Outage Performance of a Novel Relaying Protocol: Decode or Quantize and Forward

Pascal BIANCHI†, Philippe CIBLAT‡ and Walid HACHEM††

† SUPELEC, Plateau de Moulon
91192 Gif-Sur-Yvette Cedex E-mail: pascal.bianchi@supelec.fr

‡ Telecom Paris-Tech 46 rue Barrault,
75634 Paris Cedex 13 E-mail: ciblat@enst.fr

†† CNRS / Telecom Paris Tech (UMR 5141), 46 rue Barrault
75634 Paris Cedex 13 E-mail: hachem@enst.fr

Abstract
In this paper, we propose a novel half duplex relaying protocol which belongs to the class of hybrid Decode and Forward schemes, and we study its performance in the context of communications over slow fading channels. Our protocol is based on a Decode or Quantize and Forward (DoQF) approach. In slow fading contexts, the outage probability $P_o$ is of central importance. As the exact evaluation of the outage probability for all possible values of the Signal-to-Noise Ratio (SNR) is a difficult problem for most relaying protocols, we analyze the behaviour of $P_o$ as the SNR $\rho$ tends to infinity. In that case, $\rho^2 P_o$ converges to a constant $\xi$, which we will refer to as the outage gain. The proposed DoQF protocol is shown to outperform the classical DF protocol in terms of outage gain. Moreover, at high SNR, the proposed scheme is shown to be optimal in the wide class of hybrid DF protocols.

1. Introduction
In digital wireless communications over slow fading channels which are unknown at the transmitter side, antenna (or space) diversity is an efficient means for mitigating the effect of channel fades. Recently, a new means of providing this diversity has been considered: in the vicinity of the transmitter/receiver link, radio terminals in an idle state are likely to be present. These terminals have the ability to relay the transmitter’s signal towards the receiver, creating a virtual multiple antenna system which is capable of providing diversity [1, 2, 3, 4]. Classical relaying schemes include the Decode and Forward (DF), Amplify and Forward (AF), Compress and Forward (CF) schemes. Hybrid schemes allowing to combine these approaches have also been proposed in order to improve the network's performance [5].

It is worth noting that the CF protocol as well as hybrid strategies proposed in the literature [5] usually assume that the relay has a perfect knowledge of the channel between the relay and the destination. They also assume that some kind of knowledge of the channel between the source and the destination is available at the relay. On the opposite, this paper considers the context where both the channels “source to destination” and “relay to destination” are completely unknown by the relay. We propose a novel half-duplex relaying protocol which belongs to the class of hybrid Decode and Forward schemes, and we study its performance. We will refer to the latter scheme as the Decode or Quantize and Forward (DoQF) protocol. In the context of transmission over slow fading channels, this protocol is shown to outperform the classical DF scheme. Moreover, as the Signal to Noise Ratio (SNR) tends to infinity, it is shown to be asymptotically optimal in the wide class of hybrid DF protocols (as explained in section 3, we designate by hybrid DF any half duplex protocol such that the relay codes and forwards the source message as soon as it is able to decode during the first slot).

1.1. Contributions
1) A novel relaying scheme is introduced. The DoQF protocol can be considered as an augmented DF scheme, in which the relay is able to adapt its forwarding strategy as a function of the information that it received from the source. The transmission is divided in two slots with fixed durations. A “dynamic” version
of our protocol based on adaptive time slot durations will be studied in future works. As for the classical DF scheme, the relay tries to decode the message of the source based on the signal received during the first slot. If the latter step is successful, the relay retransmits as usual a coded version of this message during the second slot, based on an independent codebook. In case the relay is not able to decode the message, it does not remain inactive (contrary to the DF case). The relay quantizes the received signal vector using a well chosen distortion value. Next, a coded version of the quantized vector is forwarded to the destination using a well chosen transmit power. Data processing is used at the destination in order to recover the quantized signal forwarded by the relay, and to decode the initial source message.

2) The performance of the proposed DoQF scheme is analyzed. In the context of communications over slow fading channels, the relevant performance measure from the information theoretic point of view is the so-called outage probability, which is the probability that Shannon’s mutual information lies beneath a given threshold. In a single-relay network with single antenna terminals, the outage probability $P_o$ usually satisfies $\lim_{P \to \infty} (P^2 P_o) = \xi$ where $P$ is the Signal to Noise Ratio and $\xi$ is a constant. This equation indicates in particular that the diversity order of the single-relay network is 2. In the sequel, we call the constant $\xi$ “outage gain” factor. The outage gain associated with the proposed DoQF scheme is derived. The DoQF is proved to outperform the DF protocol. Furthermore, a lower bound on outage gains of a general class of hybrid-DF schemes is derived. The DoQF outage gain is shown to coincide with the latter bound.

3) Optimal time and power allocation minimizing the DoQF outage gain is obtained. The parameters involved in this minimization are the slots relative durations and the powers given to the source and to the relays. The minimization relies on a statistical knowledge of the channels.

In this paper, we do not make any assumption on the channels probability distributions except for the fact that the probability densities of the channels power gains do not vanish at zero. This assumption is satisfied in particular by the so-called Rayleigh and Rice channels.

### 1.2. General notations and channel assumptions

In the sequel, node 0 will coincide with the source, node 1 with the relay and node 2 is the destination. As the transmitted data frame is divided into slots, we shall denote by $X_n$ the random vector that represents the message transmitted by node $i$ during slot $n$. The signal received by node $i$ during slot $n$ will be denoted $Y_{in}$. Moreover, during slot $n$, node $i$ is corrupted by an Additive White Gaussian Noise (AWGN) vector $V_{in}$ with unit variance elements. We denote by $H_{ij}$ the complex random variable (r.v.) representing the scalar radio channel that conveys data from node $i$ to node $j$. Channel coefficients $H_{ij}$ are assumed to be perfectly known at the receiving node $j$, but are unknown at each other node of the network, including the transmitter $i$. The power gain of this channel will be $G_{ij} = |H_{ij}|^2$. All r.v. $G_{ij}$ are assumed to have densities $f_{G_{ij}}(x)$ which are right continuous at zero. We denote by $c_{ij}$ the limit $c_{ij} = f_{G_{ij}}(0^+)$ and we assume that all these limits are positive. In particular, in the Rayleigh case, $H_{ij} \sim CN(0, \sigma^2_{ij})$, and $c_{ij} = \sigma^{-2}_{ij}$; in the Ricean case, $H_{ij} \sim CN(\alpha_{ij}, \sigma^2_{ij})$ and $c_{ij} = (1/\sigma^2_{ij}) \exp(-|\alpha_{ij}|^2/\sigma^2_{ij})$. All channels $H_{ij}$ are assumed independent.

### 2. Proposed DoQF Protocol

The source (node 0) needs to send information at a rate of $R$ nats per channel use towards the destination (node 2). To this end, the source has at its disposal a frame of length $T$ and a dictionary of $[e^{RT}]$ Gaussian independent vectors with independent $CN(0, 1)$ elements each. Call $X_0$ the $T \times 1$ vector (dictionary element) transmitted by the source. The relay (node 1) listens to the source for a duration of $t_0T$ channel uses where $t_0$ is a fixed parameter. At the end of this period of time that we refer to as slot 0, the relay attempts to decode the source message. In case of success, the relay searches in a dictionary independent of the source dictionary the word corresponding to the source’s message and it transmits it during slot 1 to the destination. Let us partition the word $X_0$ transmitted by the source as $X_0 = [X_{00}, X_{01}]^T$ where the lengths of $X_{00}$ and $X_{01}$ are $t_0T$ and $t_1T$ respectively with $t_1 = 1 - t_0$. The signal of size $t_0T$ received by the relay during slot 0 writes

$$Y_{10} = \sqrt{\alpha_0} p H_{01} X_{00} + V_{10}$$

Parameter $\rho$ represents the total power spent by the source and the relay to transmit the message as we shall see in a moment. The gain $\sqrt{\alpha_0}$ is an amplitude gain applied by the source. Recall that the random vector $V_{10}$ represents the unit variance AWGN received by the relay. Figure 1 represents the transmit and receive signals respectively for each node of the network.

We now consider separately the case when the relay manages to decode the source message and the case when it does not. Data processing at the destination side is summarized by Figure 2.
The relay will be able to decode the source message if the event $\mathcal{E} = \{\omega : t_0 \log(1 + \alpha_0 \rho G_{01}(\omega)) > R\}$ is realized. In case $\mathcal{E}$ is realized, the relay will transmit during slot 1 the signal $\sqrt{\alpha_1} X_{11}$ of length $t_1 T$ where $\sqrt{\alpha_1}$ is the amplitude gain of the relay. In that case, the destination receives the signal $Y_2 = [Y_{20}^T, Y_{21}^T]^T$ given by

$$Y_2 = \sqrt{\rho} H_{\mathcal{E}} \begin{bmatrix} X_{00} \\ X_{01} \\ X_{11} \end{bmatrix} + V_2$$

where

$$H_{\mathcal{E}} = \begin{bmatrix} \sqrt{\alpha_0} H_{02} I_{t_0 T} & 0 \\ 0 & \sqrt{\alpha_0} H_{02} I_{t_1 T} \end{bmatrix} \begin{bmatrix} 0 \\ \sqrt{\alpha_1} H_{12} I_{t_1 T} \end{bmatrix}.$$

and $V_2$ is the unit variance AWGN received by the destination. Conditionally to the event $\mathcal{E}$, the outage probability is equal to

$$P_{o,1} = \mathbb{P} [\log \det(\rho H_{\mathcal{E}} H_{\mathcal{E}}^H + I) < RT|\mathcal{E}] .$$

**Case when the relay fails to decode the source message**

Now assume that $\mathcal{E}$ is not realized i.e., the relay fails to decode the source message. In this case, the relay quantizes the received signal, and then codes and forwards this quantized version during the second slot. More precisely, the following steps are used.

a) **Quantization.** Denote by $\tilde{Y}_{10}$ the quantized version of the received vector $Y_{10}$. Vector $\tilde{Y}_{10}$ is constructed as follows. Clearly, all $t_0 T$ components of vector $Y_{10}$ are independent and $\mathcal{CN}(0, \alpha_0 \rho G_{01} + 1 - \Delta^2)$ distributed. Denote by $\Delta^2 = \Delta^2(\rho)$ the desired squared-error distortion:

$$\mathbb{E} |\tilde{Y}_{10}(i) - Y_{10}(i)|^2 \leq \Delta^2$$

for each component $i$, where $\mathbb{E}$ denotes the expectation. The rate distortion Theorem for Gaussian sources [6] states that for each $Q$ such that

$$Q > \log \left( \frac{\alpha_0 \rho G_{01} + 1}{\Delta^2} \right)$$

there exists a $([e^{Q t_0 T}], t_0 T)$-rate distortion code which is achievable for the distortion $\Delta^2$. In practice, such a code can be constructed by properly selecting the quantized vector $\tilde{Y}_{10}$ among a quantizer-codebook formed by $[e^{Q t_0 T}]$ independent random vectors with distribution $\mathcal{CN}(0, (\alpha_0 \rho G_{01} + 1 - \Delta^2) I_{t_0 T})$. Vector $\tilde{Y}_{10}$ is selected among this codebook in such a way that sequences $Y_{10}$ and $\tilde{Y}_{10}$ are jointly typical w.r.t. the joint distribution $p_{\{Y, \tilde{Y}\}}$ given by

$$Y = \tilde{Y} + \Delta Z$$

where $\tilde{Y}$ and $Z$ are independent random variables with respective distributions $\mathcal{CN}(0, \alpha_0 \rho G_{01} + 1 - \Delta^2)$ and $\mathcal{CN}(0, \Delta^2)$. Condition (2) ensures that such a vector $\tilde{Y}_{10}$ exists with high probability as $T \to \infty$.

Note that parameter $Q$ can be interpreted as the number of nats used to quantize one component of the received vector $Y_{10}$. As the rhs of (2) depends on the channel gain $G_{01}$, it looks impossible at first glance to construct a fixed quantizer which is successful for any channel state. Nevertheless, recall that we are considering the case where event $\mathcal{E}$ is not realized i.e. $t_0 \log(1 + \alpha_0 \rho G_{01}(\omega)) < R$. In order to guarantee that (2) always hold, it is thus sufficient to define $Q = Q(\rho)$ as

$$Q(\rho) = \log \left( \frac{K}{\Delta^2(\rho)} \right)$$

where $K$ is any constant such that $K > e^\frac{R}{\alpha_0 \rho}$. In order to complete the definition of our quantizer, we now need to select a relevant distortion $\Delta^2(\rho)$. In the sequel, we only assume that

$$\lim_{\rho \to \infty} \Delta^2(\rho) = 0 ,$$

or equivalently, $\lim_{\rho} Q(\rho) = +\infty$. This means that at high SNR, fine quantization is applied.

Remark that, contrary to standard CF protocols, no Wyner-Ziv coding is used by the relay in the present protocol.

b) **Forwarding.** Assume that (2) holds i.e. the quantization step is successful. The relay forwards the quantized vector to the destination. More precisely, it forwards the index of the selected quantized vector among the $[e^{Q t_0 T}]$ possible ones. This requires to transmit $Q t_0 T$ nats during the second slot of length

\[\text{Figure 1: Transmit/Receive signals for source (S), relay (R) and destination (D).}\]
To that end, the relay uses an independent Gaussian codebook with rate \( Q_{t_1}/t_1 \). As previously, denote by \( X_{11} \) the corresponding codeword. The signal transmitted by the relay can be written as \( \sqrt{\phi(\rho)} X_{11} \), where \( \phi(\rho) \) denotes the power of the relay. All our results hold for any function \( \phi \) such that \( \lim_{\rho \to \infty} \phi(\rho) = +\infty \) and \( \lim_{\rho \to 0} \phi(\rho)/\rho^2 = 0 \). In practice, \( \phi(\rho) = \alpha_1 \rho \) is a possible choice, but less power can as well be transmitted (typically, \( \phi(\rho) = \log \rho \) works as well). For technical reasons which will be enlightened later, the power \( \phi(\rho) \) should be related to the distortion \( \Delta^2(\rho) \) through the following assumption:

\[
\lim_{\rho \to \infty} (\phi(\rho)^2 \Delta^2(\rho)^{\alpha}) = +\infty. \tag{4}
\]

c) Processing at destination node. We distinguish between two possible cases namely, the destination is or is not able to recover the source message based on the first slot only. The signal received by the destination node during the first slot is equal to

\[
Y_{20} = \sqrt{\alpha_0} \rho H_{02} X_{00} + V_{20}.
\]

Thus, if the event \( D = \{ \omega : t_0 \log(1 + \alpha_0 \rho G_{02}(\omega)) > R \} \) is realized, the destination is able to recover the source message based on the first slot only. In this case, the transmission is successful by definition. The delicate case is the case where \( D \) is not realized. Then the destination must use the signal received during the second slot in order to have a chance to decode the initial message:

\[
Y_{21} = \sqrt{\phi(\rho)} H_{12} X_{11} + \sqrt{\alpha_0} \rho H_{10} X_{01} + V_{21}. \tag{5}
\]

Equation (5) can be interpreted as a multiple access channel model. Based on this observation, the data processing at the destination node follows the following three steps.

**Step 1.** The destination first tries to recover the message from the relay, which corresponds to the quantized vector \( \tilde{Y}_{10} \). The source contribution is interpreted as a noise. The recovery of \( \tilde{Y}_{10} \) is successful if the event

\[
\mathcal{F} = \left\{ \omega : t_1 \log \left(1 + \frac{\phi(\rho) G_{12}(\omega)}{\alpha_0 \rho G_{02}(\omega)} + 1 \right) > Q(\rho)t_0 \right\}
\]

is realized. Otherwise, an error is declared. Fortunately, it turns out that the probability of such an error is negligible when \( \rho \) is large enough. This claim can be motivated by the following insight. An error only holds when the following events are jointly realized: 1) the relay did not decode the source message during the first slot (\( \mathcal{E} \) is not realized), 2) the destination did not decode the source message during the first slot (\( \mathcal{D} \) is not realized), 3) the destination did not manage to extract the contribution of the relay (\( \mathcal{F} \) is not realized). It can be shown that the first two events occur with a probability of the order of \( 1/\rho \) as \( \rho \to \infty \) and that the third event occurs with a probability of the order of \( 1/(\phi(\rho) \Delta^2(\rho)^{\alpha}) \). Thus, it can be proved that the 3 above events are jointly realized with a probability which is bounded by \( C/(\rho \times \rho \times \phi(\rho) \Delta^2(\rho)^{\alpha}) \), where \( C \) is a constant w.r.t. \( \rho \). At high SNR, this probability is negligible in comparison to \( 1/\rho^2 \) due to condition (4). As the outage probability tends to zero at speed \( 1/\rho^2 \), the contribution of this event to the outage probability is thus negligible. The detailed proof is omitted due to the lack of space. As a consequence, except for a class of events with negligible probability measure, the destination manages to recover the quantized vector \( \tilde{Y}_{10} \) and thus \( X_{11} \). In other words, one can consider that Step 1 is always successful at high SNR.

**Step 2.** The contribution \( X_{11} \) of the relay to (5) is cancelled. Denote by \( Y'_{21} \) the resulting signal. From (5), we obtain

\[
Y'_{21} = \sqrt{\alpha_0} \rho H_{02} X_{01} + V_{21}.
\]

**Step 3.** Finally, the destination tries to decode the initial message. After processing, the overall received signal can be written as

\[
Y_2 = \left[ \frac{Y_{20}}{\sqrt{Y_{21}^2 + 1}} \tilde{Y}_{10} \right].
\]

It is straightforward to show that

\[
Y_2 = \sqrt{\rho} H_{\mathcal{F}} [X_{00}, X_{01}]^T + \tilde{v}_{10} \tag{6}
\]

where

\[
H_{\mathcal{F}} = \begin{bmatrix}
\sqrt{\alpha_0} H_{02} I_{t_0 T} & 0 \\
0 & \sqrt{\alpha_0} H_{10} I_{t_1 T} \\
\sqrt{\alpha_0} H_{02} I_{t_0 T} & 0 \\
0 & \sqrt{\alpha_0} H_{10} I_{t_1 T}
\end{bmatrix}
\]

and where \( \tilde{v}_{10} \) is a unit variance AWGN. From the signal model (6), it is clear that in the limit of long codewords \( T \to \infty \), the source message can be recovered without error, provided that the required rate \( R \) does not exceed the average mutual information \( \frac{1}{T} I([X_{00}, X_{01}]^T : Y_2) \). Therefore, conditionally to the events \( \mathcal{E}, \mathcal{D} \) and \( \mathcal{F} \), the outage probability can be expressed as

\[
P_{o,2} = P \left[ \log \det(\rho H_{\mathcal{F}} H_{\mathcal{F}}^T + I) < RT \| \mathcal{E}, \mathcal{D}, \mathcal{F} \right]. \tag{7}
\]

The data processing at destination node is summarized by Figure 2.

3. Outage Probability Analysis
3.1. Genie-Aided Bound

Before deriving the outage gain of the proposed DoQF protocol, we derive a bound on the outage performance of a large class of hybrid DF protocols. Recall that we designate by hybrid DF any protocol such that 1) When the relay is able to decode the source message during the first slot, the message is coded by the relay using an independent Gaussian codebook, and then forwarded to the destination with power $\alpha_1 \rho$. 2) When the relay fails to decode, any other forwarding strategy is likely to be used. We also assume that the source power is equal to $\alpha_0 \rho$ during the whole transmission.

**Theorem 1** For any hybrid DF protocol, the outage gain $\xi = \lim_{\rho \to \infty} \rho^2 P_o$ is lower-bounded by $\xi_{GA-DF}$:

$$\xi_{GA-DF} = \frac{c_0 c_1}{\alpha_0} \left( \frac{1}{2} \frac{\exp(2R)}{4t_0 - 2} - \frac{t_0 \exp(R/t_0)}{2t_0 - 1} + \frac{c_2 c_2}{\alpha_0 \alpha_1} \frac{1}{2} \frac{\exp(2R)}{4t_1 - 2} - \frac{t_1 \exp(R/t_1)}{2t_1 - 1} \right).$$

The proof of the above result will be provided in an extended version of this paper. We will refer to the above bound $\xi_{GA-DF}$ as the Genie-Aided bound. Indeed, this bound would be achieved if, in case the relay fails to decode the source message, one assumes that the destination is perfectly aware of the signal received by the relay, just as in a SIMO system. More accurate statements will be given in the final paper.

3.2. Outage gain of the DoQF protocol

We now analyze the performance of the proposed scheme at high SNR. This requires to study the asymptotic behaviour of the (predominant) outage events. After some algebra, we prove that the outage probability is asymptotically equivalent to $\mathbb{P}[E] P_{o,1} + \mathbb{P}[E, D, F] P_{o,2}$, where $P_{o,1}$ and $P_{o,2}$ are defined by (1) and (7). The asymptotic analysis of this expression and the use of Lebesgue’s dominated convergence Theorem, lead to the following result.

**Theorem 2** The outage gain $\xi_{DoQF}$ of the proposed DoQF protocol coincides with the Genie-Aided bound given by (8):

$$\xi_{DoQF} = \xi_{GA-DF}.$$
3.3. Power and time optimization

The total power spent by the network (when normalized by $\rho$) is shown to be equal to $P = \alpha_0 + \alpha_1 t_1$. Therefore it is straightforward to minimize numerically the outage gain as a function of time and power parameters $(t_0, \alpha_0)$ considering a fixed power $P$. Then $t_1$ and $\alpha_1$ are obtained as $1 - t_0$ and $(P - \alpha_0)/(1 - t_0)$ respectively. The optimal resource allocation depends obviously on coefficients $c_{01}, c_{02}, c_{12}$.

4. Numerical Illustrations and Simulations

Figure 3 compares the performance of the DoQF and DF schemes as a function of the SNR. Channels are Rayleigh distributed. The corresponding channel variance is a function of the distance between terminals following a path loss model with exponent equal to 3. The relay lies at two thirds of the source-destination distance on the source-destination line segment. The required data rate is equal to 2 bits per channel use. Outage performance with equal duration time slots and equal amplitudes (curves marked with “non opt”) is compared to the performance after time and power optimization (“opt”). Substantial gain are observed between DF and DoQF, and between optimized and nonoptimized protocols.

![Figure 3: Outage performance of DF and DoQF protocols](image1)

Figure 4 represents the outage gains for DoQF and DF versus the position of the relay on the source-destination line segment. The farther the relay from the source is, the better DoQF compared to DF works. This fact can be explained as follows: if the relay is close to the destination, it will be more often in outage and the Quantization step will thus operate more often.

![Figure 4: Outage gain of DF and DoQF versus relay position](image2)

References


