Multiuser Power and Bandwidth Allocation in Ad Hoc Networks with Type-I HARQ under Rician Channel with Statistical CSI

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***Abstract*—In this paper, we address the problem of joint power and bandwidth allocation for Type-I hybrid automatic repeat request in military ad hoc networks based on orthogonal frequency division multiple access when only statistical channel state information is available to perform the Resource Allocation (RA) and when considering practical modulation and coding schemes. The novelty of our work lies in the consideration of the Rician fast fading channel during the RA process, which is a general fading model which can represent the Rayleigh fading channel and the additive white Gaussian noise channel as special cases. The objective of this paper is to propose an algorithm to minimize the sum of the transmit power of each user under different quality of service constraints. An accurate approximation of the coded packet error rate over the Rician fading channel is first proposed, and an efficient algorithm to solve the problem is developed. The effectiveness of the algorithm is demonstrated though simulations. Our results point out a significant energy saving when the Rician distribution is explicitly taken into account when performing the RA instead of considering the Rayleigh channel.**

***Index Terms*—HARQ, Resource allocation, Rician Fading, Optimization**

I. Introduction

Hybrid Automatic Repeat reQuest (HARQ) consists in the combination of Forward Error Correction (FEC) and Automatic Repeat reQuest (ARQ) protocol which allows to increase the reliability of wireless communications. This paper focuses on Resource Allocation (RA) for Type-I HARQ in military ad hoc networks assuming the use of Orthogonal Frequency Division Multiple Access (OFDMA) as the multiple access technology. We consider that only statistical Channel State Information (CSI) is available to perform the RA since military ad hoc Medium Access Control (MAC) schemes based on Time Division Duplexing (TDD) cannot handle instantaneous feedback, conversely to modern civilian cellular network systems. In addition, we take into account the effect of practical Modulation and Coding Scheme (MCS) in our derivations, which gives a practical interest to our work. Our objective is to minimize the total transmit power (the sum of the transmit power of each user) under several Quality of Service (QoS) constraints, which is motivated by the inherently limited battery lifetime in military ad hoc networks. In this context, the main novelty of our work is that we consider the Rician channel model in our RA algorithm, which is a generic channel model which can represent the Rayleigh fading channel and the Additive White Gaussian Noise (AWGN) channel as special cases. Considering this channel model appears to be interesting since military ad hoc networks can be deployed in various theatres with different propagation conditions. For instance, in rural area, it may happen that a strong Line of Sight (LOS) exists between the transmitter and the receiver and, in this case, the channel is well modelled by the so called Rician channel model [1]. Also, it is known that the error probability is lower on the Rician channel than on the Rayleigh one and hence, we expect a power saving by explicitly considering the Rician model during the RA.

The problem of the minimization of the transmit power for HARQ in the single user context has been investigated in numerous works, such as [2] where the authors minimize the average transmit power for Chase Combining (CC)-HARQ over the Rayleigh block fading channel considering capacity achieving codes. A similar work was done in [3], where the authors considered practical MCS over the Rayleigh block fading channel. An extension of this work has been presented in [4] for relaying systems. In [5], the average power consumption of Type-I HARQ over the Rayleigh block fading channel was minimized in the finite blocklength regime.

On the other hand, the RA problem for HARQ-based OFDMA systems with power concern was treated in [6], where the authors minimize the transmit power for Incremental Redundancy (IR)-HARQ in a multiuser OFDMA system when considering queuing constraints. The authors of [7] maximize the energy efficiency for IR-HARQ when channels are perfectly known by the transmitter. The Energy Efficiency (EE) of IR-HARQ in the multiuser context is also investigated in [8]. All the above mentioned works consider the use of capacity achieving codes, which gives upper bound concerning the achievable performance with practical MCS. Practical MCS were taken into account in [9] where the authors maximize the EE of Type-I HARQ when the transmitter is assumed to have full CSI information. In [10], the authors minimize the transmit power for Type-I HARQ with QoS constraints on the

Rayleigh channel with statistical CSI. This work is extended in [11] for Type-II HARQ, again considering the Rayleigh channel and statistical CSI. To the best of our knowledge, these are the only works which consider the RA problem for HARQ when the objective is to minimize the transmit power with practical MCS and statistical CSI.

From the above discussion, it is clear that there exist no work in the literature which address the problem of the total transmit power minimization of HARQ-based OFDMA systems over the Rician fading channel with statistical CSI and practical MCS. The contribution of this paper is to study this problem for Type-I HARQ. In detail, we start studying the PER of convolutional codes over the Rician fast fading channel. We then formulate the problem of the minimization of the transmit power with QoS constraints. This problem is similar to the one of [12] but, in our work, we consider the more general Rician fading channel, and we add a constraint on the maximum allowed transmit power. We propose an algorithm to optimally solve this problem and finally, the effectiveness of the proposed approach is investigated through numerical simulations.

The remainder of the paper is organized as follows. In Section II, we present the system model while in Section III we study the PER of convolutional codes over the Rician fast fading channel. Section IV is devoted to the problem formulation, and Section V to the optimal problem resolution of this problem. Numerical results are discussed in Section VI and finally, concluding we draw some concluding remarks in Section VII.

II. System Model

We consider a military clustered ad hoc network with a total bandwidth B which is divided in N_c subcarriers, and we focus on intra cluster RA. The number of links in the cluster is L, and we consider the use of OFDMA as the multi access technology. Moreover, to perfom the RA, we assume that a Cluster Head (CH) is elected in each cluster. The CH centralizes the statistical CSI of the different links, and allocate the physical resources.

The channel of each link is modelled as constant for the duration of an OFDMA symbol, and it changes independently between consecutive symbols. This assumption is justified by the use of a well designed frequency hopping pattern along with bit interleaved coded modulation [13]. The sampled channel impulse response of link l during the jth OFDMA symbol is defined by $\mathbf{h}_l(j) = [h_l(j,0), ..., h_l(j,M-1)]^T$ with $(.)^T$ denoting the transposition operator. The main originality of our work comes from the consideration of the (possible) existence of a LOS between the transmitter and the receiver, that is, the first tap of the channel impulse response, $h_l(j,0)$, is in general modelled as a complex Gaussian random variables with a non zero mean denoted by μ_l and variance $\zeta_{l,0}^2$ [1]. The other taps are considered as complex Gaussian random variables too, but their mean is zero and their variance is $\zeta_{l,i}^2$, $i \in \{1, ..., M-1\}$. Finally, we make the common assumption that the different taps are uncorrelated. All these assumptions can be summarized as $\mathbf{h}_l(j) \sim \mathcal{CN}(\boldsymbol{\mu}, \Sigma_l)$, where $\mathcal{CN}(\boldsymbol{\mu}, \Sigma_l)$ corresponds to the multi-variate complex normal distribution with mean $\boldsymbol{\mu} = [\mu_l, 0, ..., 0]^T$ and covariance matrix $\Sigma_l := \text{diag}_{M\times M}(\zeta_{l,0}^2, ..., \zeta_{l,M-1}^2)$.

With these notations, we can express the signal received on the nth subcarrier of link l at OFDMA symbol i as

$$Y_l(i,n) = \sqrt{PL_l} H_l(i,n) X_l(i,n) + Z_l(i,n), \qquad (1)$$

where PL_l is a deterministic power attenuation coefficient which depends on the distance between the emitter and the receiver, $\mathbf{H}_l(i) := [H_l(i,0), ..., H_l(i, N_c-1)]^T$ denotes the Fourier transform of $\mathbf{h}_l(i)$, $X_l(i,n)$ is the transmitted symbol on the nth subcarrier of the ith OFDMA symbol and $Z_l(i,n) \sim \mathcal{CN}(0, N_0 B/N_c)$, with N_0 the noise power spectral density. The elements of $\mathbf{H}_l(i)$ are identically distributed random variables $H_l(i,n) \sim \mathcal{CN}(\mu_{F,l,n}, \zeta_l^2)$ where $\zeta_l^2 := \text{Tr}(\Sigma_l)$ and $\mu_{F,l,n} := \mu_l e^{-j2\pi n/N_c}$. Therefore, it is known that $|H_l(i,n)|$ follows a Rician distribution with parameters $\Omega_l := |\mu_l|^2 + \zeta_l^2$ and $K_l := |\mu_l|^2/\zeta_l^2$ [14]. The Rician K_l parameter is known to be an important indicator of the link quality, indeed, $K_l = 0$ corresponds to the Rayleigh fading while $K_l \to \infty$ corresponds to the AWGN channel. Then, the higher the value of K_l, the better the quality of the channel.

We define the average Gain-to-Noise ratio (GNR) as:

$$G_l := \frac{PL_l \mathbb{E}[|H_l(i,n)|^2]}{N_0} = \frac{PL_l \Omega_l}{N_0}. \qquad (2)$$

Without any loss of generality, we assume that for all l, $\Omega_l = 1$.

As said previously, we assume that the CH only has statistical knowledge on the channel, more precisely, the CH is assumed to know only the average GNR and the Rician K_l factor of each link to perform the RA. Due to this assumption, the power allocated on each subcarrier is identical since the subcarriers are identically distributed, and we define $P_l := \mathbb{E}[|X_l(i,n)|^2]$ as the power allocated to link l to transmit one modulated symbol on one subcarrier. Moreover, we define n_l as the number of subcarrier allocated to link l and $\gamma_l := N_c/n_o$ as the associated proportion of bandwidth. It can then easily be seen that the average power consumed by link l to transmit one OFDMA symbol is $N_c \gamma_l E_l$, where

$$E_l := N_c \frac{P_l}{B}. \qquad (3)$$

The average Signal-to-Noise Ratio (SNR) of link l is given by

$$\overline{\text{snr}}_l := G_l E_l. \qquad (4)$$

We assume that each link in the cluster uses a Type-I HARQ mechanism, that is, for the link l, the stream of information bits is arranged into packets of L_l bits, and $L_{0,l}$ overhead bits, including a Cyclic Redundancy Check (CRC), are added in each packet. After that, a Medium Access Control (MAC) packet is obtained by encoding these $L_{M,l} := L_l + L_{0,l}$ bits by a FEC of rate R_l, and the MAC packet is modulated. We assume the use of either the BSPK modulation, of a 2^{m_l}-QAM modulation on link l. The modulated packet is then sent on the channel and, after reception, the receiver decodes the packet, and check the validity of the received bits using the CRC, which is assumed to be error-free. If an error is detected, the receiver discards the packet, and sends a Negative ACKnowledgement (NACK) to the transmitter, who retransmits the same packet. Otherwise, if the packet is successfully received, an ACKnowledgement (ACK) is sent to the transmitter, who sends another MAC packet. The mechanism is repeated until an ACK is sent by the receiver, or the maximum number of transmissions, denoted by P, is reached. Notice that we consider the ACK/NACK as error free, which is a common assumption in the literature [15], [16].

III. Approximation of the Packet Error Rate

One of the major difficulty when dealing with HARQ with practical MCS is the absence of closed-form expression for the coded PER. In the literature, some approximations can be found [17], [18], but they are in general valid only on frequency-flat block fading channel i.e. when the channel is modelled as a single coefficient which remains constant within the duration of a packet. In [15], [16], the notion of effective SNR is exploited to perform the RA, but these effective SNR methods require either instantaneous CSI, or at least outdated CSI, which is not available in our study. In [19], it is proposed to upper bound the PER to perform the RA based on statistical CSI in the case of the Rayleigh channel. In the present paper, we want to extend this study by considering the more general case of the Rician fading channel. To our best knowledge, this problem has never been addressed in the literature.

To approximate the PER in the Rician channel, we resort to an approach originally proposed in [20], where two observations are drawn concerning the relation between the PER at the output of the Viterbi decoder and the uncoded Bit Error Rate (BER) at the input of the decoder: *i)* this relation is almost independent of the considered modulation and more importantly *ii)* the relation is approximately linear in the logarithmic domain. From these two observations, we can approximate the actual PER of link l, $q_l(E_l)$, by $\tilde{q}_l(E_l)$ expressed as follows

$$\tilde{q}_l(\overline{\text{snr}}_l) = (\text{BER}_l(\overline{\text{snr}}_l))^{a_l} e^{b_l}, \tag{5}$$

where a_l and b_l are fitting coefficients which depend on the packet length, on the convolutional code parameters and on the Rician K_l parameter, BER_l is the BER of the lth link over the Rician fading channel with parameter K_l and $\overline{\text{snr}}_l$ is given by (4). Notice that this type of approximation has been used for the RA in the single user context in [9], where full CSI is considered to be available at the transmitter side, whereas our work considers that only statistical CSI is available.

From (5), we see that the expression of $\tilde{q}_l$ involves the BER of the modulation of the lth link in the Rician fading channel. It is well known that the BER over a fading channel is obtained by averaging the BER in the AWGN channel over all the possible values of the SNR. To perform this operation, we first express the instantaneous SNR on one subcarrier of the lth link which is given by

$$\text{snr}_l := |h_l|^2 \overline{\text{snr}}_l, \tag{6}$$

where $h_l \sim \mathcal{CN}(\mu_l, \zeta_l^2)$. Hence, from the above discussion, the average BER of link l writes

$$\text{BER}_l(\overline{\text{snr}}_l) = \mathbb{E}_{\text{snr}_l}[\text{BER}_{l,\text{AWGN}}(\text{snr}_l)], \tag{7}$$

where $\mathbb{E}_{\text{snr}_l}[.]$ is the mathematical expectation taken over the possible values of snr_l, and $\text{BER}_{l,\text{AWGN}}$ is the BER of the lth link in the Additive White Gaussian Noise (AWGN) channel. A common approximation of $\text{BER}_{l,\text{AWGN}}$ is

$$\text{BER}_{l,\text{AWGN}}(\text{snr}_l) \approx \psi_l Q(\sqrt{\beta_l \text{snr}_l}), \tag{8}$$

where ψ_l and β_l are modulation-dependent parameters whose values can be found in Table 6.1 in [21], and $Q(.)$ is the Q-function. Calculating the exact value of the expectation in (7) appears to be difficult since it would involve numerical integrations due to the presence of the Q-function. For this reason, we propose to approximate the Q-function by a combination of exponentials as suggested for example in [22] or [23]

$$Q(x) \approx \sum_{i=1}^{i_{max}} \delta_i e^{-\theta_i x^2}, \tag{9}$$

where δ_i and θ_i are fitting coefficients and i_{max} is the number of exponentials in the sum. The larger the value of i_{max}, the better the approximation. In this paper, we used the coefficients proposed in [23], where $i_{max} = 4$. The expectation in (7) can be then approximated as

$$\text{BER}_l(\overline{\text{snr}}_l) \approx \psi_l \sum_{i=1}^{i_4} \delta_i \mathbb{E}_{\text{snr}_l}[e^{-\theta_i \beta_l \text{snr}_l}]. \tag{10}$$

The expectation in (10) is exactly the moment generating function of the distribution of snr_l evaluated in $-\theta_i\beta_l$. We can easily prove that $Y_l := 2/(\overline{\text{snr}}_l \zeta_l^2)\text{snr}_l$ follows a noncentral chi-square distribution with 2 degrees of freedom and non-centrality parameter $|\mu|_l^2/\zeta_l^2$, which yields [24]:

$$\text{BER}_l(\overline{\text{snr}}_l) \approx \psi_l \sum_{i=1}^{i_4} \delta_i \frac{e^{-A_{l,i}}}{1 + \theta_i \beta_l \overline{\text{snr}}_l \zeta_l^2}. \tag{11}$$

where $A_{l,i} := \theta_i \beta_l \overline{\text{snr}}_l |\mu_l|^2 / (1 + \theta_i \beta_l \overline{\text{snr}}_l \zeta_l^2)$. The PER can be then approximated by plugging (11) into (5). The accuracy of the proposed PER approximation is checked in Section VI where we can observe that the approximation is quiet accurate, and therefore can be used to predict the PER of the system with an analytically tractable expression.

IV. Problem Formulation

In this section, we formulate the optimization problem that we want to solve. The objective of this paper is to solve the problem of the minimization of the total transmit power under two QoS constraints: a constraint on the goodput (which is the minimum useful data rate) and a constraint on the maximum allowed transmit power. For Type-I HARQ, the goodput η_l of the lth user is given by [15]

$$\eta_l(G_lE_l, \gamma_l) := \gamma_l m_l R_l \alpha_l (1 - q_l(G_lE_l)), \tag{12}$$

where $\alpha_l := L_{M,l}/L_l$. The problem that we want to solve can be then written as

Problem 1.

$$\min_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{l=1}^{L} \gamma_l E_l \tag{13}$$

$$\text{s.t.} \quad \gamma_l m_l R_l \alpha_l (1 - q_l(G_lE_l)) \geq \eta_l^{(0)}, \quad \forall l, \tag{14}$$

$$\gamma_l E_l \leq Q_{Max,l}, \quad \forall l, \tag{15}$$

$$\sum_{l=1}^{L} \gamma_l \leq 1, \tag{16}$$

$$\gamma_l > 0, E_l > 0, \quad \forall l, \tag{17}$$

where $\mathbf{E} := [E_1, \cdots, E_K]$ *and* $\boldsymbol{\gamma} := [\gamma_1, \cdots, \gamma_K]$.

In this problem, (14) is the minimum goodput constraint, (15) is the constraint on the maximum transmit energy, (16) is a structural constraint on the bandwidth and (17) are positivity constraints. One can see that this problem is identical to the one in [12], except that we add the constraint (15) and that we consider a different expression of the PER which takes into account the Rician channel.

As discussed in Section III, there is no analytical expression for $q_l(G_lE_l)$ in the coded case, therefore, we can not directly solve Problem 1. For this reason, we will not directly tackle this problem but an approximation, where we replace $q_l(G_lE_l)$ by $\tilde{q}_l(G_lE_l)$ given in (5). The resulting problem writes

Problem 2.

$$\min_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{l=1}^{L} \gamma_l E_l \tag{18}$$

$$\text{s.t.} \quad m_l R_l \gamma_l \alpha_l (1 - \tilde{q}_l(G_lE_l)) \geq \eta_l^{(0)}, \quad \forall l, \tag{19}$$

$$(15), (16), (17),$$

Our objective is now to solve Problem 2. A feasibility condition for this problem is given in [11] and, for the rest of this paper, we consider that this feasibility condition is satisfied. Actually, Problem 2 appears to be difficult to solve since the objective function (18) and constraint (15) are neither concave nor convex. Therefore, to render it convex, we first perform the following change of variable:

$$\forall l, Q_l = \gamma_l E_l. \tag{20}$$

After this change of variables, Problem 2 can be equivalently rewritten in the following form, which will be proved to be convex

Problem 3.

$$\min_{\mathbf{Q},\boldsymbol{\gamma}} \quad \sum_{l=1}^{L} Q_l \tag{21}$$

$$\text{s.t.} \quad m_l R_l \gamma_l \alpha_l (1 - \tilde{q}(G_l \frac{Q_l}{\gamma_l})) \geq \eta_l^{(0)}, \quad \forall l \tag{22}$$

$$Q_l \leq Q_{Max,l}, \quad \forall l \tag{23}$$

$$\sum_{l=1}^{L} \gamma_l \leq 1 \tag{24}$$

$$Q_l > 0, \gamma_l > 0, \quad \forall l \tag{25}$$

where $\mathbf{Q} = [Q_1, ..., Q_L]$

This problem is characterized by the following lemma.

Lemma 1. *Problem 3 is the minimization of a convex function over a convex set.*

Proof: First, we compute the second order derivative of $\log(f(x))$ where $f(x) := \exp(-ax/(1+2bx))/(1+2bx)$ where a and b are non negative constants:

$$\log(f(x))'' = \frac{4ab}{(1+2bx)^3} + \frac{4b^2}{(1+2bx)^2} > 0. \tag{26}$$

Therefore, f is strictly log convex and hence stricly convex [25]. It follows that $\tilde{q}_l$ is convex since it can be expressed as $g(x)^u$ where $g(x)$ is a non negative linear combination of convex function, and $u \geq 1$. Then, since $\tilde{\eta}_l(Q_l, \gamma_l) := \gamma_l(1 - \tilde{q}_l(G_lQ_l/\gamma_l))$ is the so-called *perspective* function of $\tilde{q}(G_lE_l)$ [25] and since the perspective of a concave function is concave, it results that $\tilde{\eta}_l(Q_l, \gamma_l)$ is concave. Finally, the other constraints (15), (16), (17) and the objective function (21) are linear and as a consequence convex. ■

Thanks to the change of variables (20), our Problem 3 falls within the convex optimization framework. In the next section, we will explain how to optimally solve this problem.

V. Optimal Problem Resolution

Since Problem 3 is a standard convex minimization problem due to Lemma 1, we know that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient to find its optimal solution [25]. These conditions write

$$1+\kappa_l+\mu_l G_l \tilde{q}_l'\left(\frac{G_l Q_l}{\gamma_l}\right)=0, \tag{27}$$

$$\lambda+\mu_l\left(-\left(1-\tilde{q}_l\left(\frac{Q_l}{\gamma_l}\right)\right)-G_l\frac{Q_l}{\gamma_l}\tilde{q}_l'\left(G_l\frac{Q_l}{\gamma_l}\right)\right)=0, \tag{28}$$

$$\kappa_l(Q_l-Q_{Max,l})=0, \tag{29}$$

$$\mu_l\left(\eta_l^{(0)}-\gamma_l m_l R_l\alpha_l\left(1-\tilde{q}\left(G_l\frac{Q_l}{\gamma_l}\right)\right)\right)=0, \tag{30}$$

$$\lambda(\sum_{l=1}^{L}\gamma_l-1)=0, \tag{31}$$

where λ, $\boldsymbol{\mu}=[\mu_1,...,\mu_L]$ and $\boldsymbol{\kappa}=[\kappa_1,...,\kappa_L]$ are the non negative Lagrangian multipliers. To solve this problem, we will proceed in two steps: first, we will consider λ as known, and we will express the optimal solution as a function of this unique multiplier. Second, we will find the optimal value of λ. Let us begin with the first step.

A. Solution for fixed λ

In what follows, our objective is to express the solution of the KKT conditions as a function of the unique multiplier λ. Given (27), we can see that $\mu_l>0$ since $\kappa_l\geq 0$ and, $\forall x\geq 0$, $\tilde{q}_l'(x)\leq 0$. Then, (30) yields

$$\gamma_l^* m_l R_l\alpha_l\left(1-\tilde{q}_l\left(G_l\frac{Q_l^*}{\gamma_l^*}\right)\right)=\eta_l^{(0)}, \tag{32}$$

which gives us a first relation between γ_l^* and Q_l^*. To continue the resolution, we have to discuss about the possible values of κ_l.

Case 1): $\kappa_l=0$ In this case, from (27), we obtain

$$\mu_l=-\frac{1}{G_l\tilde{q}_l'\left(G_l\frac{Q_l^*}{\gamma_l^*}\right)}. \tag{33}$$

By plugging (33) into (28) and using (32), we obtain

$$\gamma_l^*(\lambda)=\frac{\eta_l^{(0)}}{m_l R_l\alpha_l(1-\tilde{q}_l(F_l^{-1}(\lambda G_l)))}, \tag{34}$$

$$Q_l^*(\lambda)=\frac{\gamma_l^*(\lambda)}{G_l}F_l^{-1}(\lambda G_l), \tag{35}$$

where $F_l(x):=-(1-\tilde{q}_l(x))/\tilde{q}_l'(x)-x$ and F_l^{-1} is the inverse of F_l with respect to the composition. We denote by $\overline{I}_\kappa$ the set of the index of the links for which $\kappa_l=0$.

Case 2): $\kappa_l>0$: The complementary slackness condition (29) gives us

$$Q_l^*=Q_{Max,l}, \tag{36}$$

and we can obtain γ_l^* thanks to (32). More precisely, let us define the following function

$$\mathcal{F}_l(x):=m_l R_l\alpha_l x\left(1-\tilde{q}_l\left(\frac{G_l Q_{Max,l}}{x}\right)\right)-\eta_l^{(0)}. \tag{37}$$

Let $\gamma_l^{(0)}$ be the smallest zero of $\mathcal{F}_l$ on $(0,1)$. Due to (32), we have $\gamma_l^*=\gamma_l^{(0)}$ when $\kappa_l>0$. Moreover, combining (27) with (28), we get the following inequality

$$\lambda>F_l\left(G_l\frac{Q_{Max,l}}{\gamma_l^{(0)}}\right). \tag{38}$$

The index of the links for which $\kappa_l>0$ (i.e. the links for which (38) holds) are grouped into I_κ.

Combining the two cases, the optimal solution of Problem 3 is given by

$$(\gamma_l^*(\lambda),Q_l^*(\lambda))=\begin{cases}((34),(35)) & \text{if } l\in\overline{I}_\kappa,\\ (\gamma_l^{(0)},Q_{Max,l}) & \text{Otherwise.}\end{cases}$$

We have then exhibited the optimal solution of Problem 3 as a function of the optimal Lagrangian multiplier λ. In what follows, our objective will be to find the optimal value of λ.

B. Search for the optimal value of λ

To find the optimal value of λ, we will use the complementary slackness condition (31). In detail, we form the sum of the optimal values of the bandwidth sharing variables γ_l, $l\in\{1,L\}$, for a given value of the Lagrangian multipliers λ, which can be formulated mathematically as

$$\Gamma(\Lambda):=\sum_{l\in I_\kappa}\gamma_l^{(0)}+\sum_{l\in\overline{I}_\kappa}\mathcal{G}_l(\Lambda), \tag{39}$$

where $\mathcal{G}_l(x):=\eta_l^{(0)}/(m_l R_l\alpha_l(1-\tilde{q}_l(F^{-1}(xG_l))))$. We can prove that Γ is continuous and non increasing. We use this property to find the optimal value of λ. Since λ is non negative, there are two possibilities: *i)* $\Gamma(0)<1$: in this case, we know that the optimal value of λ is zero since increasing λ will only decrease the value of Γ, which means that (31) will never be satisfied, and *ii)* $\Gamma(0)>1$: since Γ is non increasing, the optimal value of λ must be such that (31) is verified, i.e. $\Gamma(\lambda)=1$. Finally, putting all pieces together, the optimal solution of Problem 3 is expressed in the following theorem.

Theorem 1. *If $\Gamma(0)<1$, the optimal solution of Problem 3 is given by $\gamma_l^*=\mathcal{G}_l(0)$ and $Q_l^*=\gamma_l/G_lF^{-1}(0)$ for all l.*

Otherwise, this optimal solution is given by $\gamma_l^*=\mathcal{G}_l(\lambda)$ and $Q_l^*=\gamma_l/G_lF^{-1}(\lambda G_l)$ for the links in $\overline{I}_\kappa$ i.e. such that $\lambda\leq F_l(G_lQ_{Max,l}/\gamma_l^{(0)})$, and for the other links $\gamma_l^*=\gamma_l^{(0)}$ and $Q_l^*=Q_{Max,l}$, where λ is the solution of $\Gamma(\lambda)=1$ on $\mathbb{R}^{+*}$.

Theorem 1 states that the optimal solution of Problem 3 reduces to a line search of the optimal value of λ, indeed, either this optimal value is zero and the optimal solution of Problem 3 can be computed directly, or this optimal value is strictly positive and hence a line search has to be performed to solve $\Gamma(\lambda)=1$.

K_l	0	10
a_l	9.73	9.39
b_l	18.57	19.37

TABLE I: Fitting parameters for two values of K_l

VI. Numerical Results and Discussion

In this section, we first study the accuracy of the PER approximation proposed in Section III, and second, we investigate the results of the algorithm proposed in Section V.

The accuracy of the proposed PER approximation is illustrated on Fig. 1 and 2, where we implemented the convolutional code of generators polynomial $[171, 133]_8$ over a Rician fast fading channel, along with both the BPSK and the QPSK modulation. The fitting parameters we used are given in Table I. We can see that the PER approximation is close to the PER obtained by simulation, which confirms the usefulness of this approximation.

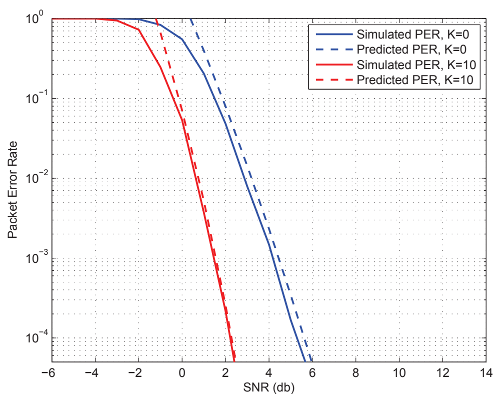

Fig. 1: PER obtained by simulation and with the approximation, BPSK modulation.

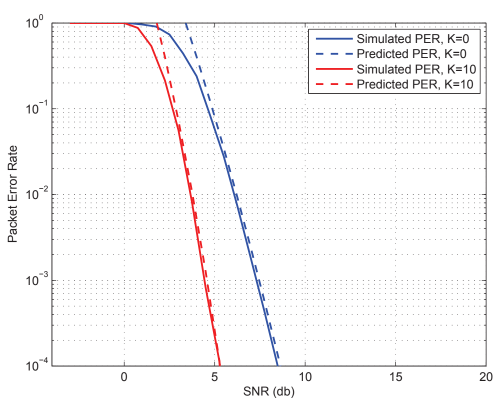

Fig. 2: PER obtained by simulation and with the approximation, QPSK modulation.

Concerning the RA algorithm, we set the number of links to 8, and each link is separated by a distance D_l uniformly drawn in $[50 \text{ m}, 1 \text{ km}]$. The bandwidth B is set to 5 MHz, the noise power spectral density N_0 to -170 dBm/Hz and the packet length L_l is 128. For simplicity, we consider $\alpha_l = 0$. The maximum allowed transmit power is set to 28 dBm and the required goodput is equal for all link, and is given by $G_p/(BL)$, where G_p is the target sum of the goodput. The carrier frequency is $f_c = 2\,400$ MHz and we put $PL_l = (4\pi f_c/c)^{-2} D_l^{-3}$ where c is the speed of light in vacuum. We performed 100 Monte Carlo simulations for each point. Finally, we use the same convolutional code as for Fig. 1 and 2 along with QPSK modulation.

To study the results of the proposed algorithm, we consider three scenarios: in the first one, the RA is performed by considering that every link experiences a Rayleigh channel, while, in the second scenario, we suppose that two of the L links experience a Rician channel with parameter $K_l = 10$ for the both. In the third scenario, all the links experience a Rician channel with parameter $K_l = 10$.

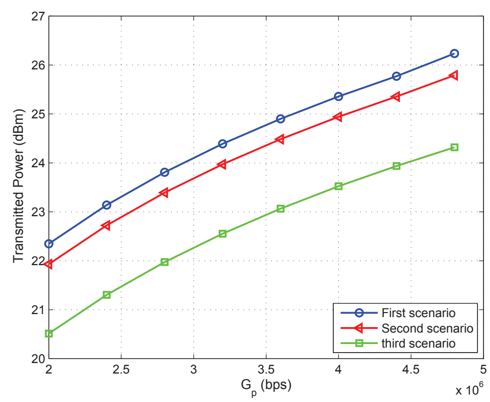

Fig. 3: Results of the proposed algorithm for various values of K_l

It can be seen from Fig. 3 that considering the Rician model in the RA process allows a significant energy saving. For instance, for $G_p = 3\,200$ kbps, 0.4 dBm (i.e. 10%) can be saved when two links are Rician (scenario 2) compared with scenario 1, while scenario 3 allows to save 1.9 dBm (i.e. 35%) compared with scenario 1. This can explained by the fact that the RA algorithm satisfies the goodput constraint with equality and, in the Rayleigh channel, more energy is required to achieve the minimum goodput constraint. Therefore, performing the RA by considering that the channel is Rayleigh distributed while its actual distribution is the Rician one induces a loss of energy, that is, considering the appropriate channel model is of great importance when performing the RA with statistical CSI.

VII. Conclusion

In this contribution, we have proposed an algorithm for the optimal RA problem for Type-I HARQ in multiuser military ad hoc networks based on OFDMA when the channel is Rician. We first proposed a simple yet accurate approximation of the

PER, and then we derived an algorithm which minimizes the total transmit power with several QoS constraints. Then, we conducted numerical simulations to illustrate the results of the proposed algorithm. These results highlight the importance of considering the appropriate channel model when performing the RA with statistical CSI. Indeed, we show that the transmit power can be significantly reduced when the RA is performed based on the actual distribution of the channel. In future work, we will study the generalization of the proposed approach to Type-II HARQ.

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