SECOND ORDER BLIND EQUALIZATION : THE BAND LIMITED CASE

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ABSTRACT

Most of the second order based fractionnally sampled blind equalizers are known to perform poorly in the context of band limited signals. In this paper, we analyse the behaviour of the subspace method in the particular context of band limited signals. As it is well known, the subspace channel estimate is obtained as the eigenvector associated to the eigenvalue 0 of a certain positive quadratic form Q. We show that apart 0, Q has quite small eigenvalues, and that this induces poor statistical performance. More importantly, we characterize the numerical kernel of Q, and show that it contains vectors constructed from certain spheroidal wave sequences. From this, we deduce that the subspace method does not allow to estimate accurately the transfer function of the channel on a certain frequency interval.

1. INTRODUCTION

Let $\{v_n\}_{n\in Z}$ be a zero mean unit variance i.i.d symbol sequence to be transmitted through a linear channel at the baud rate $1/T$. The continuous time received signal $\tilde{y}(t)$ can be written as :

$$\tilde{y}(t) = \sum_{n\in Z} v_n \tilde{h}(t - nT)$$

where the filter $\{\tilde{h}(t)\}$ results from the emission and the reception filters and from the multipath effects. In this paper, we assume without restriction that $\tilde{h}(t)$ is causal and time limited. Generally, $\{\tilde{h}(t)\}$ is unknown, and has therefore to be estimated in order to retrieve the symbols from the received signal. In most communication systems, the emitter sends periodically a training sequence known from the receiver, and which allows to estimate the unknown channel. However, the use of a training sequence has certain well known drawbacks. Therefore, a number of works have been devoted to the so-called blind equalization problem consisting in identifying the channel from the sole knowledge of the received signal $\tilde{y}(t)$. Gardner ([4]) and Tong et al ([8]) were the first to remark that it is possible to use the cyclostationnarity of $\tilde{y}(t)$ in order to identify the channel from the second order statistics of the observations. For this, they proposed to sample $\tilde{y}(t)$ at rate $2/T$ (or more generally to q/T for $q > 1$; we just consider $q = 2$ in this paper). The discrete time signal $y(n) = \tilde{y}(nT/2)$ can be written as

$$y(n) = \sum_{k=0}^{P} h_k u_{n-k} = [h(z)]u(n)$$

where u_n is defined by $u_{2n} = v_n$ et $u_{2n+1} = 0$ and where $h(z) = \sum_{k=0}^{P} h_k z^{-k}$, $h_k = \tilde{h}(k\frac{T}{2})$. $\{y(n)\}$ is cyclostationnary with cyclic frequencies 0 et $\frac{1}{2}$ and its corresponding cyclospectra are given by $S_y^0(e^{2i\pi f}) = \frac{1}{2}|h(e^{2i\pi f})|^2$ and $S_y^{\frac{1}{2}}(e^{2i\pi f}) = \frac{1}{2}h(e^{2i\pi f})h^*(e^{2i\pi(f-\frac{1}{2})})$. It is well established that if $h(z)$ et $h(-z)$ have no common zero, the knowledge of S^0 and $S^{\frac{1}{2}}$ allow to retrieve $h(z)$. Moreover, a number of time domain estimation algorithms of $h(z)$ have been proposed recently.

The above mentionned identifiability condition is of course verified in most cases. However, it has been observed that the performance of many second order statistics based identification algorithms are very poor if the channel is "ill-conditionned", i.e. if $h(z)$ and $h(-z)$ have almost common zeros ([10]). Van der Veen also considered in [9] the case of band limited channels. To precise this, we note that generally, the bandwith of the emission filter of the emitter is an interval $[-\beta/T, \beta/T]$ where $1/2 < \beta < 1$. The Fourier transform of $\tilde{h}(t)$ is thus numerically zero outside this interval, and this implies that

$$h(e^{2i\pi f}) \approx 0 \quad \text{if} \quad f \in [-1/2, -\beta/2] \cup [\beta/2, 1/2] \quad (1)$$

Van der Veen showed that in this case, the so-called Sylvester matrix introduced in the subspace method of [6] is ill conditioned, and that the estimation of the signal and the noise subspace is a difficult task.

In this paper, we study the performance of the subspace method of [6] in the band limited channel case. We first recall that the subspace estimate is defined as the eigenvector associated to the eigenvalue 0 of a certain matrix Q.

The work of the first author is supported by a DRET/CNRS fellowship

Next, we justify that this matrix has also quite small eigenvalues, and we show that the "smallest" wave spheroidal wave sequences associated to a certain interval belong to the numerical kernel of Q. This observation is interpreted in the frequency domain, and leads to the conclusion that the subspace method estimates very poorly the transfer function of the channel over the frequency interval of the bandwith of $h(e^{2i\pi f})$ on which the cyclic spectrum $S^{1/2}(e^{2i\pi f})$ is nearly zero. All these claims are essentially based on a heuristic analysis because it is quite difficult to study analytically the behaviour of (quasi) band-limited FIR transfer functions. In order to justify our analysis, we study the asymptotic covariance matrix of the subspace estimate. Using numerical evaluations, we show that the covariance of the subspace estimate is explosive in the directions of the spheroidal wave sequences, and that the variance of the estimated transfer function is very high in the above mentionned frequency interval.

We finally introduce some convenient notations : we put $\mathcal{I}_1 = [-\frac{1}{2}, -\frac{\beta}{2}] \cup [\frac{\beta}{2}, \frac{1}{2}[$, $\mathcal{I}_2^+ = [\frac{1-\beta}{2}, \frac{\beta}{2}]$, $\mathcal{I}_2^- = [-\frac{\beta}{2}, -\frac{1-\beta}{2}]$, $\mathcal{I}_2 = \mathcal{I}_2^+ \cup \mathcal{I}_2^-$ and $\mathcal{I}_3 = [-\frac{1-\beta}{2}, \frac{1-\beta}{2}]$. We note that $h(e^{2i\pi f}) \approx 0$ if $f \in \mathcal{I}_1$ and that $S^{1/2}(e^{2i\pi f}) \approx 0$ if $f \in \mathcal{I}_1 \cup \mathcal{I}_3$.

2. REVIEW OF THE SUBSPACE METHOD

We assume for convenience that the degree P of the filter $h(z)$ is odd and we set $P = 2M + 1$. Let $Y(n)$ be the 2–variate signal defined by $Y(n) = [y(2n+1)\, y(2n)]^T$. $Y(n)$ is clearly stationary, and can be written as $Y(n) = [H(z)]v_n$, where $H(z) = [H_1^T(z)\, H_2^T(z)]^T$ is the one input / two outputs FIR filter defined by

$$H_1(z^2) = \frac{h(z) - h(-z)}{2z^{-1}} H_2(z^2) = \frac{h(z) + h(-z)}{2} \quad (2)$$

Let $N \geq M$, and put $Y_N(n) = [Y^T(n) \ldots Y^T(n-N)]^T$. Then, $Y_N(n)$ can be written as $Y_N(n) = \mathcal{T}_N(h) V_{M+N}(n)$ where $\mathcal{T}_N(h)$ is the so-called $2(N+1) \times (M+N+1)$ Sylvester (block Toeplitz) matrix associated to the filter $H(z)$ defined from $h(z)$ by (2). The covariance matrix R_N of $Y_N(n)$ is thus equal to $R_N = \mathcal{T}_N(h)\mathcal{T}_N(h)^*$. It is well established that if $h(z)$ and $h(-z)$ have no common zero, the rank of the matrices $\mathcal{T}_N(h)$ and R_N is equal to $M + N + 1$. Moreover, denote by Π_N the orthogonal projection matrix on the Kernel of R_N. If $f(z) = \sum_{k=0}^{P} f_k z^{-k}$ is a degree P FIR filter, we put $\mathbf{f} = (f_0, \ldots, f_P)^T$, and consider the quadratic form Q defined by

$$\mathbf{f} \rightarrow \text{Trace}[\Pi_N \mathcal{T}_N(f)(\mathcal{T}_N(f))^*] = \mathbf{f}^* Q \mathbf{f}$$

The subspace method of [6] is based on the observation that the Kernel of Q is the one-dimensional subspace generated by the vector $\mathbf{h}$ associated to the filter $h(z)$. In practice, the matrix Q is of course unknown, but it can be estimated consistently from the observations if the additive noise is white (or more generally if the second order statistics of the noise are known up to a scalar constant). The eigenvector $\hat{\mathbf{h}}$ associated to the smallest eigenvalue of the estimate $\hat{Q}$ of Q represents a consistent estimate of $\mathbf{h}$ (up to a scalar constant). $\hat{\mathbf{h}}$ will be referred to as the subspace estimate of $\mathbf{h}$ in the sequel.

3. REVIEW ON THE SPHEROIDAL WAVE SEQUENCES.

The order $2M + 2$ spheroidal wave sequences ([7]) of the interval $\mathcal{I}_1 \cup \mathcal{I}_2$ are the (unit norm) eigenvectors $\{\mathbf{k}_j\}_{j=1,2M+2}$ associated to the eigenvalues $\{\lambda_j\}_{j=1,\ldots,2M+2}$ (with $\lambda_1 \leq \ldots \leq \lambda_{2M+2}$) of the positive Toeplitz matrix $\mathcal{K}$ defined by

$$\mathcal{K} = \int_{\mathcal{I}_1 \cup \mathcal{I}_2} \overline{D_{2M+1}(e^{2i\pi f})} D_{2M+1}^T(e^{2i\pi f}) df$$

where $D_{2M+1}(e^{2i\pi f}) = [1, e^{-2i\pi f}, \ldots, e^{-2i\pi(2M+1)f}]^T$ and $\overline{(.)}$ stands for the conjugate. They play an important role in various problems involving implicitely band limited signals (e.g. band limited spectral estimation ([5]), broadband source localization ([2]), array beamforming ([3])). It is well known that the matrix $\mathcal{K}$ is ill conditionned, and that its "numerical" rank is equal to $1 + \text{int}((2M+2)|\mathcal{I}_1 \cup \mathcal{I}_2|)$, where int(.) stands for the integer part of (.) and where $|\mathcal{I}_1 \cup \mathcal{I}_2| = \beta$ represents the size of $\mathcal{I}_1 \cup \mathcal{I}_2$. In the following, we denote by s the dimension of the numerical kernel of $\mathcal{K}$. Let $k_j(z)$ be the polynomial associated to the vector $\mathbf{k}_j$. As $\mathbf{k}_j^* \mathcal{K} \mathbf{k}_j$ is given by

$$\mathbf{k}_j^* \mathcal{K} \mathbf{k}_j = \int_{\mathcal{I}_1 \cup \mathcal{I}_2} |k_j(e^{2i\pi f})|^2 df$$

the bad conditionning of $\mathcal{K}$ implies that if $j \leq s$, then

$$k_j(e^{2i\pi f}) \approx 0 \quad \text{if} \quad f \in \mathcal{I}_1 \cup \mathcal{I}_2$$

In others words, the FIR filters $k_j(z)$ for $j = 1, s$ associated to the s "smallest" spheroidal wave sequences of $\mathcal{I}_1 \cup \mathcal{I}_2$ are nearly band limited and their bandwith coincide with the interval $\mathcal{I}_3$.

4. THE NUMERICAL KERNEL OF Q

In this section, we justify that Q is ill conditionned, i.e. that it has a "numerical" kernel. For this, we have first to derive its closed form expression in terms of Π_N. Let us put $\Pi_N = [\Pi_{0,N}, \ldots, \Pi_{N,N}]$ (where each matrix $\Pi_{k,N}$ is $2(N+1) \times 2$) and $\Pi_N(e^{2i\pi f}) = \sum_{k=0}^{N} \Pi_{k,N} e^{-2i\pi k f}$. Let us also define the matrix D_Π by

$$D_\Pi = \int_0^1 \overline{D_{M+N}(e^{2i\pi f})} D_M^T(e^{2i\pi f}) \otimes \Pi_N(e^{2i\pi f}) df \quad (3)$$

where $\otimes$ stands for the Kronecker product. Then, $Q = \mathcal{P} D_\Pi^* D_\Pi \mathcal{P}$ where $\mathcal{P} = I_{M+1} \otimes J_2$, and $J_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The matrix Q depends on Π_N, i.e. on the vectors of the kernel of R_N. Let us first analyse the properties of these vectors. For this, let $\mathbf{g} = [g_0, \ldots, g_{2N+1}]$ be a unit norm row vector of the noise subspace, and put $g(e^{2i\pi f}) = \sum_{k=0}^{2N+1} g_k e^{-2i\pi k f}$. It is easy to check that

$$g(e^{2i\pi f}) h(e^{2i\pi f}) + g(e^{2i\pi (f+1/2)}) h(e^{2i\pi (f+1/2)}) = 0 \quad (4)$$

for each f. If f belongs to $\mathcal{I}_3$, $f + 1/2$ is in $\mathcal{I}_1$ and $h(e^{2i\pi(f+1/2)}) \approx 0$. Therefore, relation (4) implies that

$$g(e^{2i\pi f}) \approx 0 \quad \text{if} \quad f \in \mathcal{I}_3$$

Using this, we now derive an approximate expression of Q. Let $(\pi_{k,N})_{k=0,2N+1}$ be the elementary columns of Π_N and let us denote by $\pi_N(e^{2i\pi f})$ the vector valued function defined by $\pi_N(e^{2i\pi f}) = \sum_{k=0}^{2N+1} \pi_{k,N} e^{-2i\pi k f}$. From what preceds, each component of $\pi_N(e^{2i\pi f})$ is nearly zero on $\mathcal{I}_3$. Therefore, $\pi_N(e^{2i\pi f}) \approx 0$ if $f \in \mathcal{I}_3$. On the other hand, using (3) and the relation

$$\Pi_N(e^{2i\pi f}) = \frac{1}{2} \left[\pi_N(e^{i\pi f}), \pi_N(e^{i\pi(f+1)}) \right] \begin{bmatrix} 1 & e^{i\pi f} \\ 1 & e^{i\pi(f+1)} \end{bmatrix}$$

one can check that

$$Q = \int_0^{1/2} \mathcal{Q}_2(e^{2i\pi f}) \mathcal{Q}_2^*(e^{2i\pi f}) df$$

$$\mathcal{Q}_2(e^{2i\pi f}) = \mathcal{Q}_1(e^{2i\pi f}) - \mathcal{Q}_1(e^{2i\pi(f+1/2)})$$
$$\mathcal{Q}_1(e^{2i\pi f}) = \overline{D_{2M+1}(e^{2i\pi f})} \left(\pi_N(e^{-2i\pi f})\right)^*$$

As $\pi_N(e^{2i\pi f}) \approx 0$ if $f \in \mathcal{I}_3$, this reduces to

$$Q = \int_{\mathcal{I}_1} \mathcal{Q}_1(e^{2i\pi f}) \mathcal{Q}_1^*(e^{2i\pi f}) df + \int_{\mathcal{I}_2^+} \mathcal{Q}_2(e^{2i\pi f}) \mathcal{Q}_2^*(e^{2i\pi f}) df$$

From this, we get immediatly that if $\mathbf{l}$ is a $(2M+2)$-dimensional vector and if $l(z) = \sum_{k=0}^{2M+1} l_k z^{-k}$ represents its associated degree $2M+1$ FIR filter, $\mathbf{l}^* Q \mathbf{l}$ is given by

$$\mathbf{l}^* Q \mathbf{l} = \int_{\mathcal{I}_1} \|\phi(e^{2i\pi f})\|^2 df + \int_{\mathcal{I}_2^+} \|\phi(e^{2i\pi f}) - \phi(e^{2i\pi(f+1/2)})\|^2$$

$$\phi(e^{2i\pi f}) = l(e^{2i\pi f}) \left(\pi_N(e^{-2i\pi f})\right)^*$$

It is therefore quite clear that $\mathbf{k}_j^* Q \mathbf{k}_j \approx 0$ for $j = 1, s$. In other words, Q is an ill conditionned matrix, and its numerical kernel contains the vectors $\{\mathbf{k}_j\}_{j=1,s}$.

In practice, the true kernel and the numerical kernel of Q are of course difficult to separate. More precisely, even if the estimate $\hat{Q}$ of Q is very accurate (if the signal to noise ratio is very high, or if the duration of the observation is large), the eigenvector associated to the smallest eigenvalue of $\hat{Q}$ will certainly contains a non zero contribution belonging to the space generated by the vectors $\{\mathbf{k}_j\}_{j=1,s}$. This of course tends to indicate that the subspace estimate has very poor statistical performance. This discussion has an interesting interpretation in the frequency domain. For each f, the estimated channel $\hat{h}(e^{2i\pi f})$ is likely to be a linear combination of $h(e^{2i\pi f})$ and of the $(k_j(e^{2i\pi f}))_{j=1,s}$. If $f \in \mathcal{I}_1 \cup \mathcal{I}_2$, the $(k_j(e^{2i\pi f}))_{j=1,s}$ are nearly zero, and $\hat{h}(e^{2i\pi f}) \approx h(e^{2i\pi f})$. However, this is no longer true on $\mathcal{I}_3$ because the $(k_j(e^{2i\pi f}))_{j=1,s}$ are of course not zero on $\mathcal{I}_3$. Therefore, the subspace method estimates accurately the channel on $\mathcal{I}_1 \cup \mathcal{I}_2$, but not on $\mathcal{I}_3$. These heuristic claims are going to be justified in the next paragraph by analysing the asymptotic covariance matrix of $\hat{\mathbf{h}}$.

We also mention that the true matrix R_N is ill conditionned ([9]). Therefore, R_N has a "numerical" kernel which is quite difficult to separate from the true kernel. In other words, the projection matrix Π_N introduced previously has to be replaced in practice by the projection matrix $\tilde{\Pi}_N$ on the larger space generated by the true kernel and the numerical kernel. The matrix Q has also to be replaced by a certain matrix $\tilde{Q}$. However, the vectors $\{\mathbf{k}_j\}_{j=1,s}$ still belong to the numerical kernel of $\tilde{Q}$. To see this, we note that our analysis of the numerical kernel of Q is entirely based on the fact that if $\mathbf{g} \in \mathrm{Ker}(R_N)$, then $g(e^{2i\pi f}) \approx 0$ if $f \in \mathcal{I}_3$. But, it is easy to check that the vectors of the "augmented" kernel of R_N still satisfy this property. Therefore, the vectors $\{\mathbf{k}_j\}_{j=1,s}$ are in the numerical kernel of $\tilde{Q}$.

We finally note that apart their own interest, the above results can be used in order to derive a relevant version of the so-called JOSC (joint optimization with subspace constraints) estimation algorithms (see [10]). The idea is to remark that the subspace approach allows to estimate the channel up to the numerical kernel of Q. In order to raise this indeterminacy, one can use a covariance matching algorithm in which the parameter is constrained to belong to the numerical kernel of $\hat{Q}$. The main benefit of this approach is to reduce the number of parameters to be estimated by minimizing the (non linear) covariance matching cost function. It is clear that any a priori information on the numerical kernel of Q can be used in order to improve the performance of such an algorithm. This last point will be treated in a forthcoming paper.

5. STATISTICAL PERFORMANCE ANALYSIS.

In this section, we justify the previous claims by studying the asymptotic covariance matrix of the subspace estimate

$\hat{\mathbf{h}}$ in the case where the observations are corrupted by an additive white gaussian noise with variance σ^2. We also assume that the symbol sequence is circular. We first recall ([1]) that if K denotes the sample size, the estimate $\hat{h}$ is asymptotically Gaussian, and that its asymptotic covariance matrix C_{ssm} defined by

$$C_{ssm} = \lim_{K\to\infty} KE\left[(\hat{\mathbf{h}} - \mathbf{h})(\hat{\mathbf{h}} - \mathbf{h})^*\right]$$

is given by

$$C_{ssm} = (\mathcal{P}D_\Pi^* D_\Pi \mathcal{P})^{\#} \mathcal{P} D_\Pi^* \Sigma D_\Pi \mathcal{P} (\mathcal{P} D_\Pi^* D_\Pi \mathcal{P})^{\#}$$

where Σ is a certain matrix (see [1] for more details). The notation $(.)^{\#}$ stands for the pseudo-inverse.

In order to justify our claims, we have to check that $\mathbf{h}$ is very badly estimated in the space generated by the vectors $\{\mathbf{k}_j\}_{j=1,s}$, i.e. that the $(\mathbf{k}_j^* C_{ssm} \mathbf{k}_j)_{j=1,s}$ are very large. In our evaluations, the channel $h(z)$ results from a spectral raised cosine shaping filter and from 2 multipaths. The roll-off of the shaping filter is $\rho = 0.25$, which corresponds to $\beta = 0.62$, i.e. $\mathcal{I}_1 = [-.5, -.31] \cup [.31, .5[$, $\mathcal{I}_2^+ = [.19, .31]$, $\mathcal{I}_2^- = [-.31, -.19]$, and $\mathcal{I}_3 = [-.19, .19]$. The signal to noise ratio is equal to $50dB$ and we set $N = 9$. We first show in table 1 that the matrix C_{ssm} is very ill conditioned, thus showing that $\mathbf{h}$ is very badly estimated in certain subspaces. In table 2, we give the $(\mathbf{k}_j^* C_{ssm} \mathbf{k}_j)_{j=1,s}$, which

$\max_{\mathbf{x}\neq 0} \frac{\|\|C_{ssm}\mathbf{x}\|\|^2}{\|\|\mathbf{x}\|\|^2}$	$\min_{\mathbf{x}\neq 0} \frac{\|\|C_{ssm}\mathbf{x}\|\|^2}{\|\|\mathbf{x}\|\|^2}$
$31.65dB$	-$50dB$

Table 1: Maximal and minimal singular values of C_{ssm}

as expected, are very large and close from the largest eigenvalue of C_{ssm}. We also plot the asymptotic variances of

$\mathbf{k}_j^* C_{ssm} \mathbf{k}_j$ with $j = 1, \ldots, s = 7$
26.4 ; 25.3 ; 22.7 ; 12.6 ; 11 ; 7 ; 3

Table 2: Values in dB

the $\hat{h}(e^{2i\pi f})$ versus f. Figure 1 shows that $h(e^{2i\pi f})$ is very badly estimated if $f \in \mathcal{I}_3$. We finally mention that the optimally weighted subspace method introduced in [1] does not provide significantly better results in our context.

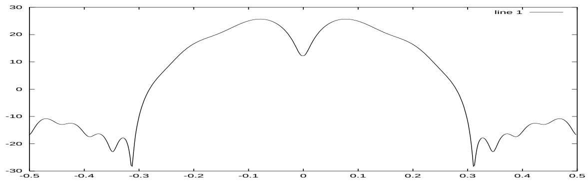

Figure 1: $\lim_{K\to\infty} KE\left[\|\|\hat{h}(e^{2i\pi f}) - h(e^{2i\pi f})\|\|^2\right]$ vs. f.

6. CONCLUSION

In this paper, we have analysed the performance of the subspace method of [6] in the context of band limited channels. We have shown that the associated quadratic form is ill conditionned, and that its numerical kernels contains certain spheroidal wave sequences. We have deduced from this that the subspace method estimates very poorly the transfer function of the channel over a certain interval. We feel that this can be used in order to develop a relevant version of the JOSC algorithms introduced in [10].

7. REFERENCES

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