Robust Spectrum Sensing Algorithms Under Noise Uncertainty

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***Abstract*—In this paper, two spectrum sensing procedures are studied. The first one is based on a modification of the Energy Detector (ED) and the second one is based on a two-sample Goodness-of-Fit (GoF) test. The two procedures require a vector of noise samples, and are robust against noise uncertainty. Exact probability of detection are given for deterministic and Gaussian signal over additive white Gaussian noise (AWGN) and Rayleigh flat fading channel for the modified ED. Simulations show that the two procedures outperform previous schemes.**

I. Introduction

The objective of Spectrum Sensing is to detect whether a given frequency band is being used or not. Several spectrum sensing schemes have been developed in the literature. These schemes can be classified in two categories. The first category includes schemes that requires prior knowledge about the signal to detect (feature-based detector), such as cyclostationarity [1], matched filtering [2] and filter-bank techniques [3]. The procedures of the second category do not require any prior knowledge about the signal. This paper focuses on this later category, and especially on the Energy Detector (ED) [4] and on the two-sample Goodness-of-Fit (GoF) based techniques [5].

The ED is one of the most popular spectrum sensing scheme since its implementation is easy. However, the ED requires to know accurately the noise power [6]. When the noise power is unknown, it has to be estimated, leading to a non-Constant False Alarm Rate (CFAR) detector. Moreover, the exact threshold of the ED can be calculated only if the distribution of the noise energy is known. Recently, the authors of [7] have proposed a method called Generalized Energy Detector (GED), and proved that when the sample size goes to infinity, the GED is CFAR. Notice that the GED requires a set of noise samples.

The GoF procedures based on a distribution comparison are more recent. One of the first paper dealing with this approach is [8], where the authors use the well known Anderson-Darling (AD) test to detect a constant signal. In [5], the authors use a two-sample Kolmogorov Smirnov (KS) test. The Cramer Von Mises test has also been used for spectrum sensing purpose [9]. An approach based on Student t-test is proposed in [10]. In [11] and [12], the authors show that whereas the AD test is more powerful than the ED to detect a constant signal, the ED outperforms the AD test when the signal varies during time. In this study, we focus on the two-sample tests, such as proposed in [5] because these tests do not require knowledge on the noise distribution, unlike the other GoF-based methods. These techniques only require a set of noise only samples, as the GED. The drawback of GoF techniques is that they are generally less powerful than the ED.

In this paper, two configurations are investigated. First, the noise is assumed to be Gaussian with unknown power. Second, the statistical distribution of the noise is unknown. In both cases, it is assumed that two sets of data are available: a set of noise only, and a set of samples of the signal that we want to sense.

The contributions of this paper are the following. First, we show that there is a mismatch between the asymptotic distribution of the statistic of the GED proposed in [7] and its distribution when the sample size is finite. We derive the exact distribution of this statistic, and provide exact expressions of the probability of detection when the transmitted signal is deterministic or Gaussian for both the Additive White Gaussian Noise (AWGN) and the Rayleigh channels. To the authors' knowledge, these performance have not been derived yet. The second contribution of this paper is to propose the use of a GoF test developed in [13] in order to improve the one proposed in [5]. It is shown by simulation that this new test outperforms previous two-sample GoF schemes.

This paper is organized as follows. In section II, the problem statement is formulated. The GED is presented in section III, where its exact performance are derived. The two-sample GoF procedure is set out in section IV. Simulations are performed to compare the performance of the two methods with existing schemes in section V under Gaussian and non-Gaussian noise. Section VI concludes this paper.

II. Problem Statement

First, let N denote the number of samples available to perform the sensing, and $\mathbf{y} = (y[1], ..., y[N])$ denote a vector of complex received samples that we want to sense. Spectrum sensing problem can be formulated as a two hypothesis testing problem where the null hypothesis H_0 corresponds to the absence of signal, $\mathbf{y} = \mathbf{n}$ where $\mathbf{n} = (n[1], ..., n[N])$ is a vector of noise samples, while the alternative hypothesis H_1 corresponds to the presence of the sum of a noise process with a non-zero energy signal $\mathbf{s} = (s[1], ..., s[N])$ in this band

$\mathbf{y} = \mathbf{s} + \mathbf{n}$. The two hypothesis can be written as

$$H_0 : \mathbf{y} = \mathbf{n},$$
$$H_1 : \mathbf{y} = \mathbf{s} + \mathbf{n}.$$

As said in the introduction, it is assumed that aside from the set of received samples $\mathbf{y}$, M noise samples $\mathbf{n}_2 = (n_2[1], ..., n_2[M])$ are also available to perform the sensing. In practice, noise samples can be collected by listening to rarely used channels [5] for instance.

III. Generalized Energy Detector

A. Procedure

The ED procedure is based on the following statistic $S := \sum_{i=1}^{N} |y[i]|^2$. The statistic S is compared with a threshold T, and H_0 is rejected if $S > T$. In this section, the noise is assumed to be circular complex, white and Gaussian with zero-mean and variance $2\sigma^2$ in order to derive analytical results. Let χ_n^2 denote the chi-square distribution with n degrees of freedom, and $F_{\chi_n^2}$ its cumulative density function (cdf). Under H_0, S/σ^2 follows a χ_{2N}^2 distribution. Then, for a given probability of false alarm $P_{fa} = \alpha$, the threshold T is computed by:

$$T = \sigma^2 F_{\chi_{2N}^2}^{-1}(1 - \alpha), \tag{1}$$

where $F_{\chi_{2N}^2}^{-1}$ is the inverse of $F_{\chi_{2N}^2}$. However, it must be highlighted that the threshold T depends on σ^2. Therefore, when σ^2 is unknown, it has to be estimated from the noise samples $\mathbf{n}_2$. Nevertheless, if the noise power is estimated, the P_{fa} obtained by simulation does not match the target P_{fa} [5]. Moreover, the ED is not a CFAR detector when the noise power is estimated since the P_{fa} obtained by simulation varies with the number of samples used to estimate the variance (see Fig. 7).

An interesting modification of S taking into account the noise samples has been proposed in [7]. The authors proved that, under H_0, the GED is asymptotically normally distributed and independent of σ^2 (and thus CFAR) when the number of samples goes to infinity. They also provide the asymptotic probability of detection of a Gaussian signal over AWGN channel.

The GED considered in this paper is written as follows:

$$S_{GED} := \frac{M}{N}\frac{S}{B}, \tag{2}$$

where $B := \sum_{i=1}^{M} |n_2[i]|^2$. Note that (2) has a different normalization than the one in [7] for technical reasons explained later.

First, we prove that the GED is CFAR regardless of the number of samples. Since $n_2[i]$ is a circular zero-mean Gaussian random variable with variance $2\sigma^2$, it is possible to write $n_2[i] = \sigma \bar{n}_2[i]$, where $\bar{n}_2[i]$ is a circular Gaussian random variable whose real and imaginary part have unit variance. Similarly, under H_0, it is possible to write $y[i] = \sigma \bar{y}[i]$ where $\bar{y}[i]$ follows a circularly-symmetric complex Gaussian distribution whose real and imaginary part have unit variance. The statistics B and S can be written $B = \sigma^2 \bar{B}$ and $S = \sigma^2 \bar{S}$, where $\bar{B} = \sum_{i=1}^{M} |\bar{n}_2[i]|^2$ and $\bar{S} = \sum_{i=1}^{N} |\bar{y}[i]|^2$. The statistic $\bar{B}$ (resp. $\bar{S}$) follows a χ_{2M}^2 (resp. χ_{2N}^2) distribution, which is independent of σ. Therefore, the ratio of S and B, which is equal to the ratio between $\bar{S}$ and $\bar{B}$, does not depend on σ^2, and thus it leads to a CFAR detector whatever the value of M and N. Moreover, the ratio of two chi-square distributed random variables normalized by their respective degrees of freedom follows a F distribution [14], which explains the normalization done in (2).

More precisely, S_{GED} is $F_{2N,2M}$ distributed and its cdf, denoted by $F_{F_{2N,2M}}$, takes the following form [14]

$$F_{F_{2N,2M}} = I\left(\frac{Nx}{M + Nx}\Big| N; M\right), \tag{3}$$

where $I(.|.,.)$ is the incomplete regularized beta function. The threshold for a given $P_{fa} = \alpha$ can then be computed by

$$T_{GED} = F_{F_{2N,2M}}^{-1}(1 - \alpha), \tag{4}$$

where $F_{F_{2N,2M}}^{-1}$ is the inverse of $F_{F_{2N,2M}}$. The threshold T_{GED} does not depend on σ^2, which renders it CFAR as a consequence. This modification of ED should be of practical interest to achieve a constant false alarm rate. As said before, the authors of [7] proved that, under H_0, the GED statistic is asymptotically normally distributed when the sample size goes to infinity. However, when the sample size is finite, the distribution of S_{GED} is different from the normal distribution, as illustrated in Fig. 1. We also represent on the same figure the difference between the target P_{fa} and the P_{fa} obtained by simulation with the asymptotic and the exact distribution. It is clear that the P_{fa} obtained by simulation using the asymptotic distribution of S_{GED} is not exactly the target P_{fa}.

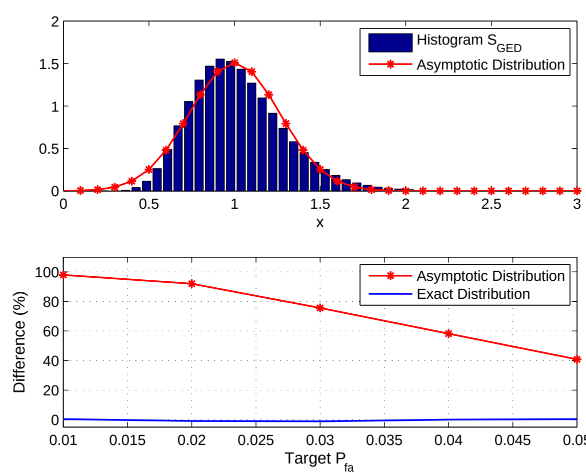

Fig. 1. Difference between asymptotic distribution and histogram of S_{GED} (top) and between P_{fa} obtained by simulation using asymptotic approximation and exact distribution (bottom), $N = 20$, $M = 50$, 10^6 trials.

B. Probability of Detection of a Deterministic Signal Over AWGN and Rayleigh Fading Channel

In this section, the signal samples $\mathbf{s}$ are considered as unknown and deterministic, with $\mathbf{s}\mathbf{s}^H = E$ where E is a constant and the subscript $(.)^H$ represents the transpose conjugate. The received samples when H_1 hypothesis is true are $y[i] = hs[i]+n[i]$, where h is the complex attenuation coefficient of the channel ($h = 1$ corresponds to AWGN channel). Under H_1 hypothesis, S/σ^2 has a non-central χ^2_{2N} distribution with non-centrality parameter $N_C(|h|^2) = |h|^2 E/\sigma^2$. It follows that S_{GED} has a non-central $F_{2N,2M}$ distribution with non-centrality parameter $N_C(|h|^2)$. Let $F_{\nu_1,\nu_2,\delta}(.)$ denote the cdf of the non-central F_{ν_1,ν_2} distribution with non-centrality parameter δ. The expression of $F_{\nu_1,\nu_2,\delta}(.)$ is [14]

$$F_{\nu_1,\nu_2,\delta}(x) = e^{-\frac{\delta}{2}} \sum_{j=0}^{\infty} \frac{\delta^j}{2^j j!} I\left(\frac{\nu_1 x}{\nu_2 + \nu_1 x} \Big| \frac{\nu_1}{2} + j; \frac{\nu_2}{2} \right). \quad (5)$$

The probability of detection P_d is then

$$P_d = 1 - F_{2N,2M,N_C(|h|^2)}(T_{GED}). \quad (6)$$

If the channel is AWGN, the probability of detection is given by (6) with $h = 1$. However, when the channel is flat fading, h varies from one realization to another. The average probability of detection is

$$\bar{P}_{d,det} = 1 - \mathbb{E}_{|h|^2}[F_{2N,2M,N_C(|h|^2)}(T_{GED})], \quad (7)$$

where $\mathbb{E}_{|h|^2}[.]$ denotes the mathematical expectation over $|h|^2$. Here, we consider the Rayleigh fading model [15]. The coefficient h follows a complex Gaussian distribution with zero-mean and variance $2\beta^2$, so $|h|^2$ follows an exponential distribution with parameters $1/(2\beta^2)$ [15]. Therefore, using (5) in (7), we have

$$\begin{aligned}\bar{P}_{d,det,R} = 1 \\ & - \frac{1}{2\beta^2} \sum_{j=0}^{\infty} \frac{\left(\frac{E}{2\sigma^2}\right)^j}{j!} g(T_{GED}, N, M, j). \\ & \int_0^{+\infty} x^j e^{-x\left(\frac{E}{2\sigma^2} + \frac{1}{2\beta^2}\right)} dx,\end{aligned} \quad (8)$$

where

$$g(T_{GED}, N, M, j) = I\left(\frac{NT_{GED}}{M + NT_{GED}} | N + j; M \right). \quad (9)$$

Direct calculation leads to

$$\bar{P}_{d,det,R} = 1 - \frac{1}{2\beta^2} \sum_{j=0}^{\infty} \frac{\left(\frac{E}{2\sigma^2}\right)^j}{C^{j+1}} g(T_{GED}, N, M, j), \quad (10)$$

where $C = E(2\sigma^2)^{-1} + (2\beta^2)^{-1}$.

C. Probability of Detection of Gaussian Signal Over AWGN and Rayleigh Fading Channel

The signal samples $s[i]$ are now considered as i.i.d zero-mean Gaussian random variables. The value $\mathbf{s}\mathbf{s}^H = E$ is not constant and varies between realizations. Without loss of generality, the real and imaginary part of the signal are considered to have unit variance. Under H_1, for a given realization, S_{GED} follows a non-central $F_{2N,2M}$ distribution with non-centrality parameter $N_C(|h|^2 E) = |h|^2 E/\sigma^2$. The probability of detection for one realization is then

$$P_d = 1 - F_{2N,2M,N_C(|h|^2 E)}(T_{GED}). \quad (11)$$

If the channel is AWGN, $|h| = 1$ and the average probability of detection is

$$\bar{P}_{d,g,AWGN} = 1 - \mathbb{E}_E\left[F_{2N,2M,N_C(S)}(T_{GED})\right], \quad (12)$$

where $\mathbb{E}_E[.]$ is the mathematical expectation over E. Since E follows a χ^2_{2N} distribution, calculation leads to

$$\begin{aligned}\bar{P}_{d,g,AWGN} = 1- \\ & \frac{1}{2^N \Gamma(N)} \sum_{j=0}^{\infty} \frac{\left(\frac{1}{2\sigma^2}\right)^j}{j! C_2^{j+N}} \Gamma(j+N). \\ & g(T_{GED}, N, M, j),\end{aligned} \quad (13)$$

where $C_2 = (2\sigma^2)^{-1} + 1/2$. Let us now consider a Rayleigh fading channel. Let A denote the product between $|h|^2/\beta^2$ and E. Since $|h|^2/\beta^2$ follows a χ^2_2 distribution and E follows a χ^2_{2N} distribution, and h and E are independent, A is the product of two independent χ^2 distributed random variables. The distribution of A is given by [16]

$$f_A(x) = \frac{x^{-\frac{1}{2}+\frac{N}{2}} K_{1-N}(\sqrt{x})}{2^N \Gamma(N)}, \quad (14)$$

where $K_\nu(.)$ is the modified Bessel function of the second kind of order ν.

The non-centrality parameter of S_{GED} is $\beta^2 A/\sigma^2$. The average probability of detection of a Gaussian signal over Rayleigh fading channel is, using the expression (5)

$$\begin{aligned}\bar{P}_{d,g,R} = 1- \\ & \mathbb{E}_A\left[\sum_{j=0}^{\infty} \frac{\left(\frac{\beta^2 A}{2\sigma^2}\right)^j}{j!} e^{-\frac{A\beta^2}{2\sigma^2}} g(T_{GED}, N, M, j) \right],\end{aligned} \quad (15)$$

where $\mathbb{E}_A[.]$ is the expectation over A. The equation (15) transforms into

$$\begin{aligned}\bar{P}_{d,g,R} = 1- \\ & \sum_{j=0}^{\infty} \frac{\left(\frac{\beta^2}{2\sigma^2}\right)^j}{j!} g(T_{GED}, N, M, j) \\ & \int_0^{+\infty} x^j e^{-x\frac{\beta^2}{2\sigma^2}} f_A(x) dx.\end{aligned} \quad (16)$$

After some algebra, using [17, eq. (6.643)], the average probability of detection is

$$\bar{P}_{d,g,R} = 1 - \frac{1}{2^N (N-1)!}\left(\frac{\beta^2}{2\sigma^2}\right)^{\frac{-N}{2}} e^{\frac{\sigma^2}{4\beta^2}} \sum_{j=0}^{\infty} g(T_{GED}, N, M, j). \Gamma(j+N) W_{-j-\frac{N}{2},\frac{1-N}{2}}\left(\frac{\sigma^2}{2\beta^2}\right), \tag{17}$$

where $W_{\lambda,\mu}(z)$ is the Whittaker function [17]. The probability of detection derived in this part have been validated through various simulations. These expressions should be useful for system designers to simply evaluate the performance of the GED.

IV. Two-Sample GoF Test

In this section, the assumption that the noise has a Gaussian distribution is relaxed. Two-sample GoF tests are non-parametric hypothesis tests, whose objective is to compare the distributions of two sets of data. These tests are based on the measurement of a distance between the empirical cdf of the two sets of data. The empirical cdf of a given set of data $(x_1, ..., x_N)$ is defined as

$$F_N(y) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{1}(x_i \leq y), \tag{18}$$

where $\mathbb{1}(A)$ is the indicator function whose value is one if A is true, and zero otherwise. The two-sample KS test is based on the statistic

$$D_{n,n'} = \sup_y |F_{1,n}(y) - F_{2,n'}(y)|, \tag{19}$$

where n (resp. n') is the number of samples in the first (resp. second) set of data, and $F_{1,n}(.)$ (resp. $F_{2,n'}(.)$) is the empirical cdf of the first (resp. second) set of data. The statistic $D_{n,n'}$ is compared with a threshold, and the hypothesis that the two sets the same distribution is rejected if $D_{n,n'}$ is greater than this threshold. The analytical expression of the threshold can be found in [5], where authors use a two-sample KS test to compare the distribution of samples from $\mathbf{y}$ and $\mathbf{n}_2$. Notice that the two-sample GoF test can be applied to both complex samples or their magnitude. We will consider here tests based on the samples magnitude since it has been shown by simulation in [7] that testing magnitudes allows better performance than testing complex samples.

In [13], Zhang proposes several two-sample tests that outperform the KS one and we thus investigate here the application of these tests to the spectrum sensing context. Especially, we focus on the test based on the statistic Z_C, which is defined as follows. First, let $\mathbf{x}_1 = (x_{1,1}, \cdots, x_{1,M})$ with $x_{1,i} = |n_2[i]|$, $\mathbf{x}_2 = (x_{2,1}, \cdots, x_{2,N})$ with $x_{2,i} = |y[i]|$, and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)$ of length $P = M + N$. Let $X_{1(j)}$ (resp. $X_{2(j)}$) denote the jth order observation of $x_{1,i}, i = 1, ..., M$ (resp. $x_{2,i}, i = 1, ..., N$) where the jth order observation of a set of data is the jth largest element of this set. Finally, let R_{kj} denote the rank of $X_{k(j)}$ in $\mathbf{x}$. The statistic Z_C is defined as, [13, eq. (9.3) pp. 42]

$$Z_C = \frac{1}{P}\sum_{k=1}^{2}\sum_{j=1}^{n_k} \log\left(\frac{n_k}{j-0.5} - 1\right)\log\left(\frac{P}{R_{kj}-0.5} - 1\right), \tag{20}$$

where $n_1 = M$ and $n_2 = N$. The statistic Z_C is compared with a threshold, and the hypothesis that $\mathbf{x}_1$ and $\mathbf{x}_2$ have the same distribution (H_0 hypothesis) is accepted if the statistic is greater than the threshold. Let G_0 denote the cdf of $\mathbf{x}_1$ and $\mathbf{x}_2$ under H_0. In [13], it is proved that the test statistic does not depend on G_0 which implies the threshold is insensitive to noise distribution and allows to compute a unique lookup table giving the threshold with respect to the target P_{fa}. For example, when $M = 50$, $N = 20$ and $P_{fa} = 0.01$, the threshold is 2.9214.

V. Simulation Results

In this section, the performance of the two proposed approaches are compared by simulations with the ED and with the two-sample test described in [5]. The channel is assumed to be AWGN, and the transmitted signal is complex and Gaussian. To perform the ED, the noise variance is estimated from the M noise samples. Let $\hat{\sigma}^2 = B/(2M)$ denote the estimation of the noise variance. The threshold of the ED is then computed substituting σ^2 by its estimation in (1). First, the noise is considered as Gaussian, then, this assumption is relaxed and a Gaussian mixture is considered.

A. Performance Under Gaussian Noise

1) Fixed P_{fa}: First, the P_{fa} is set to 0.01, with $N = 20$ and $M = 50$. The SNR in dB varies from -8 dB to 4 dB by step 2 dB. The number of trials for every values of the SNR is $20,000$. The probability of false alarm and the probability of detection obtained by simulation are shown as a function of the SNR in Fig. 2 and Fig. 3. As observed in [5], when the variance of the noise is unknown and has to be estimated, the resulting probability of false alarm using ED is higher than the target probability of false alarm. For this reason, we also represent the probability of detection of ED with a target P_{fa} of 0.0014 since this value allows the P_{fa} obtained by simulation to be 0.01.

It can be seen that the ED with the target $P_{fa} = 0.0014$ gives the same probability of detection as the GED. However, the GED allows to find analytically a threshold to achieve this P_{fa}. The GED outperforms the two-sample GoF tests. Moreover, the GED exhibits a very good agreement between theory and simulation. The two-sample GoF test based on Z_C outperforms the two-sample KS test.

2) ROC curve: In this section, the SNR has been fixed to 0 dB with $N = 20$ and $M = 50$. The P_{fa} varies between 0.01 and 0.1 by step 0.01. The number of trials for every values of the P_{fa} is $20,000$. The ROC curve is shown in Fig. 5 as a function of the target P_{fa}. As previously, the P_{fa} obtained by simulation for the ED is higher than target P_{fa} (Fig. 4).

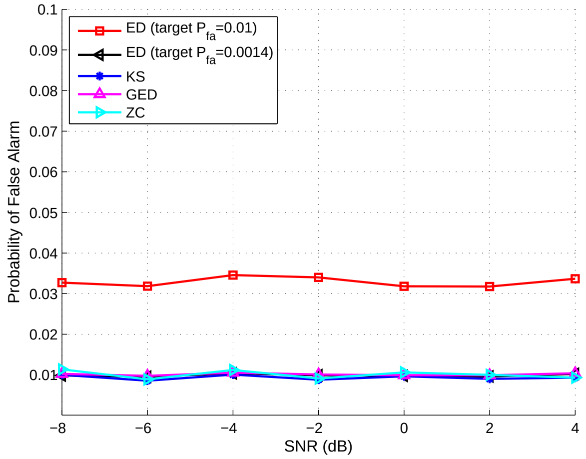

Fig. 2. Probability of false alarm obtained by simulation of different detectors with target $P_{fa} = 0.01$, $N = 20$ and $M = 50$.

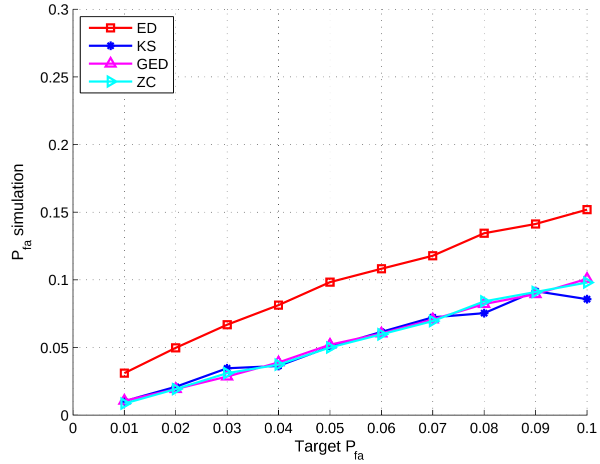

Fig. 4. Probability of false alarm of different detectors obtained by simulation with $N = 20$ and $M = 50$, SNR $= 0$ dB.

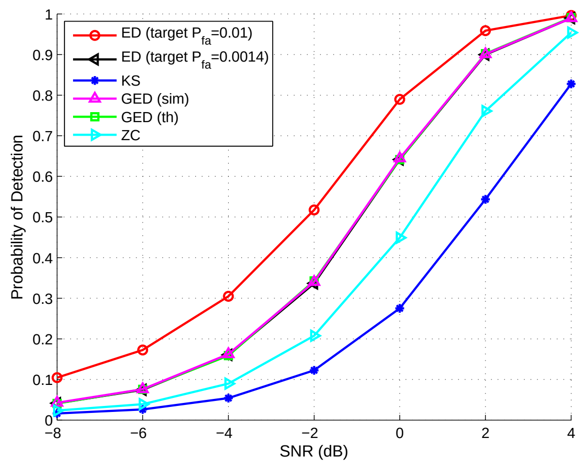

Fig. 3. Probability of detection of different detectors in AWGN channel with target $P_{fa} = 0.01$, $N = 20$ and $M = 50$.

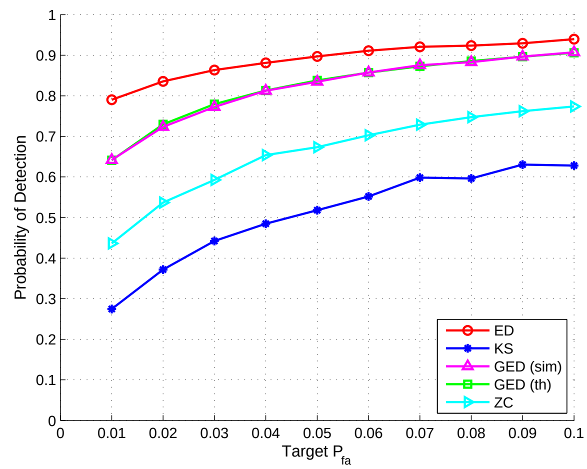

Fig. 5. ROC curve of different detectors in AWGN channel with $N = 20$ and $M = 50$, SNR $= 0$ dB.

For this reason, the probability of detection as a function of the P_{fa} obtained by simulation is also shown on Fig. 6.

Once again, the GED outperforms the two-sample GoF, and Z_C outperforms the two-sample KS test. Again, there is a very good agreement between theory and simulation for the GED.

3) Influence of M: The influence of M is studied. The value of M varies between 10 and 40 by step 5 and $20,000$ iterations are performed for every values of M. The P_{fa} is set to 0.01, and the SNR to 0 dB. Fig. 7 shows that the ED is not CFAR when the power of the noise is estimated since the P_{fa} obtained by simulation depends on M. The GED and Z_C outperform previous schemes for every values of M, as it can be seen on Fig. 8.

B. Performance Under Non-Gaussian Noise

In this section, the noise is assumed to be non-Gaussian. The considered distribution is a Gaussian mixture [18], which is sometimes used to model man-made noise [19]. The real and the imaginary part of the noise are i.i.d, according to the following mixture. The number of component in the mixture is set to 3. The means of the components are $\mu_1 = 0$, $\mu_2 = 4$ and $\mu_3 = 8$ respectively. The weights of the different components are $w_1 = 0.6$, $w_2 = 0.2$ and $w_3 = 0.2$ respectively. The variance is set to $\sigma_1^2 = 1$ for the first component, $\sigma_2^2 = 0.5$ for the second one and $\sigma_3^2 = 0.5$ for the third one. The probability density function (pdf) of the described Gaussian mixture is illustrated in Fig. 9. For the simulation, $20,000$ trials are performed for every value of the SNR. The target probability of false alarm is 0.01. The probability of detection and the probability of false alarm are shown in Fig. 10 and Fig. 11.

When the noise is non-Gaussian, the ED and the GED are unusable because they give P_{fa} higher than the target P_{fa}. It can be explained because the calculation of the thresholds for those two detectors is designed assuming Gaussian noise.

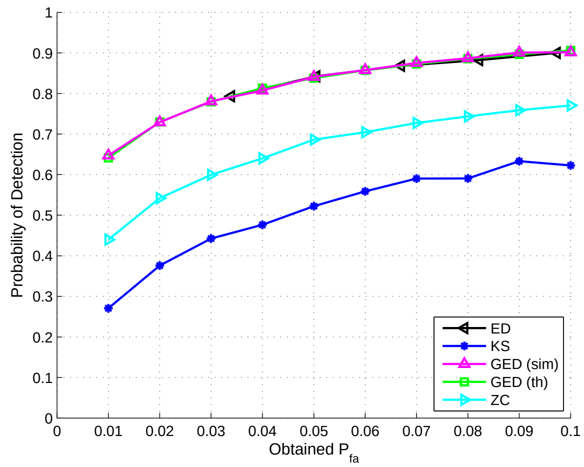

Fig. 6. ROC curve of different detectors in AWGN channel with $N = 20$ and $M = 50$, SNR $= 0$ dB.

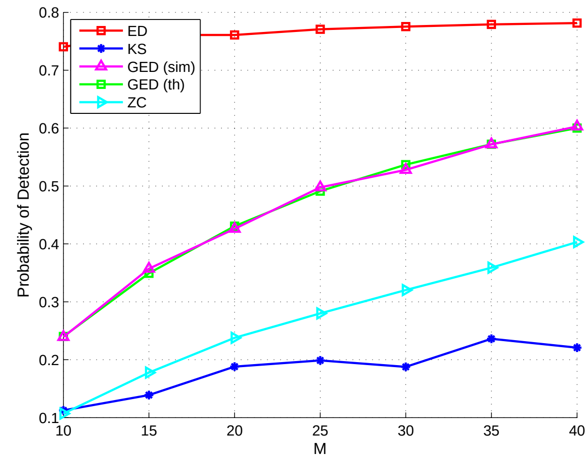

Fig. 8. Influence of M on the probability of Detection of different detectors, target $P_{fa} = 0.01$, $N = 20$, SNR $= 0$ dB.

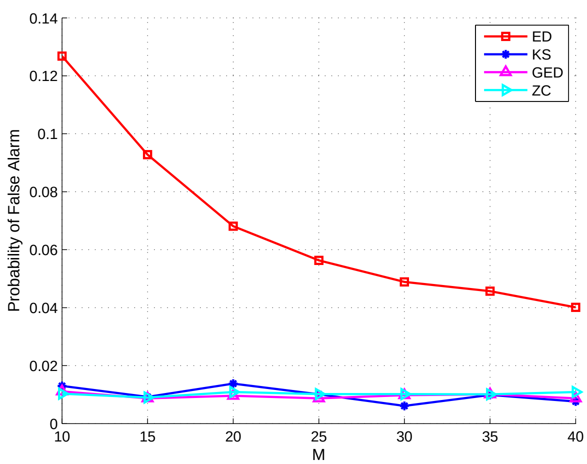

Fig. 7. Influence of M on the probability of false alarm of different detectors, target $P_{fa} = 0.01$, $N = 20$, SNR $= 0$ dB.

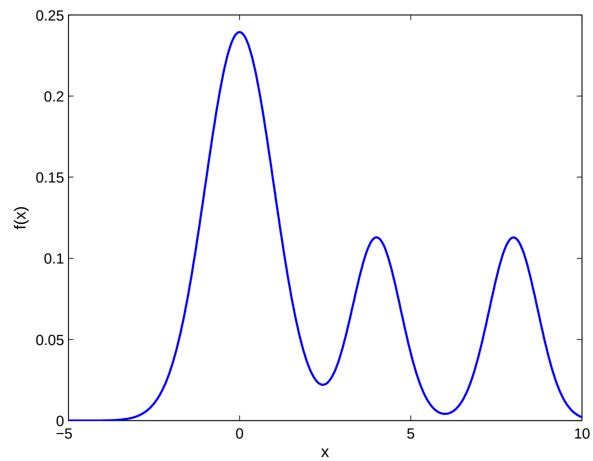

Fig. 9. Probability density function of the described Gaussian mixture.

However, the two-sample GoF tests do not assume any distribution for the noise. These two approaches are CFAR detectors even when the noise is non-Gaussian. It can be seen that the test based on Z_C outperforms the two-sample KS test.

VI. Conclusion

In this paper, we studied two CFAR detectors based on modified ED and two-sample GoF tests for blind spectrum sensing under the hypotheses that a vector of noise samples is available. When the noise is Gaussian, the threshold for the GED is simple to obtain, and this detector allows better performance than the two-sample GoF schemes on AWGN channel. The threshold for the GoF test has to be obtained by Monte Carlo simulation and is more computationally expensive. However, under non-Gaussian noise, the proposed GoF test outperforms the conventional one from literature.

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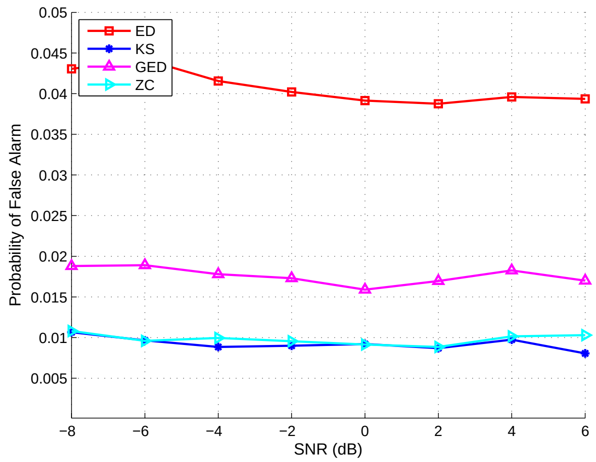

Fig. 10. Probability of false alarm obtained by simulation with target $P_{fa} = 0.01$ under non-Gaussian noise, $M = 50$, $N = 20$.

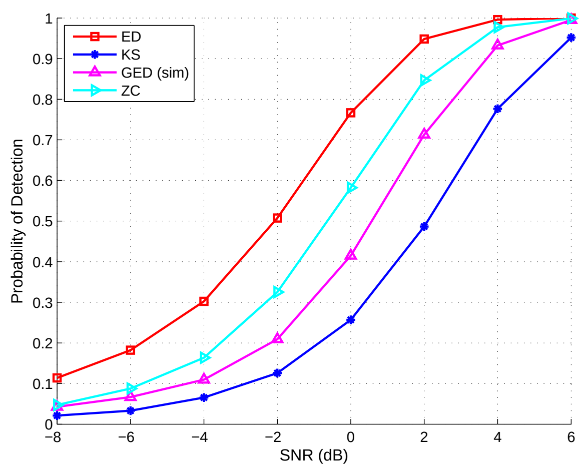

Fig. 11. Probability of detection of different detectors with target $P_{fa} = 0.01$ under non-Gaussian noise, $M = 50$, $N = 20$.

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