LOCAL INTERPOLATION IN MULTIRESOLUTION DECOMPOSITION OF IMAGES

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ABSTRACT

A simple interpolation method from low frequencies to high frequencies in a two-band filter bank is shown to be a very efficient way for enhancing signal zooming or for erasing blocking artifacts.

1. INTRODUCTION

The multiresolution decomposition of images has proven to be a very effective transformation in image coding. Its decorrelative and spatial properties have allowed to obtain better compression results than orthogonal block transformations as the well-known DCT. The filter design has been paid much attention from the research community leading to very good results [1]. However, residual spatial redundancies are still present across the different frequency bands, mainly at edges and in textured areas. Unfortunately, they cannot be eliminated via filter optimization. Several solutions have been proposed in order to have that spatial redundancy decreased. One of these consists in introducing spatially variant filters [3] [5] in order to adapt locally the filter characteristics to the signal content .

This paper introduces a new and simple way for generating that spatial adaptability of the filters. It proposes to interpolate the high-frequency subband of a two-band filter bank from the low frequencies. It will be shown at first that a linear interpolation between the two subbands can be reduced to a modification of the filters. From this result, a local adaptation of the filters will be envisaged for two applications: signal zooming and reduction of blocking artifacts in multigrid decomposition of images [2].

2. MULTIRESOLUTION INTERPOLATION

The separable multiresolution analysis of images is obtained by iterating the two-band subband decomposition on the resulting low-frequency signal, successively along the lines and the columns [6]. So, the transformed signal is composed of a low-frequency component together with high-frequency subbands distributed according to a logarithmic scale along the frequency axes (Figure 5). In the remainder of the paper, the developments will be explained for unidimensional decompositions, the extension to separable image transformations being straightforward.

The multiresolution transformation decorrelates effectively images as long as they can be considered as a stationary random process. However a spatial correlation remains across the successive frequency bands, mainly at edges and in textured areas (Figure 5). A solution for decreasing this redundancy consists in introducing a linear interpolation filter between the two channels of a two-band filter bank as shown in Figure 1. Thanks to the symmetrical subtraction-addition structure of this analysis/synthesis system, its eventual perfect reconstruction property is preserved.

It can be shown that the filter bank together with its interpolation structure is equivalent to another linear filter bank as depicted in Figure 2. Therefore the optimization of the filter $F(z)$ will not be more effective than the global optimization of the equivalent filter bank. The interest in the structure shown in Figure 1 lies on:

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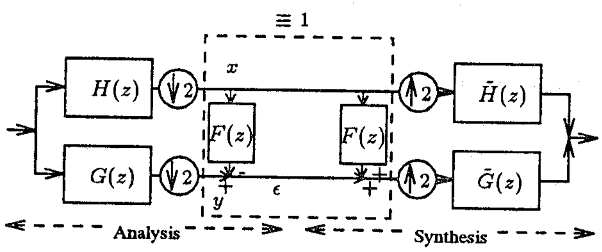

Figure 1: Interpolation filters in the filter bank.

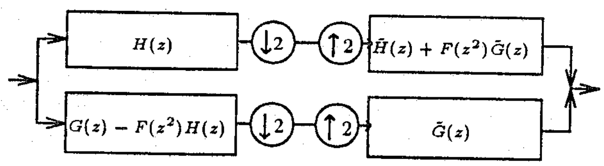

Figure 2: Equivalent filter bank.

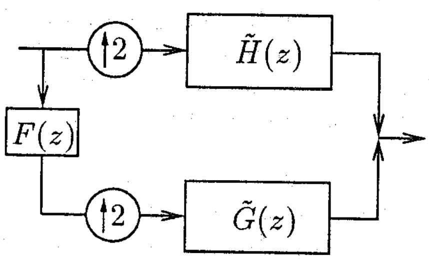

Figure 3: A zooming structure based on the scheme of Figure 1.

1. The ability to change locally the interpolation filter $F(z)$ and consequently to *modulate* the filter bank. It is worth noting that, given a perfect reconstruction filter bank (Figure 1), the interpolation filter $F(z)$ can be optimized, even locally, without loosing the perfect reconstruction property while classical time-varying systems impose to optimize the filter bank globally (Figure 2), keeping the constraint in mind.

2. The opportunity of an interpolation process at the synthesis when the high-frequency content has been lost or corrupted locally (as in multigrid decompositions) or globally (as in signal zooming). As no interpolation is required at the analysis, the resulting filter bank is not symmetric anymore and only the filter $\tilde{H}(z)$ is changed.

2.1. Filter optimization

2.1.1. Local adaptation of the filter bank

In the case of an adaptation of the filter bank thanks to the introduction of an interpolation filter, each pixel is to be associated to a particular interpolation filter $F_i(z)$. If we denote $x(i)$ and $y(i)$ the low and high-frequency coefficients respectively, $F_i(z)$ can be computed as the filter that minimizes the power of the interpolation error denoted $\epsilon(i)$ and expressed by

$$\epsilon(i) = y(i) - \sum_k x(i-k) f_i(k) \qquad (1)$$

Then the coefficients of each interpolation filter are found as the solution of the following system

$$\frac{\partial \epsilon^2(i)}{\partial f_i(k)} = 0 \quad \forall\, k$$

2.1.2. Statistical adaptation of the interpolation filter

In practical implementations, the number of interpolation filters must be limited. To fulfill this requirement, the pixels are classified in a few clusters according to their local statistics. Then an interpolation filter $F_i(z)$ is to be optimized for each cluster so defined by solving the following system:

$$\frac{\partial \sigma_\epsilon^2}{\partial f_i(k)} = 0 \quad \forall\, k \qquad (2)$$

The statistical clustering can be made general enough to allow the use of the same bank of interpolation filters for all the images.

3. DISCUSSION AND APPLICATIONS

3.1. Improving the coding efficiency

The interpolation structure of Figure 1 with local adaptation of the filter bank has shown very poor entropy reductions ($\sim 10\%$). Two reasons can be invoked:

1. The interpolation filter is placed after the subsamplers and suffers from the aliasing present at that point.
2. But the main reason lies in the fact that the interpolation decreases the largest high-frequency coefficients only. These are very important from a visual point of view but do not contribute a lot to the entropy of the high-frequency subband (Laplace distribution).

Eventually, as the objective entropy gain is poor, this structure will not be useful as a compression tool. However, its ability to predict visual characteristics, as edges, allows us to use it in other applications as signal zooming or reduction of blocking artifacts in multigrid decompositions of images.

3.2. Signal zooming

Signal zooming means the increase of the resolution thanks to the addition of **a priori unknown** detail information. The easiest way to zoom a signal consists in the replication of the existing pixels. This approach can be interpreted in the multiresolution framework as the insertion of zero-valued high-frequency subbands into Haar synthesis filters. The interpolation structure presented in this paper can be used to guess, at the synthesis, the unknown high-frequency coefficients instead of inserting zeros, what will result in an improvement of the contours quality. Of course, we could use any other type of filters than Haar filters what results in Figure 3.

We should optimize the interpolation filters by using (1) and (2), what is not possible. Actually, by definition, we do not know the detail information which we would like to guess. Thus, the error (1) can never be expressed. Nevertheless, the similarity that we can find between two successive low-pass approximations of a multiresolution analysis lets us suggest to use the same interpolation bank along the whole zooming process. These filters would have been computed on the basis of a single decomposition. This approach can be associated to the identification of self-similarity inside an image. A local property is identified at a given scale and propagates through the various resolutions via the interpolation process.

Results are shown in Figure 4, the top left image is the reconstruction of Lena from the single low-pass subband of a 3-stage multiresolution decomposition using the Haar filter bank (all the details were put to zero) while the top right image results from the adjunct of the proposed interpolation method where we divided the pixels in two clusters only (contours or not contours).

3.3. Postprocessing in multigrid decomposition of images

The multigrid decomposition of images [2] (quadtree or binary tree) can also be interpreted in the multiresolution framework. Actually this transformation can equivalently be defined as a non-uniform quantization of the high-frequency coefficients of a Haar multiresolution analysis. A coarse quantization will lead to blocking artifacts that could be reduced thanks to the use of adapted filters. Actually, a correction can be brought to these high-frequency coefficients thanks to an interpolation from the low to the high-frequency band. The results related to the application of this method are illustrated in Figure 4 that shows the initial image to be postprocessed (at the bottom left) and the postprocessed image (at the bottom right). It is worth noticing the reduced blocking effects while we used the same clustering and the same interpolation filter as for zooming.

We can note that the same structure can be used to reduce the effect of a coarse quantization in a classical multiresolution compression scheme.

4. CONCLUSION

We have shown that the combination of a two-band filter bank with a simple subtractive-additive interpolation structure is equivalent to a modulated two-band filter bank. The local optimization of the interpolation filter enables the prediction of the highest frequencies, i.e. the contours, which are important visual features. If this structure is not very useful for compression purposes, it can be used fruitfully for zooming or image enhancement since a very rough optimization leads to noticeable performances.

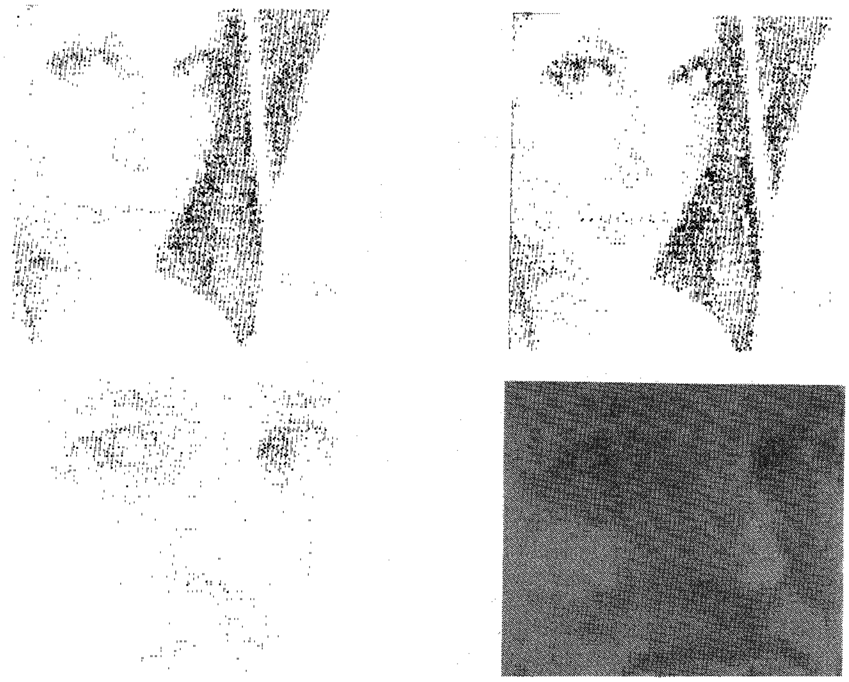

Figure 4: Examples of zoom (top) and postprocessing (bottom) results

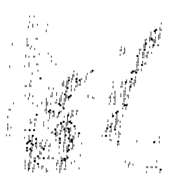

Figure 5: A multiresolution decomposition showing spatial redundancy across subbands.

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