Energy Efficient Resource Allocation for HARQ with Statistical CSI in Multiuser Ad Hoc Networks

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***Abstract*—We address the problem of energy efficient power and bandwidth allocation for Hybrid Automatic Repeat reQuest (HARQ) in multiuser ad hoc networks with statistical Channel State Information (CSI) and practical Modulation and Coding Schemes (MCS) under Quality of Service (QoS) requirements. Using an upper-bound of the packet error rate, we propose an algorithm to maximize the Sum of the Energy Efficiency (SEE) of the different users. We also propose two suboptimal less-complex algorithms, one based on alternating optimization and the other on a high Signal-to-Noise Ratio (SNR) approximation. It is shown that the proposed algorithms allow a substantial gain in term of SEE compared with conventional algorithms.**

I. Introduction

Hybrid Automatic Repeat reQuest (HARQ) is a powerful mechanism that combines Forward Error Correction (FEC) and Automatic Repeat reQuest protocol (ARQ), which allows to take advantage of temporal diversity to ensure high quality communications over time varying channel. The HARQ mechanism is now well studied, and is used in different standards such as 4G Long Term Evolution (LTE). This paper focuses on Resource Allocation (RA) for HARQ in a multiuser context assuming the use of Orthogonal Frequency Division Multiple Access (OFDMA) as the multi access technology. We also consider the use of practical Modulation and Coding Schemes (MCS), and we assume that only statistical Channel State Information (CSI) is available to perform the RA [1].

When dealing with RA, it is important to choose a relevant optimization criterion. Recently, a metric called Energy Efficiency (EE), which is defined as the ratio between the efficiency and the energy consumed per transmitted bit [2], has gained interest in the scientific literature. This metric allows to capture the best achievable trade-off between data rate and energy consumption [3]. The present paper uses the EE as the criterion to optimize, and more precisely, our objective is to maximize the Sum of the EE (SEE) of the different users.

In the single user context, the EE of HARQ has been analyzed by considering capacity-achieving codes and so using information-theoretic tools in [4]–[7] or by considering practical MCS in [8]–[12]. For instance, in [11], the authors optimize the power distribution between the retransmission rounds offline based on statistical CSI.

In the multiuser context, the RA problem for HARQ has been especially addressed from an information theoretic point of view in [13]–[15] and from a practical point of view, *i.e.* with realistic MCS and statistical CSI, in [1], [16], [17]. More precisely, only [14], [15], the authors focus on metrics related to EE. For instance, in [14], the authors maximize the EE of Incremental Redundancy (IR) HARQ with respect to the power and bandwidth when channels are perfectly known by the transmitters. But, to our knowledge, there is no work which focus on the maximization of the EE for HARQ considering practical MCS and statistical CSI in the multiuser context. Therefore the objective of this paper is to start investigating this problem and to propose algorithms to maximize the SEE under Quality of Service (QoS) constraints. In this contribution, we derive the optimal algorithm and two suboptimal algorithms with less-complexity. These proposed algorithms rely on Geometric Programming (GP) and fractional programming. Numerical simulations show that the proposed algorithms yield substantial gains on the SEE compared with the algorithm which minimizes the transmit power [17] and the one which maximizes the sum of the goodput. In addition, we take into account the circuitry consumption in our derivations [10].

The rest of the paper is organized as follows. In Section II, we present the system model. Section III is devoted to the problem formulation, and Section IV to the optimal problem resolution. The two suboptimal algorithms are proposed in Section V, and numerical results are discussed in Section VI. Finally, concluding remarks are drawn in Section VII.

II. System Model

A. Channel Model and HARQ Mechanism

We consider a clustered ad hoc network with K links, where the total available bandwidth B is divided in N_c subcarriers. In this paper, the considered multi access technology is OFDMA, but our work can extend straightforwardly to any multiple access multicarrier scheme and to single-carrier frequency division multiplexing. We assume that, in each cluster, there is a Cluster Head (CH) which collects the statistical CSI and performs the RA. We focus on intra cluster RA, and do not consider inter-cluster interference. Each link in the cluster is modelled as a time-varying Rayleigh channel which remains constant within an OFDMA symbol, and changes independently between two symbols. This assumption corresponds to a high-mobility system, a fully interleaved fading channel or a system with frequency hopping [1]. We denote by $\mathbf{h}_k(j) = [h_k(j,0), ..., h_k(j,M-1)]^T$ the sampled channel impulse response of link k during the jth OFDMA symbol, where $(.)^T$ stands for the transposition operator. We assume that each tap of the channel is an independent random variable such that $\mathbf{h}_k(j) \sim \mathcal{CN}(0, \Sigma_k)$, where $\mathcal{CN}(0, \Sigma_k)$ stands for the multi-variate complex normal distribution with zero mean and covariance matrix $\Sigma_k := \text{diag}_{M\times M}(\zeta_{k,0}^2, ..., \zeta_{k,M-1}^2)$.

The received signal on link k on the nth subcarrier at OFDMA symbol i is

$$Y_k(i,n) = H_k(i,n)X_k(i,n) + Z_k(i,n), \tag{1}$$

where $\mathbf{H}_k(i) := [H_k(i,0), ..., H_k(i, N_c-1)]^T$ is the Fourier transform of $\mathbf{h}_k(i)$, $X_k(i,n)$ is the transmitted symbol on the nth subcarrier of the ith OFDMA symbol and $Z_k(i,n) \sim \mathcal{CN}(0, N_0B/N_c)$, with N_0 the noise power spectral density. The elements of $\mathbf{H}_k(i)$ are identically distributed random variables $H_k(i,n) \sim \mathcal{CN}(0, \zeta_k^2)$ where $\zeta_k^2 := \text{Tr}(\Sigma_k)$. We can now define the average Gain-to-Noise ratio (GNR) as

$$G_k := \frac{\mathbb{E}[|H_k(i,n)|^2]}{N_0} = \frac{\zeta_k^2}{N_0}. \tag{2}$$

It is assumed that the CH only knows the average GNR of each link to perform the RA.

A HARQ scheme is used at the Medium Access Layer (MAC). The stream of information bits is arranged into packets of L_k bits, and $L_{0,k}$ overhead bits are added in each packet. A packet is sent on the channel at most P times, and the content of the P transmitted packets, called MAC packets, depends on the considered HARQ scheme. For type-I and Chase Combining (CC) HARQ, the MAC packets are identical and are obtained by encoding the bits by a FEC of rate R_k, while, for IR-HARQ, the bits are encoded by a FEC called mother code, and then the encoded stream is split into redundancy blocks following the rate compatible coding principle. We assume that the rate of the mother code is R_k/P and that the length of the MAC packets are identical. After the pth reception, the receiver decodes the information bits. When type-I HARQ is used, only the pth received packet is used to decode the information bits. In case of CC-HARQ, the receiver performs the maximum ratio combining with the p received packets before decoding. Finally, when IR-HARQ is used, the receiver concatenates all the received redundancy blocks, and decodes the information bits.

B. Energy Consumption Model

We suppose that a 2^{m_k}-QAM modulation is used. Since the length of the packet is $L_{s,k} := L_k + L_{0,k}$, the number of modulated symbols to transmit is therefore $N_{s,k} := L_{s,k}/(m_k R_k)$. Let n_k be the number of subcarriers allocated by the CH to the kth link, and $\gamma_k := n_k/N_c$ be the corresponding proportion of bandwidth. The number of OFDMA symbols required by the kth link to transmit a MAC packet is then $N_{O,k} := N_{s,k}/n_k$. Since only statistical CSI is available to perform the RA and all subcarriers are identically distributed, the same power is used on all the subcarriers. We then define $P_k := \mathbb{E}[|X_k(i,n)|^2]$ as the power allocated to the kth link to transmit one modulated symbol on one subcarrier.

The total energy consumed to send one MAC packet is the sum of the transmission energy, the energy consumed by the receiver and the energy to process the ACKnowledgment (ACK) or the Negative ACK (NACK). The energy spent by the kth link to send one OFDMA symbol is

$$E_{O,k} := N_c \gamma_k \frac{E_k}{\kappa_k} + E_{ctx,k}, \tag{3}$$

where $E_k := P_k N_c/B$, κ_k is the efficiency of the power amplifier and $E_{ctx,k} := P_{ctx,k} N_c/B$ is the energy consumption of the circuitry of the transmitter. Since $N_{O,k}$ OFDMA symbols are required to send one MAC packet, the energy consumption to send one MAC packet is

$$E_{tx,k} := N_{O,k} E_{O,k}. \tag{4}$$

Similarly, the energy consumed by the receiver to process the MAC packet is $E_{rx,k} := N_{O,k} E_{crx,k} + E_{dec,k}$, where $E_{crx,k} := P_{crx,k} N_c/B$ is the energy consumption of the circuitry of the receiver, and $E_{dec,k}$ is the decoding energy.

We denote by $E_A = E_N$ the energy to send and process a ACK or a NACK. The total energy to process a MAC packet of the kth link is $E_{MAC,k} = E_{tx,k} + E_A + E_{rx,k}$, and finally, the energy consumed for one transmitted bit is

$$E_{bit,k} := \frac{R_k}{L_{s,k}}(E_{tx,k} + E_A + E_{rx,k}). \tag{5}$$

C. Energy Efficiency

The EE is defined as the ratio between the efficiency and the energy consumed per transmitted bit. It is expressed as

$$\text{EE}_k(E_k, \gamma_k) := \frac{\theta_k}{E_{bit,k}}, \tag{6}$$

where θ_k is the efficiency of the kth link, *i.e.* the number of information bit reliably transmitted per transmitted bit. From [18], we know that for the considered HARQ schemes

$$\theta_k = R_k \frac{L_k}{L_{s,k}} \frac{D_k(E_k)}{S_k(E_k)}, \tag{7}$$

where $D_k(E_k) := (1 - q_{k,P}(E_k))$ and $S_k(E_k) := (1 + \sum_{p=1}^{P-1} q_{k,p}(E_k))$ with $q_{k,p}(E_k)$ the probability that the first p transmissions are all received in error.

By plugging (5) and (7) into (6), we obtain

$$\text{EE}_k(E_k, \gamma_k) = \frac{L_k D_k(E_k)}{S_k(E_k)(A_k E_k + B_k \gamma_k^{-1} + C_k)}, \tag{8}$$

where $A_k := (L_{s,k})/(m_k R_k \kappa_k)$, $B_k := L_{s,k}(P_{ctx,k} + P_{crx,k})/(B m_k R_k)$ and $C_k := E_A + E_{dec,k}$.

In eq. (8), the first main difficulty is the absence of closed-form for $q_{k,p}(E_k)$ when using practical MCS. To overcome this issue, we will upper-bound $q_{k,p}(E_k)$ as in [17]. We have $q_{k,p}(E_k) \le \pi_{k,p}(E_k)$ where $\pi_{k,p}(E_k)$ is the probability not to decoding the information packet given the first p MAC packets. Moreover, for uncoded communications, block codes and convolutional codes over the Rayleigh fast fading channel, $\pi_{k,p}(E_k)$ is also upper-bounded as follows [17]

$$\pi_{k,p}(E_k) \le \tilde{\pi}_{k,p}(E_k) := \frac{g_{k,p}}{\text{SNR}_k^{d_{k,p}}}, \tag{9}$$

where $g_{k,p}$ is a fitting coefficient which depends on the MCS and the packet length, $\text{SNR}_k := G_k E_k$ and $d_{k,p}$ is the diversity gain. More precisely, in theory, for type-I HARQ, $d_{k,p} = d_{k,1}$ with $d_{k,1}$ the free distance of the code, for CC-HARQ, $d_{k,p} = p d_{k,1}$ and, for IR-HARQ, $d_{k,p}$ is the free distance associated with the pth transmission. In practice, $d_{k,p}$ is a fitting parameter which depends on the MCS and packet length. This upper-bound is tight for medium-to-high Signal-to-Noise Ratio (SNR).

III. Problem Formulation

We would like to solve Problem 1 which addresses the maximization of the SEE under some QoS constraints.

Problem 1.

$$\max_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{k=1}^{K} \frac{L_k D_k(E_k)}{S_k(E_k)(A_k E_k + B_k \gamma_k^{-1} + C_k)}, \tag{10}$$

$$\text{s.t.} \quad \gamma_k m_k R_k \alpha_k \frac{D_k(E_k)}{S_k(E_k)} \geq \eta_k^{(0)}, \quad \forall k, \tag{11}$$

$$\gamma_k E_k \leq Q_{max,k}, \quad \forall k, \tag{12}$$

$$\sum_{k=1}^{K} \gamma_k \leq 1, \tag{13}$$

$$\gamma_k > 0, E_k > 0, \quad \forall k, \tag{14}$$

where $\mathbf{E} := [E_1, \cdots, E_K]$, $\boldsymbol{\gamma} := [\gamma_1, \cdots, \gamma_K]$, *and* $\alpha_k := L_k / L_{s,k}$.

The QoS constraints are the following ones: (11) is the goodput constraint (in bits/s/Hz), (12) is the maximum transmit energy constraint (proportional to the maximum transmit power constraint in W), (13) is the bandwidth constraint and (14) are positivity constraints.

As said previously, the terms $q_{k,p}(E_k)$ in $D_k(E_k)$ and $S_k(E_k)$ prevent us to obtain a closed-form expressions of the involved functions. Therefore, by replacing $q_{k,p}(E_k)$ with its upper-bound given in (9), we obtain an other optimization problem described below.

Problem 2.

$$\max_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{k=1}^{K} \frac{L_k \tilde{D}_k(E_k)}{\tilde{S}_k(E_k)(A_k E_k + B_k \gamma_k^{-1} + C_k)}, \tag{15}$$

$$\text{s.t.} \quad m_k R_k \gamma_k \alpha_k \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)} \geq \eta_k^{(0)}, \quad \forall k, \tag{16}$$

$$(12), (13), (14),$$

where $\tilde{D}_k(E_k) := 1 - g_{k,P}(E_k G_k)^{-d_{k,P}}$ *and* $\tilde{S}_k(E_k) := 1 + \sum_{p=1}^{P-1} g_{k,p}(E_k G_k)^{-d_{k,p}}$.

The goal of the paper is now to solve Problem 2. A necessary and sufficient condition for the feasibility of Problem 2 can be found in [17]. For the rest of this paper, we assume that this condition holds.

We observe that the objective function given by (15) is the sum of ratios whose numerator is concave and denominator is convex as long as $d_{k,p} \geq 1$. If in addition the constraints are convex, an efficient algorithm has been proposed in [19] and successfully employed for RA optimization in [20] and [21]. Unfortunately, going back to our Problem 2, we see that the constraint given by (12) is not convex preventing thus to use [19]. In order to solve Problem 2, we propose in the next Section to operate a change of variables to render constraint (12) convex. This change of variable is inspired by the GP [22].

IV. Optimal Problem Resolution

In order to obtain convex constraint for (12) in Problem 2, we perform the following change of variables $x_k := \log(E_k)$ and $y_k := \log(\gamma_k)$. Problem 2 can then be rewritten in the following equivalent problem.

Problem 3.

$$\max_{\mathbf{x},\mathbf{y}} \quad \sum_{k=1}^{K} \frac{f_k(x_k)}{g_k(x_k, y_k)}, \tag{17}$$

$$\text{s.t.} \quad \eta_k^{(0)} e^{-y_k} \left(1 + \sum_{p=1}^{P-1} a_{k,p} e^{-x_k d_{k,p}}\right) - m_k R_k \alpha_k \left(1 - a_{k,P} e^{-x_k d_{k,P}}\right) \leq 0, \quad \forall k, \tag{18}$$

$$e^{x_k + y_k} - Q_{max,k} \leq 0, \quad \forall k, \tag{19}$$

$$\sum_{k=1}^{K} e^{y_k} - 1 \leq 0, \tag{20}$$

where $\mathbf{x} := [x_1, ..., x_k]$, $\mathbf{y} := [y_1, ..., y_k]$, $f_k(x_k) := L_k(1 - a_{k,P} e^{-x_k d_{k,P}})$, $g_k(x_k, y_k) := (1 + \sum_{p=1}^{P-1} a_{k,p} e^{-x_k d_{k,p}})(A_k e^{x_k} + B_k e^{-y_k} + C_k)$, *and* $a_{k,p} := g_{k,p} / G_k^{d_{k,p}}$.

It is easy to check that Problem 3 is still the maximization of the sum of ratios whose numerator is concave and denominator is convex, **but** now the constraints are all convex. Therefore it is possible to apply the algorithm proposed in [19]. Actually, the optimal solution of Problem 3 can be characterized through Theorem 1 whose proof can be found in [20].

Theorem 1. *Let* $(\mathbf{x}^*, \mathbf{y}^*)$ *denote the optimal solution of Problem 3. There exist* $\mathbf{u}^* := [u_1^*, ..., u_K^*]$ *and* $\boldsymbol{\beta}^* := [\beta_1^*, ..., \beta_K^*]$ *such that* $(\mathbf{x}^*, \mathbf{y}^*)$ *is an optimal solution of the following problem*

$$\max_{\mathbf{x},\mathbf{y}} \quad \sum_{k=1}^{K} u_k^* \left(f_k(x_k) - \beta_k^* g_k(x_k, y_k)\right), \tag{21}$$

$$\text{s.t.} \quad (18), (19), (20).$$

In addition, $(\mathbf{x}^*, \mathbf{y}^*)$ *also satisfies the following system of equations*

$$u_k^* g_k(x_k^*, y_k^*) - 1 = 0, \tag{22}$$

$$\beta_k^* g_k(x_k^*, y_k^*) - f_k(x_k^*) = 0. \tag{23}$$

Theorem 1 enables to transform Problem 3 into a concave maximization problem in $\mathbf{x}$ and $\mathbf{y}$ thanks to the subtractive form of (21) and so the absence of ratio. The issue is obviously to obtain $\mathbf{u}^*$ and $\boldsymbol{\beta}^*$. Therefore, to solve (18)-(23), we will proceed into two steps as suggested in [19]–[21]. During the first step, we solve (18)-(21) for fixed $\mathbf{u}$ and $\boldsymbol{\beta}$. During the second step, $\mathbf{x}$ and $\mathbf{y}$ are fixed and $\mathbf{u}$ and $\boldsymbol{\beta}$ are updated using the Modified Newton (MN) method. We iterate both steps until convergence.

1) Solution of the first step: The first step consists in solving (18)-(21) for given $(\mathbf{u}, \boldsymbol{\beta})$, which is equivalent to solve the following problem

$$\min_{\mathbf{x},\mathbf{y}} \quad \sum_{k=1}^{K} u_k \left(L_k a_{k,P} e^{-x_k d_{k,P}} + \beta_k g_k(x_k, y_k) \right), \quad (24)$$
$$\text{s.t.} \quad (18), (19), (20).$$

This problem is convex and so can be efficiently solved by numerical algorithms *i.e.* the interior point method [22].

2) Solution of the second step: In order to find the optimal $(\mathbf{u}^*, \boldsymbol{\beta}^*)$, we define $\psi_k(\mathbf{u}, \boldsymbol{\beta}) := -f_k(x_k) + \beta_k g_k(x_k, y_k)$ and $\psi_{k+K}(\mathbf{u}, \boldsymbol{\beta}) := -1 + u_k g_k(x_k, y_k)$ for $k = 1, ..., K$. According to Theorem 3 in [20], we know that the optimal solution $(\mathbf{u}^*, \boldsymbol{\beta}^*)$ is unique and is achieved if and only if $\boldsymbol{\psi}(\mathbf{u}, \boldsymbol{\beta}) = 0$, where $\boldsymbol{\psi}(\mathbf{u}, \boldsymbol{\beta}) = [\psi_1(\mathbf{u}, \boldsymbol{\beta}), \psi_2(\mathbf{u}, \boldsymbol{\beta}), ..., \psi_{2K}(\mathbf{u}, \boldsymbol{\beta})]$. Then, we can use the MN method to update $(\mathbf{u}, \boldsymbol{\beta})$ [21]. Finally, the optimal procedure to solve Problem 2 is given by Algorithm 1, which converges to the optimal solution of Problem 2 [21].

Algorithm 1 Optimal Resolution

1: Initialize $\mathbf{E}^{(0)}$, $\boldsymbol{\gamma}^{(0)}$, $\mathbf{u}^{(0)}$ and $\boldsymbol{\beta}^{(0)}$
2: Set $\epsilon > 0$, $i = 0$
3: **while** $\boldsymbol{\psi}(\boldsymbol{\beta}^{(i)}, \mathbf{u}^{(i)}) > \epsilon$ **do**
4: $\quad i = i + 1;$
5: $\quad$ Find $(\mathbf{E}^{(i)}, \boldsymbol{\gamma}^{(i)})$ using (24) and change of variables
6: $\quad$ Compute $(\mathbf{u}^{(i)}, \boldsymbol{\beta}^{(i)})$ with the MN method
7: **end while**

V. Suboptimal Problem Resolution

A. Algorithm based on Alternating Optimization

We hereafter propose to reduce the complexity using a suboptimal resolution of Problem 2 based on alternating optimization [23]. In this approach, the problem is solved by alternating between the both variables $\mathbf{E}$ and $\boldsymbol{\gamma}$ until convergence.

1) Maximization w.r.t $\mathbf{E}$*:* First, we consider $\boldsymbol{\gamma}$ as fixed and feasible, and we maximize the objective function with respect to $\mathbf{E}$. The resulting problem is separable, and, from (15), the K subproblems write

$$\max_{E_k} \quad L_k \frac{\tilde{D}_k(E_k)}{\tilde{S}_k(E_k)(A_k E_k + F_k)}, \quad (25)$$
$$\text{s.t.} \quad \eta_k^{'(0)} \tilde{S}_k(E_k) - m_k R_k \alpha_k \tilde{D}_k(E_k) \leq 0, \quad (26)$$
$$E_k - E'_{max,k} \leq 0, \quad (27)$$
$$E_k > 0, \quad (28)$$

where $F_k := B_k \gamma_k^{-1} + C_k$, $\eta_k^{'(0)} := \eta_k^{(0)} \gamma_k^{-1}$ and $E'_{max,k} := Q_{max,k} \gamma_k^{-1}$.

These subproblems are neither concave nor convex due to the form of (25). However, once again, it is easy to prove that the numerator of (25) is concave and the denominator is convex as long as $d_{k,p} \geq 1$. The subproblem thus corresponds to the maximization of a ratio between concave and convex functions over a convex compact set. This subproblem can be efficiently solved by using the Dinkelbach's algorithm [3].

Actually, the Dinkelbach's algorithm requires to solve the following problem

$$\max_{E_k} \quad \mathcal{F}_k(E_k) \quad (29)$$
$$\text{s.t.} \quad (26), (27), (28),$$

where $\mathcal{F}_k(E_k) := L_k \tilde{D}_k(E_k) - \lambda_n \tilde{S}_k(E_k)(A_k E_k + D_k)$ with $\lambda_n \in \mathbb{R}^+$.

We can prove that there exists a unique $E_{min,k} \in (g_{k,P}^{1/d_{k,P}} G_k^{-d_{k,P}}, E'_{max,k}]$ such that the goodput constraint is satisfied as long as $E_k \geq E_{min,k}$. The optimal solution of (26)-(29) is denoted by E_k^* and is exhibited in Theorem 2 whose proof is omitted due to page limitation.

Theorem 2. *The optimal solution of* (26)-(29) *takes the following form:*

1) If $\mathcal{F}'_k(E_{min,k}) < 0$*, then* $E_k^* = E_{min,k}$.

2) If $\mathcal{F}'_k(E'_{max,k}) > 0$*, then* $E_k^* = E'_{max,k}$.

3) Else, E_k^* *is the solution of* $\mathcal{F}'_k(E_k^*) = 0$ *in* $[E_{min,k}, E'_{max,k}]$*, which is unique. This case can be efficiently solved using the bisection method.*

Finally the Dinkelbach's algorithm is summarized by Algorithm 2.

Algorithm 2 Dinkelbach's Algorithm

1: Set $\epsilon > 0; n = 0; \lambda_n = 0$
2: **do**
3: $\quad$ Find E_k^* by applying Theorem 2.
4: $\quad \mathcal{G}_k(\lambda_n) = L_k \tilde{D}_k(E_k^*) - \lambda_n \tilde{S}_k(E_k^*)(A_k E_k^* + D_k)$
5: $\quad \lambda_{n+1} = \frac{L_k \tilde{D}_k(E_k^*)}{\tilde{S}_k(E_k^*)(A_k E_k^* + D_k)}$
6: $\quad n = n + 1;$
7: **while** $\mathcal{G}_k(\lambda_n) > \epsilon$

2) Maximization w.r.t $\boldsymbol{\gamma}$*:* When $\mathbf{E}$ is fixed, Problem 2 writes

$$\max_{\boldsymbol{\gamma}} \quad \sum_{k=1}^{K} \frac{\gamma_k H_k}{\gamma_k J_k + M_k} \quad (30)$$
$$\text{s.t.} \quad \gamma_k \geq \gamma_{min,k}, \quad \forall k \quad (31)$$
$$\gamma_k \leq \gamma_{max,k}, \quad \forall k \quad (32)$$
$$\sum_{k=1}^{K} \gamma_k \leq 1 \quad (33)$$

where $\gamma_{min,k} := \eta_k^{(0)} \tilde{S}_k(E_k)/(m_k R_k \alpha_k \tilde{D}_k(E_k))$, $\gamma_{max,k} := Q_{max,k}/E_k$, $H_k := L_k \tilde{D}_k(E_k)$, $J_k := (A_k E_k + C_k)\tilde{S}_k(E_k)$ and $M_k := B_k \tilde{S}_k(E_k)$.

Obtaining (30)-(33) leads to the maximization of a concave function over a convex set. After calculations, we obtain the optimal bandwidth allocation as

$$\gamma_k^*(\lambda) = \left[-\frac{M_k}{J_k} + \frac{\sqrt{H_k M_k \lambda}}{\lambda J_k} \right]_{\gamma_{min,k}}^{\gamma_{max,k}}, \quad (34)$$

where $[x]_a^b := \min\{b, \max\{a, x\}\}$ and λ is the non-negative optimal Lagrange multiplier. To find λ, we define the following function

$$\Gamma(\Lambda) := \sum_{k=1}^{K} \gamma_k^*(\Lambda). \quad (35)$$

We can prove that $\Gamma(\Lambda)$ is a continuous non-increasing function of Λ. Therefore, the optimal bandwidth allocation can be found using the following theorem

Theorem 3. *If $\sum_{k=1}^{K} \gamma_{max,k} \leq 1$, then $\lambda = 0$ is optimal and the optimal bandwidth allocation is given by $\forall k, \gamma_k^* = \gamma_{max,k}$. Otherwise, let λ^* denote the solution of $\Gamma(\lambda^*) = 1$, which is unique on $\mathbb{R}^{+*}$. The optimal bandwidth allocation is given by $\forall k, \gamma_k^* = \gamma_k^*(\lambda^*)$.*

Theorem 3 gives an efficient way to find the optimal solution of (30)-(33). Moreover, it is easy to prove that the suboptimal algorithm based on alternating optimization convergences since it creates a monotonically increasing bounded sequence of SEE.

B. Algorithm based on High SNR Approximation

Going back to Problem 2, we remark that the difficulty arises from the ratio into the sum in the objective function. In order to remove this ratio, we first consider the high-SNR regime in (15) by considering that at the last retransmission, we have no error, *i.e.*, $q_{k,P}(E_k) = 0$. The numerator of the ratio becomes equal to 1 and so is constant. The problem leads thus to the minimization of the harmonic mean of the denominator. The second approximation is to replace the harmonic mean with the arithmetic mean. Therefore we focus on the following optimization problem.

$$\min_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{k=1}^{K} L_k^{-1} \tilde{S}_k(E_k).(A_k E_k + B_k \gamma_k^{-1} + C_k), \quad (36)$$
$$\text{s.t.} \quad (12), (13), (14), (16),$$

which is a geometric program that can be efficiently solved by doing a change of variable and using the interior point method.

VI. Numerical Results

In this section, we illustrate the results of our three proposed algorithms by showing the achieved maximum SEE as a function of the required goodput and the circuitry power consumption. We also compute the SEE obtained by two other algorithms with conventional criteria: the one from [17] which minimizes the transmit power, and the one maximizing the sum rate for which the rate is replaced by the goodput (obtained by minor modifications in Algorithm 1). We implemented a CC-HARQ scheme using a convolutional code of rate $1/2$ along with QPSK modulation. The number of link is $K = 8$ and the link distances $D_{ist,k}$ are uniformly drawn in $[50\text{ m}, 1\text{ km}]$. We set $B = 5$ MHz, $N_0 = -170$ dBm/Hz, $L_k = 128$, $P = 3$ and $\alpha_k = 1$. The carrier frequency is $f_c = 2400$ MHz and we put $\zeta_k^2 = (4\pi f_c/c)^{-2} D_{ist,k}^{-3}$ where c is the speed of light in vacuum. We assume that the required goodput is equal for all link, and is equal to $G_p/(BK)$, where G_p is the target sum of the useful data rate (in bits/s) [1]. We also consider that $E_A = E_N = 0$ and that for all k, $P_{ctx,k} = P_{crx,k} = P_c$, $E_{dec,k} = 0$ and $\kappa_k = 0.5$. The maximum transmit power is set to 29 dBm. We will refer to the optimal algorithm as A1, the algorithm based on alternating optimization as A2 and the one based on the high-SNR approximation as A3. All points have been obtained through 100 Monte Carlo trials.

The convergence of A1 and A2 is illustrated on Fig. 1. In average, only 3 iterations are needed for A2 to converge, while A1 is longer to converge. At each iteration, A1 needs to solve a convex optimization problem, which can be done in polynomial time. Concerning A2, each iteration is less complex than the one of A1 since the iterations optimal solutions are in quasi closed form. It can be seen that A2 leads to slightly lower SEE than A1 after 50 iterations, which is normal since A1 is optimal and A2 suboptimal. However, the performance degradation of A2 compared with A1 is negligible.

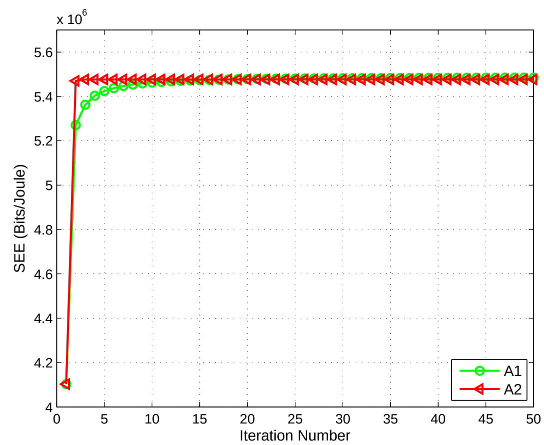

Fig. 1: Convergence of A1 and A2, $G_p = 3700$ kbps, $P_c = 0.4$ W.

In Fig. 2, we plot the SEE of the different considered algorithms versus G_p. As expected, our algorithms achieve a higher SEE compared with the conventional ones. For instance, when $G_p = 3600$ kbps, the SEE of the proposed algorithms is about 20% higher than those obtained with the transmit power minimization, and 80% higher than those obtained with the goodput maximization. We can remark that A2 gives almost the same result as A1, and that A3 gives slightly lower SEE than A1 and A2.

Finally, in Fig. 3, we plot the SEE versus P_c. First, as expected the proposed algorithms always outperform the conventional ones whatever the value of P_c. Second, the SEE

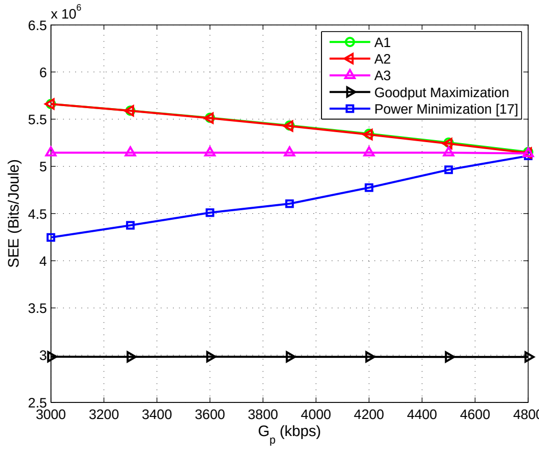

Fig. 2: Performance of the proposed algorithms, $P_c = 0.4$ W.

decreases as P_c increases. Last, when P_c is sufficiently large, the sum rate maximization is more energy efficient than the power minimization. Indeed, when P_c is large, the energy per transmitted bit is dominated by the circuitry consumption, and then it is more energy efficient to have a high goodput in order to avoid retransmissions since the goodput includes the effect of the packet error rate.

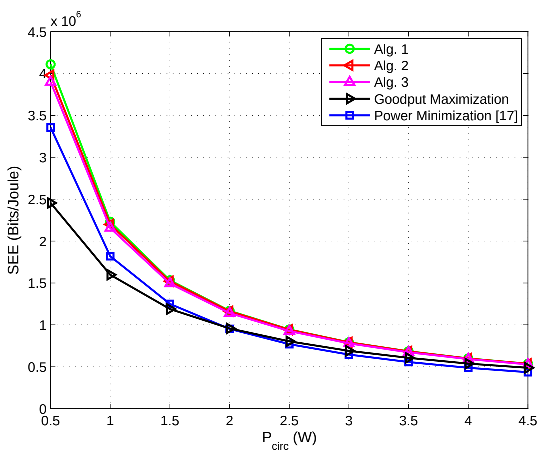

Fig. 3: Influence of P_c, $G_p = 3600$ kbps.

VII. Conclusion

In this paper, we proposed one optimal algorithm, and two suboptimal algorithms to maximize the SEE under QoS constraints for HARQ in OFDMA networks. Simulations results show that the suboptimal algorithms exhibit a slight performance degradation but are less complex. Moreover, as expected, the proposed algorithms perform much better than conventional RA algorithms with conventional criteria (transmit power minimization and maximum sum rate).

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