A second order statistics based algorithm for blind recognition of OFDM based systems

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***Abstract*— An opportunistic radio is a radio able to detect the spectrum unused bands, and to adapt its transmission parameters in order to transmit within these free bands. An opportunistic terminal has also to be able to detect opportunistic access points and to recognize their used standards. As most standards are now based on OFDM modulation with distinct intercarrier spacing, this parameter can be estimated to build standard recognition algorithm. We hence propose in this paper an algorithm for blind estimation of the intercarrier spacing of an OFDM modulation based on the second order statistics of the received signal. The algorithm construction is explained in detail. Some theoretical results are derived and numerical simulations show the gain in regard to the state of art methods.**

I. Introduction

The cognitive radio concept has been first introduced by [1] and consists in developing flexible terminals able to adapt their transmission parameters to their spectral environment. One of the classical application is the opportunistic access to the spectrum resources. An opportunistic access point has advanced sensing capabilities to detect spectrum holes and to adapt its transmission parameters to transmit within these holes. An opportunistic terminal has hence to be able to detect the opportunistic access points, and to recognize their used standards in non data aided contexts.

This paper focuses on this issue and we therefore assume that the opportunistic access point uses a standard based on an OFDM modulation. In practice, this assumption is not restrictive since most popular modulation schemes are based on this modulation (e.g. WiFi [2], WiMAX [3], DVB-T [4], 3GPP/LTE [5]). As each standard has a different inter-carrier spacing value (e.g. 15.625kHz, 10.94kHz, 312.5kHz, 1.116kHz, 15kHz for Fixed WiMAX, Mobile WiMAX, WiFi, DVBT, 3GPP/LTE respectively), this property can be used to perform the standard classification. Consequently, this paper focus on the blind intercarrier estimation of an OFDM signal. Obviously, the proposed results also apply to military applications.

Only a few methods can be found in the litterature dealing with this estimation issue. Mainly in [6], [7], [8], [9] which exploit the correlation induced by the cyclic prefix of an OFDM signal. Indeed, the cyclic prefix of an OFDM signal is added by copying the last D samples at its beginning. The correlation functions of these signals thus exhibit a peak at a time lag equal to the useful time of the OFDM signal. As the useful time is also the inverse of the intercarrier spacing, the system recognition can be performed accordingly. It is nevertheless straightforward to understand that if the cyclic prefix is short (in regard to the useful time of the OFDM symbol) or if the length of the channel impulse response is close to the cyclic prefix length, then the correlation peak is strongly decreased and the method may fail.

In the litterature, an important property is omitted to detect the useful time of an OFDM signal which is that the correlation peak is periodic since it appears for each OFDM symbol. We propose in this paper a new cost function that jointly exploit the correlation induced by the cyclic prefix and its periodicity. In regard to existing methods, the proposed method is equivalent to jointly estimate the useful time and the symbol time of an OFDM signal instead of performing this estimation in a two steps algorithm [9]. As it will be shown in the simulation section, this joint estimation significantly increase the performance of the intercarrier spacing estimation.

The paper is organized as follows: in section II, we recall the signal model and we explain how we perform the estimation. In section III, we give some theoretical results on the behaviour of the proposed algorithm and in section IV we give some numerical estimation of its performance and we compare it with the performance of the state of art algorithms. We conclude in section V.

II. Cost function for blind estimation of the intercarrier spacing

We consider the OFDM signal generated by the transmitter. Its time-continuous expression is as follows

$$x_a(t) = \sum_{k\in\mathbb{Z}} \sum_{n=0}^{N-1} \frac{a_{k,n}}{\sqrt{N}} e^{-2i\pi \frac{n}{NT_c}(t-DT_c-kT_s)} g_a(t-kT_s) \quad (1)$$

where N is the number of subcarriers, D is the cyclic prefix length and $1/T_c$ is the information symbol rate in absence of guard interval (i.e. $D=0$). The inter-carrier spacing is then equal to $1/NT_c$. $\{a_{k,n}\}$ is the transmitted sequence of symbols assumed to be independent and identically distributed (i.i.d). We also assume that all the carriers are used to transmit data. The shaping filter $g_a(t)$ is assumed to be equal to 1 if $0 \le t < T_s = (N+D)T_c$ and 0 otherwise.

In the following, we assume that $D > 0$. It is then straightforward to check that:

$$\forall k \in \mathbb{Z}, \forall t \in [0, DT_c]\ ,\ x_a(kT_s+t+DT_c) = x_a(kT_s+t) \quad (2)$$

For sake of simplicity, we firstly consider a flat fading channel. Theoretical results in more general contexts are derived in next sections.

We also assume that the received signal is sampled at a rate T_e that satisfies the Shannon condition (e.g. $T_e < T_c$). The receiver collects the following samples:

$$y(m) = \sqrt{E_s} x_a(mT_e) + \sigma b(m)$$

where E_s is the signal power, $b(m)$ is the additive white Gaussian noise with zero-mean and unit-variance, and σ^2 is the noise power.

Because of the correlation induced by the cyclic prefix, the correlation function of the received signal, $R_y(n,m) = \mathbb{E}\{y(n+m)y^*(n)\}$, is a sum of 3 terms:

$$\begin{aligned} R_y(n,m) &= R_y(n,0)\delta(m) + R_y(n,\alpha_0)\delta(m-\alpha_0) \\ &+ R_y(n,-\alpha_0)\delta(m+\alpha_0) \end{aligned} \tag{3}$$

where $\alpha_0 = NT_c/T_e$. We recall that the standard used by the terminal can be recognized thanks to the value of α_0. Therefore our main objective is to estimate this parameter.

In the following, we assume that α_0 is an integer. This assumption is purely technical, and if it is not satisfied the proposed algorithm still works. The first term $R_y(n,0)$ simplifies to $E_s + \sigma^2$ and does not depend on n. The second term simplifies to (see Eq. (1)):

$$R_y(n,\alpha_0) = E_s \sum_{k\in\mathbb{Z}} g(n+\alpha_0-k\alpha_0(1+\beta_0))g^*(n-k\alpha_0(1+\beta_0)) \tag{4}$$

where $\beta_0 = D/N$ and $g(n) = g_a(nT_e)$. Note that $\alpha_0(1+\beta_0)$ is the number of samples encompassed in an OFDM symbol, and $\alpha_0\beta_0$ is the number of samples encompassed in the cyclic prefix. The intercarrier spacing estimation of the received signal y will rely on $R_y(n,\alpha_0)$.

As $R_y(n,\alpha_0)$ is a pseudo-periodic function (or a periodic function if $\alpha_0(1+\beta_0)$ is an integer), it can be written as a Fourier series:

$$R_y(n,\alpha_0) = \sum_p R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0) e^{2i\pi \frac{np}{\alpha_0(1+\beta_0)}} \tag{5}$$

In Eq. (5), $R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)$ is the cyclic correlation coefficient of the signal y at the cycle frequency $p/\alpha_0(1+\beta_0)$. $R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)$ is computed as:

$$\lim_{M\to\infty} \frac{1}{M} \sum_{m=0}^{M-1} \mathbb{E}\{y(m+\alpha_0)y^*(m)\} e^{-2i\pi \frac{mp}{\alpha_0(1+\beta_0)}} \tag{6}$$

In the literature (e.g., [6], [7], [8], [9]), the methods aim to estimate both α_0 and $\alpha_0(1+\beta_0)$. They proceed therefore into two steps as follows:

1) They first estimate α_0 as:

$$\widehat{\alpha_0} = \operatorname*{argmax}_{\alpha} \left| R_y^{(0)}(\alpha) \right|$$

This is equivalent to compute the time average autocorrelation function of the received signal and to search for the peak

2) Then, once α_0 has been correctly estimated, they perform the estimation $\alpha_0(1+\beta_0)$ as the first positive cycle frequency of $R_y(n,\alpha_0)$

In this paper, we propose to perform this estimation jointly instead of successively. We therefore introduce the cost function $J_y^{(N_b)}(\alpha,\beta)$:

$$J_y^{(N_b)}(\alpha,\beta) = \frac{1}{2N_b+1} \sum_{p=-N_b}^{N_b} \left| R_y^{(p/\alpha(1+\beta))}(\alpha) \right|^2$$

where N_b is the number of cycle frequencies taken into account to compute this function, and α and β are the tested values of the useful time ($\widehat{NT_c/T_e}$) and the ratio $\widehat{D/N}$. Note that α belongs to a set of strictly positive integers, and β belongs to the set $\{1/4, 1/8, 1/16, 1/32\}$. Note also that if $N_b = 0$, $J_y^{(0)}(\alpha,\beta)$ does not depend on β and we boil down to the first step of the state of art methods.

Thanks to Eq. (3), one can easily check that, whatever the value of β, $J_y^{(N_b)}(\alpha_0,\beta) > 0$ and:

$$\forall \alpha > 0 \ , \ \alpha \neq \alpha_0, J_y^{(N_b)}(\alpha,\beta) = 0$$

Further, as $J_y^{(N_b)}(\alpha_0,\beta) \leq J_y^{(N_b)}(\alpha_0,\beta_0)$, and as β takes only a few values, the estimation performance of α_0 can be improved by also searching for the correct value of β. We hence propose to perform the estimation of α_0 as follows:

$$\widehat{\alpha_0} = \operatorname*{argmax}_{\alpha} \left\{ \max_{\beta\in\{\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32}\}} J_y^{(N_b)}(\alpha,\beta) \right\}$$

Furthermore, the following result holds:

Theorem 1: The cost function $J_y^{(N_b)}(\alpha,\beta)$ is insensitive to frequency offsets.

Proof: With a frequency offset, the received samples write:

$$z(m) = \sqrt{E_s} x_a(mT_e) e^{2i\pi m\Delta f} + \sigma b(m)$$

where Δf is the discrete time equivalent frequency offset. The correlation function of z at time lag α writes:

$$R_z(n,\alpha) = R_y(n,\alpha) e^{2i\pi\alpha\Delta f}$$

and, whatever the values of p and β, $R_z^{(p/\alpha(1+\beta))}(\alpha) = e^{2i\pi\alpha\Delta f} R_y^{(p/\alpha(1+\beta))}(\alpha)$. Hence $J_z^{(N_b)}(\alpha,\beta) = J_y^{(N_b)}(\alpha,\beta)$. ∎

In the following, we assume that the received signal has no frequency offset. In the next section, we derive some theoretical results on the behaviour of the cost function $J_y^{(N_b)}(\alpha,\beta)$ in more general contexts. Before that, we give

III. Some theoretical results on the behaviour of the cost function $J_y^{(N_b)}(\alpha,\beta)$

The performance of the proposed algorithm depends on three factors:

- The value taken by the cost function at point (α_0,β_0).
- The behaviour of the cost function at points $\alpha \neq \alpha_0$. In flat fading channels, we expect the cost function to

vanish as long as $\alpha > 0$ and $\alpha \neq \alpha_0$. In multi-path fading channels, this property is not true anymore.
- In practice, the cost function $J_y(\alpha, \beta)$ is not known and has to estimated. This implies some additionnal estimation noise which impacts the performance of the algorithm.

In this section, these three points will be theoretically analyzed.

A. Impact of N_b on $J_y^{(N_b)}(\alpha_0, \beta_0)$

In this subsection, we inspect the influence of N_b on the numerical value of the cost function at the point (α_0, β_0).

Theorem 2: The modulus of the cyclic correlation coefficient of $R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)$ writes as:

$$\left|R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)\right| = \left|\frac{E_s}{\alpha_0(1+\beta_0)} \frac{\sin\left(\pi \frac{\beta_0}{1+\beta_0} p\right)}{\sin\left(\pi \frac{p}{\alpha_0(1+\beta_0)}\right)}\right|$$

Proof: Thanks to Eqs. (4) and (6), we can rewrite $R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)$ as:

$$R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0) = \frac{E_s}{\alpha_0(1+\beta_0)} \sum_{n=\alpha_0}^{\alpha_0(1+\beta_0)-1} e^{-2i\pi \frac{np}{\alpha_0(1+\beta_0)}}$$

It is then straightforward to deduce the expected result. ■

Note that , when $p > \frac{1}{\beta_0}$, the values taken by $R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)$ are small in regard to the value taken around $p = 0$. We assume in the following that $N_b < \frac{1}{\beta_0}$. $J_y^{(N_b)}(\alpha_0, \beta_0)$ writes then in term of N_b as:

$$\frac{1}{2N_b+1}\left(\frac{E_s}{\alpha_0(1+\beta_0)}\right)^2 \sum_{p=-N_b}^{N_b} \left|\frac{\sin\left(\pi \frac{\beta_0}{1+\beta_0} p\right)}{\sin\left(\pi \frac{p}{\alpha_0(1+\beta_0)}\right)}\right|^2$$

As long as $N_b < \frac{1}{\beta_0}$, $J_y^{(N_b)}(\alpha_0, \beta_0)$ is a decreasing function of N_b. This result means that a good choice of N_b to ensure a great value of $J_y^{(N_b)}(\alpha_0, \beta_0)$ is $N_b = 0$.

In multi-path channel context, the received signal writes:

$$z(m) = \sqrt{E_s} \sum_{l=0}^{L-1} h(l) x_a((m-l)T_e) + \sigma b(m) \qquad (7)$$

where $\{h(l)\}_l$ are the equivalent channel impulse response of length L. We assume that $L < \alpha_0\beta_0$, i.e., the channel impulse response is shorter than the cyclic prefix.

Theorem 3: The cyclic correlation coefficient of the signal z, $R_z^{(\frac{p}{\alpha_0(1+\beta_0)})}(\alpha_0)$ writes

$$R_y^{(\frac{p}{\alpha_0(1+\beta_0)})}(\alpha_0) \int_0^1 H(\nu) H^*\left(\nu - \frac{p}{\alpha_0(1+\beta_0)}\right) d\nu$$

where $H(\nu) = \sum_l h(l) e^{-2i\pi l\nu}$.

Proof: The correlation function of the signal $z(p)$ writes as:

$$R_z(n, \alpha_0) = \sum_l |h(l)|^2 R_y(n-l, \alpha_0)$$

Using the Fourier decomposition of $R_y(n-l, \alpha_0)$ given by (5), we deduce:

$$R_z^{(\frac{p}{\alpha_0(1+\beta_0)})}(\alpha_0) = R_y^{(\frac{p}{\alpha_0(1+\beta_0)})}(\alpha_0) \sum_l |h(l)|^2 e^{-2i\pi \frac{pl}{\alpha_0(1+\beta_0)}}$$

Using the Parseval equality on the sum in this latter equation leads to the expected result. ■

From Theorem 3, we deduce that as long as p takes low values and the coherence bandwidth of the channel is large enough, the choice of N_b can be done as if the channel was flat. Consequently, to increase the value of $J_y^{(N_b)}(\alpha_0, \beta_0)$, we would like to choose N_b as small as possible.

B. Impact of N_b on $J_y^{(N_b)}(\alpha, \beta)$ when $\alpha \neq \alpha_0$

We consider the model of received signal given by Eq. (7) and we now evaluate the autocorrelation function of the signal $z(m)$. As the channel impulse response length L is smaller than $\alpha_0\beta_0$, the signal $z(m)$ is correlated at points $\alpha = v$ and $\alpha = \alpha_0 + v$ where $|v| < \alpha_0\beta_0$. Indeed, its autocorrelation function writes in term of the autocorrelation function of the signal $y(m)$ as:

$$R_z(n, v) = \sum_l h(l) h^*(l-v) R_y(n-l+v, 0) \qquad (8)$$

$$R_z(n, \alpha_0 + v) = \sum_l h(l) h^*(l-v) R_y(n-l+v, \alpha_0) \qquad (9)$$

The following results hold:

Theorem 4: If $|v| < \alpha_0\beta_0$ and as $\alpha \neq \alpha_0$, $J_z^{(N_b)}(\alpha, \beta)$ simplifies as follows

$$J_z^{(N_b)}(\alpha, \beta) = \frac{1}{2N_b+1}\left|R_z^{(0)}(\alpha)\right|^2$$

Theorem 5: If $v \neq 0$, and as long as it does not exist $(p_1, p_2) \in \{-N_b, \cdots, N_b\}^2$ such as $\frac{p_1}{(\alpha_0+v)(1+\beta)} = \frac{p_2}{(\alpha_0)(1+\beta_0)}$, $J_z^{(N_b)}(\alpha_0 + v, \beta)$ simplifies as follows

$$J_z^{(N_b)}(\alpha_0 + v, \beta) = \frac{1}{2N_b+1}\left|R_z^{(0)}(\alpha_0 + v)\right|^2$$

We just give the proof of the second theorem. The first one can be proved using the same method.

Proof: Thanks to (9), it is straight forward to check that $R_z(n, \alpha_0 + v)$ is a time periodic function of n of period $\alpha_0(1+\beta_0)$. As $J_z^{(N_b)}(\alpha_0 + v, \beta)$ tests the cycle frequencies $\frac{p}{(\alpha_0+v)(1+\beta)}$ which are not the cycle frequencies of the signal z at the time lag $\alpha_0 + v$, we can deduce that:

$$\forall p \neq 0, \left|R_z^{(p/(\alpha_0+v)(1+\beta))}(\alpha_0 + v)\right|^2 = 0 \qquad (10)$$

■

From these both theorems, we deduce that decreasing the value of $J_y^{(N_b)}(\alpha, \beta)$ for $\alpha \neq \alpha_0$ needs to choosing a large value for N_b.

C. Impact of N_b on the estimation accuracy of $\hat{J}_y^{(N_b)}(\alpha,\beta)$

In practice, the cost function $J_y^{(N_b)}(\alpha,\beta)$ is not known and has to be estimated. We denote $\hat{J}_y^{(N_b)}(\alpha,\beta)$ the estimate of $J_y^{(N_b)}(\alpha,\beta)$ given by:

$$\hat{J}_y^{(N_b)}(\alpha,\beta) = \frac{1}{2N_b+1}\sum_{p=-N_b}^{N_b}\left|\hat{R}_y^{(p/\alpha(1+\beta))}(\alpha)\right|^2$$

where

$$\hat{R}_y^{(p/\alpha(1+\beta))}(\alpha) = \frac{1}{M}\sum_{m=0}^{M-1} y(u+\alpha)y^*(m)e^{-2i\pi\frac{mp}{\alpha(1+\beta)}} \quad (11)$$

where M is the number of received samples. In this section, we analyze some properties of $\hat{J}_y^{(N_b)}(\alpha_0,\beta_0)$ under the following assumptions:

- M large enough in order to obtain asymptotic behaviors.
- Low signal-to-noise ratio (SNR), i.e. $E_s << \sigma^2$.
- Flat fading channel.

1) Behaviour of $\hat{J}_y^{(N_b)}(\alpha_0,\beta_0)$: We first address the behaviour of the cost function estimate at point α_0,β_0: $\hat{J}_y^{(N_b)}(\alpha_0,\beta_0)$. Thanks to (11) and to the large number law, $\hat{R}_y^{(p/\alpha(1+\beta))}(\alpha)$ is asymptotically normal. Further, as

$$\lim_{M\to\infty}\mathbb{E}\left\{\hat{R}_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)\right\} = R_y^{(p/\alpha_0(1+\beta_0))}(\alpha_0)$$

which does not vanish, we can deduce (see [10] for more details) that $\sqrt{M}\left(\hat{J}_y^{(N_b)}(\alpha_0,\beta_0) - J_y^{(N_b)}(\alpha_0,\beta_0)\right)$ is also asymptotically normal, with zero-mean and variance $\sigma_{\hat{J}}$. The impact of N_b on $J_y^{(N_b)}(\alpha_0,\beta_0)$ has already been discussed in section III-A. The impact on its variance is described by the following result:

Theorem 6: As long as $E_s << \sigma^2$, $\sigma_{\hat{J}}^2$ the asymptotical variance writes as:

$$\sigma_{\hat{J}}^2 = \mathcal{O}\left(\frac{1}{2N_b+1}\right)$$

We omit the proof of this result in this paper. For additionnal information, the reader can refer to [10].

2) Behaviour of $\hat{J}_y^{(N_b)}(\alpha,\beta)$ when $\alpha \neq \alpha_0$: We now focus on the impact of N_b on the mean and variance of $\hat{J}_y^{(N_b)}(\alpha,\beta)$ when $\alpha \neq \alpha_0$.

Theorem 7: The asymptotical mean of $\hat{J}_y^{(N_b)}(\alpha,\beta)$ equals:

$$\lim_{M\to\infty} M\mathbb{E}\{\hat{J}_y^{(N_b)}(\alpha,\beta)\} = \left(E_s+\sigma^2\right)^2 + \mathcal{O}\left(E_s^2\right)$$

As long as $E_s << \sigma^2$, the asymptotic mean of $\hat{J}_y^{(N_b)}(\alpha,\beta)$ does not depend on N_b.

Proof: This result is obtained thanks to some calculations. Indeed, $M\mathbb{E}\{\hat{J}_y^{(N_b)}(\alpha,\beta)\}$ writes as $\frac{1}{2N_b+1}\sum_{n=-N_b}^{N_b} M\mathbb{E}|\hat{R}_y^{(p/\alpha(1+\beta))}(\alpha)|^2$. In terms of the received signal $y(m)$, $M\mathbb{E}|\hat{R}_y^{(p/\alpha(1+\beta))}(\alpha)|^2$ equals:

$$\frac{1}{M}\sum_{\substack{m_1\\m_2}}\mathbb{E}\{y(m_1+\alpha)y^*(m_1)y^*(m_2+\alpha)y(m_2)\}e^{-\frac{2i\pi k(m_1-m_2)}{\alpha(1+\beta)}} \quad (12)$$

Writing the fourth order moment in terms of the fourth order cumulant, $\mathbb{E}\{y(m_1+\alpha)y^*(m_1)y^*(m_2+\alpha)y(m_2)\}$ expands to:

$$\begin{aligned} & \mathrm{cum}(y(m_1+\alpha), y^*(m_1), y^*(m_2+\alpha), y(m_2)) \quad (13)\\ + \; & \mathbb{E}\{y(m_1+\alpha)y^*(m_1)\}\mathbb{E}\{y^*(m_2+\alpha)y(m_2)\}\\ + \; & \mathbb{E}\{y(m_1+\alpha)y(m_2)\}\mathbb{E}\{y^*(m_1+\alpha)y^*(m_1)\}\\ + \; & \mathbb{E}\{y(m_1+\alpha)y^*(m_2+\alpha)\}\mathbb{E}\{y^*(m_1)y(m_2)\}\end{aligned}$$

As $y(m+\alpha)$ and $y(m)$ are independent when $\alpha > 0$ and $\alpha \neq \alpha_0$, the first term vanishes. The second term vanishes since $\alpha \neq \alpha_0$. The third term vanishes since $y(m)$ is circular. The fourth order moment rewrites hence, in terms of the autocorrelation function of the received signal:

$$\begin{aligned} & \mathbb{E}\{y(m_1+\alpha)y^*(m_1)y^*(m_2+\alpha)y(m_2)\}\\ = \; & R_y(m_2+\alpha, m_1-m_2)R_y^*(m_2, m_1-m_2)\end{aligned}$$

We deduce from this result that Eq. (12) does not vanish only if $m_1 = m_2$, $m_1 = m_2+\alpha_0$ or $m_1 = m_2-\alpha_0$. If $m_1 = m_2$, we obtain the sum simplifies to $(E_s+\sigma^2)^2$. For the other cases, we get the term $\mathcal{O}(E_s)$. This concludes the proof. ∎

We now focus on the asymptotic variance of $\hat{J}_y^{(N_b)}(\alpha,\beta)$ defined as:

$$\lim_{M\to\infty} M^2\mathbb{E}\left|\hat{J}_y^{(N_b)}(\alpha,\beta) - \mathbb{E}\left\{\hat{J}_y^{(N_b)}(\alpha,\beta)\right\}\right|^2$$

The following result holds:

Theorem 8: As long as $\alpha \neq \frac{\alpha_0}{2}$, the asymptotic variance of $\hat{J}_y^{(N_b)}(\alpha,\beta)$ is given by:

$$\mathcal{O}\left(\frac{\left(E_s+\sigma^2\right)^4}{2N_b+1}\right) + \mathcal{O}\left(E_s^4\right)$$

Proof: We just give a sketch of proof: To compute this variance we first write $\mathbb{E}\left|M\hat{J}_y^{(N_b)}(\alpha,\beta)\right|^2$ in terms of the cyclic coefficients. We then apply the decomposition (13) to the cyclic coefficients (instead of applying it to the signal y). We get:

$$\begin{aligned} & M^2\mathbb{E}\left|\hat{J}_y^{(N_b)}(\alpha,\beta) - \mathbb{E}\left\{\hat{J}_y^{(N_b)}(\alpha,\beta)\right\}\right|^2\\ = \; & \frac{M^2}{(2N_b+1)^2}\sum_{k_1,k_2}\left|\mathbb{E}\left\{\hat{R}_y^{(k_1/\alpha(1+\beta))}(\alpha)\hat{R}_y^{(k_2/\alpha(1+\beta))}(\alpha)\right\}\right|^2\\ + \; & \frac{M^2}{(2N_b+1)^2}\sum_{k_1,k_2}\left|\mathbb{E}\left\{\hat{R}_y^{(k_1/\alpha(1+\beta))}(\alpha)\left(\hat{R}_y^{(k_2/\alpha(1+\beta))}(\alpha)\right)^*\right\}\right|^2\end{aligned}$$

The result only requires to compute both expectations. The first one $\mathbb{E}\{\hat{R}_y^{(k_1/\alpha(1+\beta))}(\alpha)\hat{R}_y^{(k_2/\alpha(1+\beta))}(\alpha)\}$ vanishes except if $\alpha = \frac{\alpha_0}{2}$. The second one can be computed as the expectation that has been computed for the asymptotic mean. ∎

As a conclusion, we have shown that N_b has a negative impact on the value reached by the cost function $J_y^{(N_b)}$ at point (α_0,β_0) but a positive impact on the value reached by the cost function $J_y^{(N_b)}$ at point (α,β) with $\alpha \neq \alpha_0$ and also a positive impact on the variance of the estimation noise. The choice of N_b is hence a trade-off. This fact is illustrated in the next section devoted to numerical simulations.

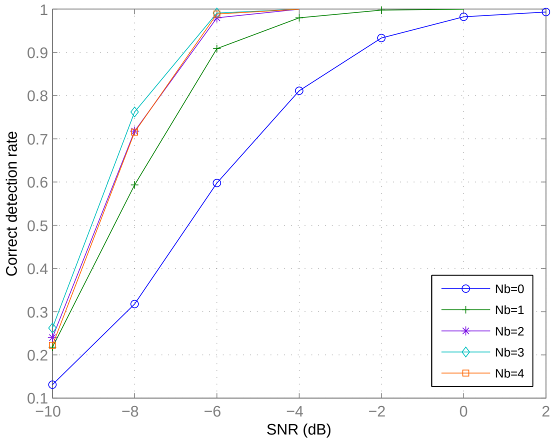

Fig. 1. Correct detection rate vs. SNR ($D/N = 1/4$)

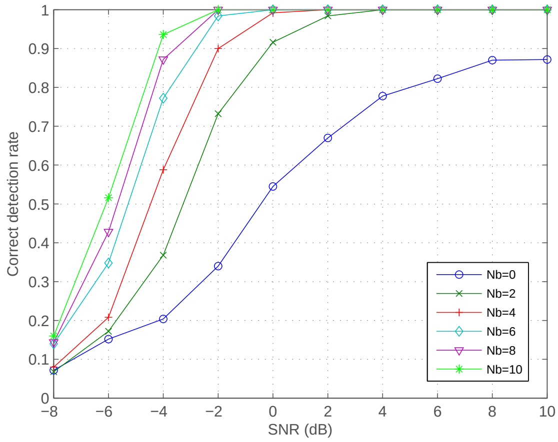

Fig. 2. Correct detection rate vs. SNR ($D/N = 1/32$)

IV. Simulations

We have evaluated the performance of our algorithm thanks to Monte-Carlo simulations as the number of realisations where the estimate of α_0 match the correct value of α_0 up to 1%. Another possible criterion is the mean square error between our estimate and the correct value but as we expect to recognize a system we had rather focus on the good detection rate. As in practice the smallest gap between the intercarrier spacing of the different standard is 1%, we also used this value for the estimation of the performance of our algorithm.

Two cases have been considered. For each case, we used an oversampling rate $T_c/T_e = 2$, 5 OFDM symbols have been generated and a multi-path fading channel has been simulated with a length equal to $L = \alpha_0\beta_0/4$. A Gaussian noise has also been added to the simulated received samples before applying the estimation algorithm. Its variance is defined as:

$$\sigma^2 = \frac{T_c}{T_e}\frac{1}{M}\sum_{m=0}^{M-1}\left|\sum_{l=1}^{L} h(l)s_a((m-l)T_e)\right|^2 10^{-\mathrm{SNR}/10}$$

where λ_l and τ_l are the magnitude and delay of the l^{th} path respectively. The performance has been evaluated over 1000 realizations. Note that the case $N_b = 0$ corresponds to the existing method.

On Figure 1, we have generated OFDM signals with $\beta_0 = D/N = 1/4$. For these signals, $N = 64$ carriers have been used to transmit data. Obviously, the performance of α_0 estimation is significantly increase with $N_b = 2$ or $N_b = 3$. We can also observe that the performance obtained with $N_b = 4$ is worst than the one obtained with $N_b = 3$ which illustrates the conclusion of the previous section.

On Figure 2, we have generated OFDM signals with $\beta_0 = D/N = 1/32$ and $N = 2048$. Once again, taking into account several cyclic frequencies significantly improve the performance of α_0's estimation. Further in these contexts, the existing method is not able to perform a correct estimation of α_0 in 100% of cases whereas our method does.

V. Conclusion

In this paper, we have introduced a new method based on the second order statistics of an OFDM signal to perform the estimation of the intercarrier spacing. The algorithm consists in jointly detecting the correlation induced by the cyclic prefix and its periodicity. We have therefore introduced a cost function that depends on N_b the number of cyclic frequencies considered to perform the detection. Several theoretical results on the impact of this parameter have been derived and it has been shown that this parameter should be chosen as a trade-off between the different factors that impact on the estimation performance. These results have been illustrated with some simulations where the gain induced by our algorithm in regard to the existing method has been highlighted.

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