Influence of Guard Band on Channel Capacity for Optical Transmission Systems

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Abstract: We analyze the influence in non-dispersive and dispersive fiber of guard band between WDM channels on the data rate limit given by the information-theoretic Shannon capacity concept when nonlinear Kerr effect occurs.

1. Introduction

Due to traffic growth in optical transmission systems, coherent detection is now employed leading to the deployment of QPSK or high-order QAM modulations and modern forward-error correcting codes. In order to exhibit the data rate limit of such optical communications systems, the Shannon capacity is relevant. Only a few works have been done for expressing in closed-form the Shannon capacity [1–5]. Unlike wireless communications, nonlinear impairments (*e.g.*, the Kerr effect) can not be neglected and induce a hard task. In [1, 2], closed-form expressions are provided assuming Gaussian distributed input, Volterra series based Nonlinear Schrödinger Equation (NSE) solutions and low nonlinear impairments. In [3, 4], perturbation theory for solving the NSE and for calculating the different involved entropies has been developed and leads to an approximate closed-form expressions for the capacity whatever the input distribution. In [5], numerous numerical illustrations have been given by solving numerically (but not theoretically) the NSE and then by simulating the capacity. All these works unveil that the capacity in presence of Kerr effect decreases if the input power is too high. However, these works do not take into account the specific spectrum property of current WDM-based systems. Therefore, we hereafter propose to analyze the influence of the guard band (between each WDM channel and induced by the mux-demux technology) on the Shannon capacity when Kerr effect is involved.

2. Closed-form expressions for the Shannon Capacity

The NSE is mainly directed by two parameters: β the coefficient for dispersion and γ the coefficient for nonlinearity [6]. We have followed the same approach as done in [1, 2] for deriving the capacity in closed-form. For the sake of simplicity, we have considered only one span, unlike [1, 2]. The power spectral density (d.s.p.), denoted by $f \mapsto S_x(f)$ of the continuous-time input signal $x(t)$ has the following form

$$S_x(f) = \frac{P_x}{B} \sum_{k=-N}^{N} \Pi_B(f - k(B+B_g)) \tag{1}$$

where $\Pi_B(.)$ is the rectangular function of bandwidth B, the index k corresponds to the k-th WDM channel. Each WDM channel really uses B out of $(B+B_g)$. Each WDM is thus separated by a guard band of B_g. P_x is the power of each WDM channel.

When $\beta = 0$, the dispersion effect is not taken into account, and then the NSE can be solved with exact closed-form expression. Therefore, we firstly focus on such a context denoted by No-Disp (Non-dispersive case). In [1], if the Kerr effect is not too high (*i.e.*, L and P_x small enough), one can find the following expression for the data rate for the central WDM channel, *i.e.*, the capacity (per channel use) multiplied by the whole bandwidth occupied by a WDM channel:

$$R_{\text{No-Disp}} = \int_{-B/2}^{B/2} \log_2 \left(1 + \frac{S_x(f)/(1+\gamma^2[(2N+1)P_x]^2 L_{\text{eff}}^2)}{P_n + 2\gamma^2 L_{\text{eff}}^2 T_x(f)/(1+\gamma^2[(2N+1)P_x]^2 L_{\text{eff}}^2)^3} \right) df \tag{2}$$

where $L_{\text{eff}} = (1-e^{-\alpha L})/\alpha$ [6] with L the length of the span and α the coefficient for attenuation, and where P_n is the ASE noise variance. Let $C_x(\tau) = \mathbb{E}[x(t+\tau)x(t)^*]$ be the autocorrelation function of the (stationary) input signal where the superscript $(.)^*$ stands for complex conjugation. We define $T_x(f) = \int C_x(\tau)\, |C_x(\tau)|^2 e^{-2i\pi f \tau} d\tau$. In [1], T_x has been calculated when $B_g = 0$. Our contribution here consists in calculating in closed-form T_x when $B_g \neq 0$. After tedious algebraic manipulations, we have obtained that $T_x(f) = \frac{P_x^3}{B} \left[\frac{1}{2} \sum_{k=N+1}^{3N} (3N+1-k)(3N+2-k)(Q_k(f)+Q_{-k}(f)) + \sum_{k=1}^{N} \left(3N^2+3N+1-k^2\right)(Q_k(f)+Q_{-k}(f)) + \left(3N^2+3N+1\right) Q_0(f) \right]$ with $Q_k(f) = \Pi_B \star \Pi_B \star \Pi_B((f+k(B+B_g))/B)$ and $\star$ stands for the convolution product.

Let us now move on the dispersive case: $\beta \neq 0$. Then the NSE does not have closed-form solution anymore. Only approximations for its solutions can be developed. As in [2], we will use the (first-order) Volterra series based solutions. After very long derivations, we have obtained that

$$R_{\text{Disp.}} = \int_{-B/2}^{B/2} \log_2 \left(1 + \frac{S_x(f)(1+4\gamma^2[(2N+1)P_x]^2 L_{\text{eff}}^2)}{P_n + M_x(f)} \right) df \tag{3}$$

where $M_x(f) = 2(2\pi\gamma)^2 \iiint \delta(f - f_1 + f_2 - f_3)|K(f; f_1, f_2, f_3)|^2 S_x(f_1) S_x(f_2) S_x(f_3) df_1 df_2 df_3$ with $K(f; f_1, f_2, f_3) = |e^{-L(\alpha+2i\beta\pi^2(f^2-f_1^2+f_2^2-f_3^2))} - 1|^2/|\alpha + 2i\beta\pi^2(f^2 - f_1^2 + f_2^2 - f_3^2)|^2$. In [2], surprisingly, a different expression for Eq. (3) has been found: the term $4\gamma^2[(2N+1)P_x]^2 L_{\text{eff}}^2$ at the numerator was omitted. In order to check which equation is the correct one, we have considered the so-called opaque channel (the output signal is null whatever the input signal). According to our reasoning, the capacity is zero as expected, unlike the approach given in [2].

3. Numerical illustrations

We have fixed $B = 50$GHz and $N = 40$ (there are 81 WDM channels). Moreover the fiber transmission is characterized by $P_n = 10^{-5}$mW/channel (ASE noise), $\alpha = 0.2$dB/km, $\gamma = 1.22\ \text{W}^{-1} \cdot \text{km}^{-1}$, and $L = 80$km. In Fig. 1 and 3, we plot the Shannon capacity versus the input power per WDM channel and various guard band values keeping the same useful band for the non-dispersive and dispersive cases respectively. We remark that the model is in perfect agreement with the numerical evaluation of the capacity while the input power is small enough. The dispersion of the fiber is beneficial from the information-theoretic point of view. In Fig. 2 and 4, we plot the gain (in percentage) for the maximum data rate due to the presence of the guard interval. This gain is only incremental. Consequently the Kerr effect has much more negative impact inside the useful band than outside it. Moreover, the higher the dispersion is, the smaller the guard band influence is.

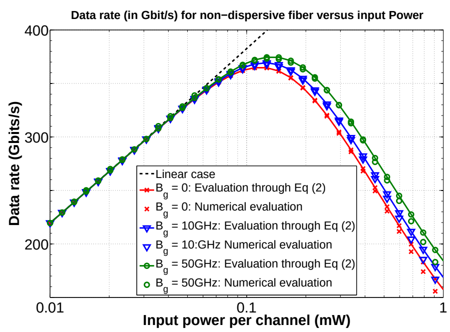

Fig. 1. Data rate vs input power in non-dispersive-case

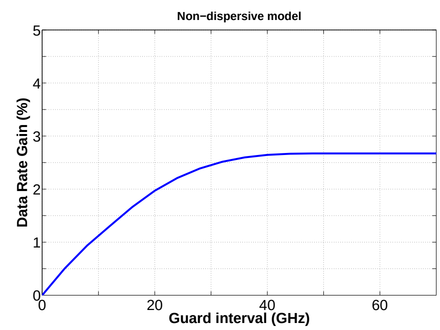

Fig. 2. Data rate gain in non-dispersive case

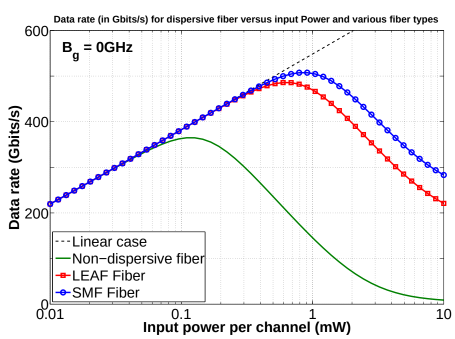

Fig. 3. Data rate vs input power in dispersive-case

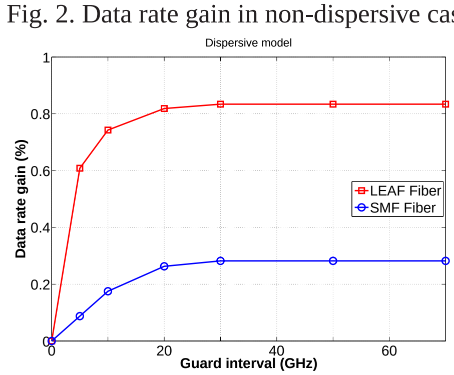

Fig. 4. Data rate gain in dispersive case

4. Conclusion

Based on theoretical Shannon capacity derivations, our numerical evaluations show that the guard band in WDM systems does not have a great influence on the channel capacity especially for realistic fibers inducing high dispersion.

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