\xi_{\mathrm{DF}} = \lim_{\rho\to\infty} \rho^{N+1} P_{\mathrm{o}}$ and to minimize it with respect to the powers $(\alpha_i)_{i=0,\ldots,N+1}$ and the slot durations $(t_i)_{i=0,\ldots,N+1}$. The constraints on these parameters are the positivity constraints, the time constraint

$$t_1 + \cdots + t_N < 1 \tag{3}$$

where we put $t_0 = 1-(t_1+\cdots+t_N)$, and the power constraint at high SNR. Let $\beta_0 = \alpha_0$ and $\beta_n = \alpha_n t_n$ for $n = 1, \ldots, N$ be the mean powers spent by nodes 0 to N. Similarly to the single relay case, power constraint can be written

$$\beta_0 + \beta_1 + \cdots + \beta_N \le 1 \ .$$

Let us write the outage gain factor as $\xi_{\mathrm{DF}} = \xi_{\mathrm{DF}}(t_1, \ldots, t_N, \beta_0, \ldots, \beta_N)$. It is given by the following proposition, which generalizes Proposition 1:

Proposition 2: The outage gain factor $\xi_{\mathrm{DF}}(t_1, \ldots, t_N, \beta_0, \ldots, \beta_N)$ for the DF protocol described in this section is given by

$$\begin{aligned}\xi_{\mathrm{DF}}(t_1, \ldots, t_N, \beta_0, \ldots, \beta_N) = \\ c_{0,N+1}\sum_{n=1}^{N+1}\frac{\prod_{m=1}^{n-1} c_{0,m}}{\beta_0^n}\left(\prod_{m=n}^{N}\frac{c_{m,N+1}}{\beta_m}\right) \\ \left(\prod_{m=1}^{n-1}\left(\exp\left(\frac{R}{1-\sum_{k=m}^{N} t_k}\right) - 1\right)\right)\mathcal{I}_n\end{aligned}$$

with

$$\begin{aligned}\mathcal{I}_n = \int_{\mathbb{R}_+^{N-n+2}} \mathbf{1}\left\{\sum_{m=n}^{N+1} v_m \le R\right\} \\ \exp\left(\frac{v_n}{t_n} + \cdots + \frac{v_N}{t_N} + (N-n+2)v_{N+1}\right)\prod_{m=n}^{N+1} dv_m \ .\end{aligned} \tag{4}$$

The function $\xi_{\mathrm{DF}}(t_1, \ldots, t_N, \beta_0, \ldots, \beta_N)$ is convex in the convex set $\mathcal{S}_N \times (0, \infty)^{N+1}$ where $\mathcal{S}_N$ is the subset of $(0, \infty)^N$ delineated by the constraint (3).

In order to obtain ξ_{DF} in practice, one has to compute the integrals $\mathcal{I}_n$ given by Equation (4). To that end, one can use the following lemma:

Lemma 1: Let $J_K(a_0, \ldots, a_K, R) : \mathbb{R}^{K+1} \times \mathbb{R}_+ \to \mathbb{R}_+$ be the function defined as

$$\begin{aligned}J_K(a_0, \ldots, a_K, R) = \int_{\mathbb{R}_+^{K+1}} \mathbf{1}\{x_0 + \cdots + x_K \le R\} \\ \exp(a_0 x_0 + \cdots + a_K x_K)\prod_{k=0}^{K} dx_k.\end{aligned}$$

When parameters $a_0, \cdots, a_K$ are all distinct, $J_K(a_0, \ldots, a_K, R)$ is given by

$$J_K(a_0, \ldots, a_K, R) = \sum_{k=0}^{K}\frac{\eta(k, K)}{a_k}(\exp(a_k R) - 1)$$

where $(\eta(0, i), \ldots, \eta(i, i))_{i=0,\ldots,K}$ is a triangular array of real numbers given by the following recurrence: $\eta(0, 0) = 1$, $\eta(k, i) = \eta(k, i-1)/(a_k - a_i)$ for $k = 0, \ldots, i-1$, and $\eta(i, i) = -\sum_{k=0}^{i-1}\eta(k, i)$.

As $\mathcal{I}_n = J_{N+1-n}(t_N^{-1}, \ldots, t_n^{-1}, N-n+2, R)$, Lemma 1 provides an easy way to compute the expression of ξ_{DF}.

Further remarks:

- Generalizing the single relay case, at the minimum of ξ_{DF} the β_i belong to the hyperplane $\beta_0 + \cdots + \beta_N = 1$. By consequence, the problem reduces to minimizing the convex function with $2N$ parameters $\xi_{\mathrm{DF}}(t_1, \ldots, t_N, \beta_0, \ldots, \beta_{N-1}, 1 - \sum_{i=0}^{N-1} \beta_i)$ on the constraint set $\sum_{i=1}^{N} t_i < 1$ and $\sum_{i=0}^{N-1} \beta_i < 1$. The function ξ_{DF} goes to infinity at the frontier of this set. The minimum is in its interior and can be found for instance by a descent method.
- We advocate the fact that the proof and the result of Proposition 2 can be rather easily modified and adapted to DF protocols other than the one described here such as the so called repetition or the space-time protocols considered in [2].

III. The AF Protocol

A. Outage Probability in the Single Relay case

One AF protocol frequently considered in the literature is the following [4], [14]: the source transmits its codeword during the whole frame of length T. The relay saves in its memory the signal it receives from the source during the first half of the frame. Then the relay applies a gain to this signal and transmits it during the second half of the frame. Here, we consider a slightly more general model: the relay does not necessarily consider the signal received from the source during the first $T/2$ channel uses. Instead, it just considers a section of this signal of length $t_1 T$ with $t_1 \leq 1/2$, and one of our purposes will be to find the value of t_1 that minimizes the outage gain factor. As is shown on figure 2 (with $N = 1$), in general we now have three slots instead of two. The lengthes of these slots are $t_0' T$, $t_1 T$ and $t_1 T$ respectively, with $t_0' + 2t_1 = 1$.
During slots 0 and 1, the destination receives Y_{20} and Y_{21} with dimensions $t_0' T$ and $t_1 T$ respectively. These signals are given by $Y_{2i} = \sqrt{\alpha_0 \rho} H_{02} X_{0i} + V_{2i}$ for $i = 0, 1$, where $\alpha_0 \rho$ is the power spent by the source as in the previous sections. During slot 1, the relay receives the signal Y_{11} with length $t_1 T$ given by the equation $Y_{11} = \sqrt{\alpha_0 \rho} H_{01} X_{01} + V_{11}$. During slot 2, the relay transmits $\sqrt{\gamma_1} Y_{11}$ towards the destination where γ_1 is the power gain applied by the relay. We assume as above that $\alpha_1 \rho$ is the power transmitted by the relay. As $\mathbb{E}[|Y_{11}|^2 \| H_{01}] = \alpha_0 \rho G_{01} + 1$, the gain γ_1 is given by

$$\gamma_1 = \frac{\alpha_1 \rho}{\alpha_0 \rho G_{01} + 1} \ .$$

During slot 2, the destination receives the signal $Y_{22} = \sqrt{\alpha_0 \rho} H_{02} X_{02} + \sqrt{\alpha_0 \gamma_1 \rho} H_{01} H_{12} X_{01} + \sqrt{\gamma_1} H_{12} V_{11} + V_{22}$ with length $t_1 T$. Using these expressions, the mutual information between the source and the destination can be shown to be

$$\begin{aligned} I = {} & t_1 T \log \Big(1 + \alpha_0 \rho G_{02} \\ & + \frac{\alpha_0 \rho G_{02} (\alpha_0 \rho G_{02} + 1)(\alpha_0 \rho G_{01} + 1) + \alpha_0 \alpha_1 \rho^2 G_{01} G_{12}}{1 + \alpha_0 \rho G_{01} + \alpha_1 \rho G_{12}} \Big) \\ & + t_0' T \log(1 + \alpha_0 \rho G_{02}) \ . \end{aligned}$$

Our purpose is to obtain the outage gain factor ξ_{AF} given by $\xi_{\mathrm{AF}} = \lim_{\rho \to \infty} \rho^2 \mathbb{P}[I \leq RT]$ where R is the targeted data rate, and to minimize it subject to the time constraints $t_1 \leq 1/2$, $t_0' + 2t_1 = 1$, and to the power constraint $\beta_0 + \beta_1 \leq 1$ where $\beta_0 = \alpha_0$ and $\beta_1 = \alpha_1 t_1$. We have the following proposition:

Proposition 3: The outage gain factor ξ_{AF} for the protocol described in this paragraph is

$$\begin{aligned} & \xi_{\mathrm{AF}}(t_1, \beta_0, \beta_1) = \\ & \frac{c_{01} c_{02}}{2 \beta_0^2} \left(\frac{1 - t_1}{3t_1 - 1} \exp\left(\frac{2R}{1 - t_1} \right) - \frac{2t_1}{3t_1 - 1} \exp\left(\frac{R}{t_1} \right) + 1 \right) \\ & \quad + \frac{c_{12} c_{02} t_1}{\beta_0 \beta_1} \Bigg(\frac{t_1}{3t_1 - 1} \left(\exp(3R) - \exp\left(\frac{R}{t_1} \right) \right) \\ & \qquad\qquad - \frac{1}{3} \left(\exp(3R) - 1 \right) \Bigg) \end{aligned}$$

Moreover the function $\xi_{\mathrm{AF}}(t_1, \beta_0, \beta_1)$ is convex on $(0, 1/2] \times \mathbb{R}_+^2$.

B. Outage Probability in the N–Relay case

Generalizing the single relay protocol studied in the previous section, we consider now the N-relay protocol described by Figure 2. According to this protocol, the data frame of

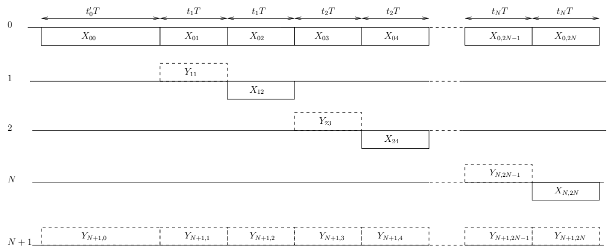

Fig. 2. AF Protocol for N relays

length T is divided into $2N+1$ slots. Slot 0 has a length equal to $t_0' T$. During this slot, the destination is the only node that listens to the source. Relay n (where $n = 1, \ldots, N$) listens to the source during slot $2n-1$ which has the length $t_n T$. During slot $2n$ which has the same length $t_n T$, relay n transmits an amplified version of the signal received by that relay during slot $2n - 1$. Let us note that a version of this protocol with $t_0' = 0$ and $t_n = 1/(2N)$ has been considered in [14] from the point of view of the so called Diversity Multiplexing Tradeoff (DMT).
Due to the fact that $t_0' = 1 - 2 \sum_{n=1}^{N} t_n$, the time and power constraints are respectively written as

$$2 \sum_{n=1}^{N} t_n \leq 1 \quad \text{and} \quad \sum_{n=0}^{N} \beta_n \leq 1 \ .$$

Writing the outage gain factor as $\xi_{\mathrm{AF}} = \xi_{\mathrm{AF}}(t_1, \ldots, t_N, \beta_0, \ldots, \beta_N)$, we have the following proposition, which generalizes Proposition 3:

Proposition 4: The outage gain factor for the AF protocol described in this section is given by

$$\xi_{\mathrm{AF}}(t_1,\ldots,t_N,\beta_0,\ldots,\beta_N) =$$

$$c_{0,N+1} \sum_{\Theta\subset\{1,\ldots,N\}} \frac{1}{\beta_0^{|\Theta|+1}} \prod_{n\in\Theta} \frac{c_{0,n}}{t_n} \prod_{m\in\overline{\Theta}} \frac{c_{m,N+1}}{\beta_m} \frac{1}{1-\sum_{n\in\Theta} t_n}$$

$$\int_{\mathbb{R}_+^{N+1}} \mathbf{1}\left\{\sum_{i=0}^{N} x_i \le R\right\}$$

$$\exp\left(\frac{N+1+|\overline{\Theta}|}{1-\sum_{n\in\Theta} t_n} x_0 + \sum_{i=1}^{N} \frac{x_i}{t_i}\right) \prod_{i=0}^{N} dx_i \quad (5)$$

where $\overline{\Theta} = \{1,\ldots,N\} - \Theta$ and $|.|$ designates the number of elements of a set.
This fonction is convex on the convex set $\mathcal{S}_N \times (0,\infty)^{N+1}$ where $\mathcal{S}_N$ is the subset of $(0,\infty)^N$ delineated by the constraint $\sum_{n=1}^{N} t_n \le 1/2$.

We note that the derivation of the integrals at the RHS of (5) is fairly simple thanks to Lemma 1 again. Notice also that when $N = 1$, the sum over Θ reduces to a sum over the two sets $\Theta = \emptyset$ and $\Theta = \{1\}$, and recovering Proposition 3 is straightforward.

IV. Numerical Illustrations and Simulations

In this section, some of the results of Propositions 1 to 4 are illustrated. Figure 3 shows an example of the performance of the DF and AF protocols described above for $N = 2$ relays. The channel distributions are the Rice distributions, *i.e.*, $H_{ij} = \mathcal{CN}(a_{ij}, \sigma_{ij}^2)$. The decay profile for all channels is described by the equations $|a_{ij}|^2 = C_1 d_{ij}^{-2}$ and $\sigma_{ij}^2 = C_2 d_{ij}^{-3}$ where d_{ij} is the distance between nodes i and j, and the constants C_1 and C_2 are chosen in such a way that $|a_{0,N+1}|^2 = \sigma_{0,N+1}^2 = 1/2$. The relays are at one third and two thirds of the source-destination distance on the source-destination line segment. The required data rate is equal to 2 bits per channel use. Outage performance with equal duration time slots and equal amplitudes (curves marked with "non opt") is compared to the performance after time and power optimization ("opt"). The SNR gain due to optimization is substantial in DF. This gain is maintained when we leave the asymptotic regime in the SNR. In Figure 4, we are in presence of one relay. This figure shows the SNR gain due to optimization of ξ_{DF} and ξ_{AF} as a function of the distance between the relay and the source. Here, channels are Rayleigh channels with $\sigma_{ij}^2 \propto d_{ij}^{-3}$. The dashed curves represent the SNR gain obtained by simulation for an outage probability set to 10^{-3}. We notice that the optimization is all the more useful as the relay is far from the source, and this effect is more pronounced when the DF protocol is used.

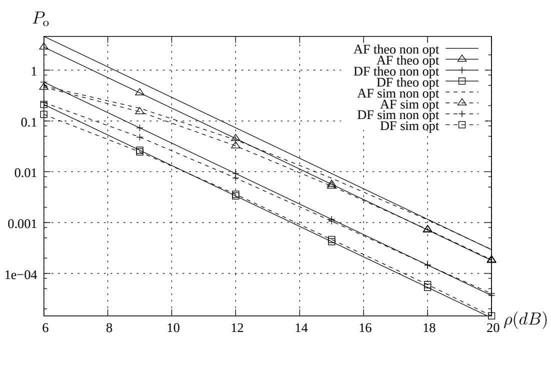

Fig. 3. Outage performance of DF and AF protocols, $N = 2$ relays

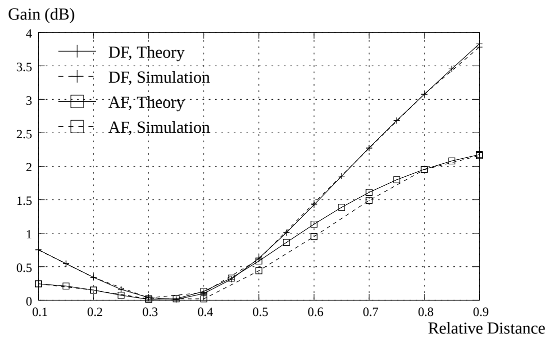

Fig. 4. Merit of optimization, $N = 1$ relay

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity–Part I: System Description," *IEEE Trans. on Com.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[2] J.N. Laneman and G.W. Wornell, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. on IT*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[3] J.N. Laneman, D.N.C. Tse, and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. on IT*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[4] R. Nabar, H. Bölcskei, and F. Kneubühler, "Fading Relay Channels: Performance Limits and Space-Time Signal Design," *IEEE JSAC*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[5] H. Li and Q. Zhao, "Distributed Modulation for Cooperative Wireless Communications," *IEEE SP Mag.*, vol. 23, no. 5, pp. 30–36, Sept. 2006.

[6] Y. Xie, C.N. Georghiades, and A. Arapostathis, "Minimum Outage Probability Transmission With Imperfect Feedback for MISO Fading Channels," *IEEE Trans. on WC*, vol. 4, no. 3, pp. 1084–1091, May 2005.

[7] M. Kang and M.S. Alouini, "Capacity of MIMO Ricean Channels," *IEEE Trans. on WC*, vol. 5, no. 1, pp. 112–122, Jan. 2006.

[8] H. Shin, M.Z. Win, J.H. Lee, and M. Chiani, "On the Capacity of Doubly Correlated MIMO Channels," *IEEE Trans. on WC*, vol. 5, no. 8, pp. 2253–2265, Aug. 2006.

[9] M.O. Hasna and M.-S. Alouini, "Optimal Power Allocation for Relayed Transmissions Over Rayleigh-Fading Channels," *IEEE Trans. on WC*, vol. 3, no. 6, pp. 1999–2004, Nov. 2004.

[10] E.G. Larsson and Y. Cao, "Collaborative Transmit Diversity with Adaptive Radio Resource and Power Allocation," *IEEE Comm. Letters*, vol. 9, no. 6, pp. 511–513, June 2005.

[11] J. Shi, G. Yu, Zh. Zhang, Y. Chen, and P. Qiu, "Partial Channel State Information Based Cooperative Relaying and Partner Selection," in *WCNC*, 2007.

[12] W. Hachem, P. Bianchi, and Ph. Ciblat, "Outage Probability Based Power and Time Optimization for Relay Networks," submitted to *IEEE. Trans. on SP*.

[13] N. Prasad and M.K. Varanasi, "Diversity and Multiplexing Tradeoff Bounds for Cooperative Diversity Protocols," June 2004, p. 268.

[14] K. Azarian, H. El Gamal, and P. Schniter, "On the Achievable Diversity-Multiplexing Tradeoff in Half-Duplex Cooperative Channels," *IEEE Trans. on IT*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.