Training Design for CFO Estimation in OFDM over Correlated Multipath Fading Channels

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***Abstract*— Carrier frequency offset (CFO) estimation is a key challenge in multicarrier systems such as OFDM. Often, this task is carried out using a preamble made of a number, say J, of repetitive-slots (RS). Here, we address the issue of optimal RS preamble design using the Cramér-Rao bound. We show that the optimal value of J is a trade-off between the multipath diversity gain and the number of unknowns to be estimated. In the case of correlated channel taps, we show that uniform power loading of the active subcarriers is not optimal (in contrast with the uncorrelated case) and a better power loading scheme is proposed. The theoretical results are supported by computer simulations.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has become the standard of choice for wireless LAN's such as IEEE 802.11a, and is being considered for several IEEE 802.11 and 802.16 standards. The popularity of OFDM arises from the balanced transceiver complexity, and the time-frequency granularity that it offers. However, synchronization continues to be a critical challenge. Here, we focus on carrier frequency offset (CFO) synchronization, assuming perfect frame and timing synchronization. As CFO destroys the orthogonality between the subcarriers leading to intercarrier interference (ICI) and to significant performance degradation, an accurate CFO estimation is therefore needed.

Data-aided CFO estimation in current OFDM systems employs a preamble made of a number, say J, of repetitive slots (RS) [1][2]. This preamble is obtained by using one OFDM symbol after deactivating all subcarriers except those whose frequencies are integer multiples of J [3]. It has been shown that the RS-based CFO maximum likelihood (ML) estimator is identical to the NSC-based ML estimator in the absence of virtual subcarriers (VSC) [4][5][1]. Here, we address the issue of optimal preamble design using the Cramér-Rao bound as the metric. This involves optimizing J and the power loading. We show that the optimal value of J is a trade-off between the multipath diversity gain (in a sense to be defined later in the paper) and the number of unknowns to be estimated. In the case of uncorrelated channel taps, uniform power loading is optimal. In the case of correlated channel taps, we show that uniform power loading of the active subcarriers is no longer optimal and a better power loading scheme is proposed. The theoretical results are supported by computer simulations.

Notation: Superscripts * and ^{T} will denote conjugate transposition and transposition. $\mathcal{R}\left[\cdot\right]$ and $\mathrm{Tr}\left\{\cdot\right\}$ denote the real part and the trace operators, respectively.

II. Signal Model and Preliminaries

We consider a single OFDM pilot symbol, which can be written as

$$x(n) = \frac{1}{\sqrt{K}} \sum_{k\in\mathbb{K}} s_k e^{j2\pi nk/N}, \qquad n = 0, \cdots, N-1, \quad (1)$$

where N denotes the total number of subcarriers, $\mathbb{K}$ is the subset of active subcarriers with K denoting its cardinality, and s_k is the pilot symbol transmitted over the kth subcarrier. We assume that the power of the transmitted OFDM pilot symbol is fixed and set to one without loss of generality. This implies $\sum_{k\in\mathbb{K}} |s_k|^2 = K$, but the power distribution among the active subcarriers is not constrained to be uniform.

The frequency-selective channel is modeled as an FIR filter with impulse response $\mathbf{h} = [h_0, ..., h_{L-1}]^T$, and frequency-domain response $H_k := \sum_{l=0}^{L-1} h_l e^{-j2\pi kl/N}$. We let $\mathbf{H}_c$ denote the circulant matrix with first column, $[h_0, h_1, ...h_{L-1}, 0, ..., 0]^T$. In order to analyze the performance of CFO estimation, we will assume the following statistical model:

(AS1) The channel impulse response vector $\mathbf{h}$ is a zero-mean circularly symmetric Gaussian vector with covariance matrix $\mathbf{R}_h = \mathbb{E}[\mathbf{h}\mathbf{h}^H]$.

We assume a standard cyclic prefix (CP) based OFDM system with CP length $L_{cp} \geq L-1$. In the absence of CFO, the symbol-rate sampled noise-free signal can, after removing the CP, be written as $\mathbf{y} = \mathbf{H}_c\mathbf{x}$ where $\mathbf{x} = [x(0), \cdots, x(N-1)]^T$. In the presence of CFO and noise, this signal becomes

$$y(n) \;\; = \;\; \frac{1}{\sqrt{K}} e^{j2\pi\nu n/N} \sum_{k\in\mathbb{K}} s_k H_k e^{j2\pi nk/N} + w(n)$$

with $n = 0, \cdots, N-1$ and where ν is the normalized CFO and $w(.)$ is AWGN with variance σ^2.

III. Repetitive Slots-Based CFO Estimation

In OFDM systems, pilot symbols are usually transmitted prior to the information frame. For example in IEEE802.11a,

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[1]VSC are the subcarriers at the edges of the allocated frequency band that are deactivated in order to avoid interference with adjacent channels.

the preamble consists of a number of repetitive slots. Using this preamble structure, a ML CFO estimator was proposed in [6][4]. Correlation-based estimation methods were proposed in [1], [2], [4]. In this section, we briefly review these approaches (see [5] for a detailed overview).

A. RS-based Signal Model

We assume that the preamble is a single OFDM block made of J identical sub-blocks of length $M = N/J$ each, with M an integer. Such a pilot OFDM symbol is obtained by deactivating all subcarriers whose frequencies are not multiple of J, i.e,

$$\mathbb{K} = \{mJ, m = 0, \cdots, M-1\} - \mathcal{V} \tag{2}$$

where $\mathcal{V}$ is the set of VSC. The size of $\mathbb{K}$ satisfies $K \leq M$. The case where the preamble is made up of a sequence of identical OFDM blocks can be treated similarly.

The RS structure of the preamble allows for a simple estimation of the CFO thus avoiding the computational complexity of the joint CFO-channel estimation. Further, for $K < L$, the channel cannot be identified while CFO may still be identified; indeed in this case there would be more unknowns than equations.

Using the RS structure and the results in Section II, the received signal can be rewritten as (with $n = m + \ell Q$ and $m = 0, ..., M-1;\ \ell = 0, ..., J-1$)

$$y(m + \ell M) = z(m)\ e^{j2\pi\nu\ell/J} + w(m + \ell M)\ , \tag{3}$$

where $z(m) = e^{j2\pi\nu m/N} \sum_{n=0}^{N-1} H_c(m,n)x(n)$, and the last equality follows from the repetitive slot structure $x(m + \ell M) = x(m)$, $m = 0, ..., M-1$, $\ell = 0, ..., J-1$. Although $z(m)$ depends upon ν, the ν dependent factor can be absorbed into $\mathbf{x}$, and hence ignored. Estimating ν is now in a standard form: harmonic retrieval in additive noise for multivariate variables. The vector $\mathbf{z} = [z(0), ..., z(M-1)]^T$ may be modelled as an unknown non-random vector. Note that the acquisition range increases with J and is given by $[-J/2, J/2)$. The identifiability issue is addressed in Section IV.

B. Deterministic Maximum Likelihood Method

In [6], [4] and [5], the vector $\mathbf{z}$ was modeled as an unknown $(M \times 1)$ deterministic parameter vector, and the following RS-based DML (RS-DML) estimator was derived:

$$\hat{\nu}_{\mathrm{RS}} = \arg\max_{\nu} \sum_{m=0}^{M-1} \zeta_\nu(m) \tag{4}$$

where

$$\zeta_\nu(m) = \frac{1}{J} \left| \sum_{\ell=0}^{J-1} e^{-j2\pi\ell\nu/J} y(m + \ell M) \right|^2 . \tag{5}$$

The estimator of (4) has a simple interpretation: we estimate the CFO by peak-picking the sum of M periodograms, where each periodogram is obtained by sub-sampling the data by a factor of M. Larger values of J increase the acquisition range, but with a corresponding reduction in M, the number of averaged periodograms and concomitant loss of accuracy. Performance issues are studied in Section IV.

The above estimator can also be expressed as [4], [5]

$$\hat{\nu}_{\mathrm{RS}} = \arg\max_{\nu} \sum_{\ell=1}^{J-1} \mathcal{R}\left[r(\ell M) e^{-j2\pi\ell\nu/J} \right] \tag{6}$$

where $r(\tau)$ is the autocorrelation sequence

$$r(\tau) = \sum_{n=0}^{N-\tau-1} y^*(n) y(n+\tau)\ .$$

When $J = 2$, the estimator can be given in closed-form as

$$\hat{\nu}_{\mathrm{RS}} = \frac{1}{\pi} \arg\{r(N/2)\}\ . \tag{7}$$

which is the estimator proposed in [1]. If $J > 2$, no closed-form solution is available for the optimization problem in eq. (6). An approximate ML estimator which is computationally simpler than the above ML estimator was proposed in [4].

IV. Performance Analysis

Here, we analytically assess the performance of the RS-ML estimator using the Cramér-Rao bound (CRB), which is an algorithm-independent measure of estimation performance. Recall that ML estimators asymptotically achieve the CRB. We derive the conditional CRB (conditioned on the channel), the average (over the channel) CRB, and the Modified CRB (CRB). In deriving these bounds, we assume, as in the RS-ML method, that the relationship between $\mathbf{z}$, ν, the channel and the s_k's, is not exploited during the estimation procedure, i.e. $\mathbf{z}$ in eq. (3) is considered to be an arbitrary vector. The unknown parameter vector is then $[\mathbf{z}_R^T, \mathbf{z}_I^T, \nu]^T$ where $\mathbf{z}_R$ and $\mathbf{z}_I$ are respectively the real and imaginary parts of $\mathbf{z}$.

A. Conditional CRB

Here, the unknown parameter vector is considered to be deterministic. Since the noise is circularly symmetric, Gaussian and white, the conditional CRB (CCRB) on CFO estimation is, after some derivations, found to be

$$\mathrm{CCRB}_{\mathrm{RS}}(\nu) = \frac{1}{\gamma_h} \quad \frac{3}{2\pi^2 N (1 - 1/J^2)} \tag{8}$$

where γ_h is the conditional (on the channel) signal-to-noise ratio (SNR)

$$\gamma_h := \frac{\frac{1}{K} \sum_{k \in \mathbb{K}} |H_k|^2 |s_k|^2}{\sigma^2} \tag{9}$$

with $\mathbb{K}$ given in eq. (2). The CCRB is useful to predict the performance of CFO estimation for a particular channel. It is worth pointing out that the CCRB is not a function of the phases of the s_k's[2].

It is instructive to rewrite the CCRB as follows

$$\mathrm{CCRB}_{\mathrm{RS}}(\nu) = \xi_h \quad f(J) \quad \mathrm{CRB}_{\mathrm{AWGN}}(\nu) \tag{10}$$

[2]If the nonlinearity of high power amplifiers is an issue, the phase of the s_k's should be chosen to minimize the peak-to-average-power ratio of the transmitted signal.

where

$$\xi_h = \mathbb{E}[\gamma_h]/\gamma_h \quad (11)$$

$$f(J) = \frac{1-1/N^2}{1-1/J^2} \quad (12)$$

and

$$\mathrm{CRB}_{\mathrm{AWGN}}(\nu) = \frac{3}{2\pi^2 N(1-1/N^2)\overline{\gamma}}$$

is the CRB on the estimation of the frequency, ν, of a single exponential, $\{\exp(j2\pi\nu n/N),\ n=0,\cdots,N-1\}$, in AWGN with equivalent SNR given by

$$\overline{\gamma} := \mathbb{E}[\gamma_h] = \frac{\frac{1}{K}\sum_{k\in\mathbb{K}}\sigma_H^2(k)|s_k|^2}{\sigma^2} = \frac{\mathrm{Tr}\left\{\mathbf{R}_h\mathbf{G}\right\}}{\sigma^2} \quad (13)$$

where the expectation is with respect to the channel, and $\mathbf{G}$ is an $(L\times L)$ symmetric Toeplitz matrix whose (ℓ,ℓ') element is $g(\ell,\ell') = \frac{1}{K}\sum_{k\in\mathbb{K}}|s_k|^2\exp(j2\pi k(\ell-\ell')/N)$.

The parameter ξ_h captures the variations (or randomness) of the channel; its distribution is a function of $\mathbf{R}_h$, J and the power distribution among the active subcarriers. The function $f(J)$ measures the above-mentioned amplitude uncertainty. The latter monotonically decreases with J. If $J=N$, $f(N)=1$, which is the minimum uncertainty; in this case the amplitude of the noise-free received signal is constant. Note that $f(1)=\infty$; indeed in this case the complex amplitude of the noise-free received signal has no repetitive structure, thus $\mathrm{CCRB}_{\mathrm{RS}}(\nu)=\infty$, i.e., the CFO is non-identifiable if the preamble has no repetitive structure. However, if NSC-based estimation is used, then, the CFO could be identifiable even if there is no repetitive structure provided that some of the subcarriers are deactivated [4][5]. The CCRB associated with the NSC approach can be found in [3].

Although the amplitude uncertainty, $f(J)$, decreases with J, setting J to the maximum value, N, is in general not a good choice since ξ_h depends on J. In fact $\overline{\xi}=\mathbb{E}[\xi_h]$ increases with J, as shown by the closed form expressions presented next, and as also confirmed by simulations. In Subsection V-C, we investigate the issue of optimal choice of J.

B. Average CRB

The average CRB (ACRB) is given by

$$\mathrm{ACRB}_{\mathrm{RS}}(\nu) := \mathbb{E}[\mathrm{CCRB}_{\mathrm{RS}}(\nu)] = \overline{\xi}f(J)\mathrm{CRB}_{\mathrm{AWGN}}(\nu) \quad (14)$$

where the expectation is with respect to the channel.

Deriving closed-form expressions for $\overline{\xi}$ and thus ACRB in general does not seem tractable except for the special cases listed below. In the general case Monte-Carlo simulations can be used to accurately evaluate the ACRB.

$\overline{\xi}=\infty$ cases. Under (AS1), this occurs if $i)$ $J=N$ (i.e. $K=1$) regardless of L and $\mathbf{R}_h$, $ii)$ $L=1$ (i.e. flat fading) regardless of J, or $iii)$ $\mathrm{rank}(\mathbf{R}_h)=1$ (i.e. fully correlated paths) regardless of J. Indeed in all the above case γ_h is exponentially distributed, which implies that $\mathbb{E}[1/\gamma_h]=\infty$ and thus $\mathrm{ACRB}_{\mathrm{RS}}(\nu)=\infty$. Hence, for Rayleigh fading channels, in order for CFO estimation to be consistent, multipath diversity must not only be available (i.e. $L>1$ and $\mathrm{rank}(\mathbf{R}_h)>1$) but also captured through the choice of J, which dictates the number of subcarrier frequencies modulated.

$\mathbf{R}_h=\sigma_H^2\mathbf{I}$, uniform power loading and no VSC case. In this case, $K=M=N/J$ and multipath diversity is available iff $L>1$. The elements of $\mathbf{G}$ are given by $g(\ell,\ell')=\delta(<\ell-\ell'>_K)$ where $<\cdot>_K$ denotes arithmetic modulo K. Thus, γ_h can be written as $\gamma_h = \sigma^{-2}\sum_{i=0}^{\min(K-1,L-1)}|\sum_{j=0}^{\lfloor(L-1)/K\rfloor}h_{i+jK}|^2$ where $h_\ell=0$ if $\ell\geq L$. If L/K is an integer when $K<L$ or if $K\geq L$, $[2\sigma^2/(\sigma_H^2\max(1,L/K))]\gamma_h$ is a chi-square variable with $2\min(K-1,L-1)+2$ degrees of freedom and its inverse has an inverse-chi-square distribution with the same degrees of freedom. Therefore, $\overline{\xi}$ is found to be[3]

$$\overline{\xi} = \frac{L}{\min(K-1,L-1)\max(1,L/K)} \quad (15)$$

where we assume that $K\leq L$ with L/K an integer or that $K\geq L$. In the above expression, $\min(K-1,L-1)$ can be interpreted as the multipath diversity order captured by activating the subcarriers in $\mathbb{K}$. As pointed out in the previous case, $\overline{\xi}=\infty$ if $K=1$ or $L=1$. Further, it is worth pointing out that if we capture full multipath diversity, i.e, $K\geq L$ (i.e. $J\leq M/L$), then $\overline{\xi}=L/(L-1)$, which is independent of J. This implies that since $f(J)$ decreases with J, the value for J that minimizes the ACRB necessarily satisfies $J\geq N/L$. Further, in this case, the ACRB monotonically decreases with L, which confirms that multipath diversity improves the accuracy of CFO estimates. Finally, when $L\gg 1$ and $K\geq L$, the ACRB gets close to the (RS-based) CRB obtained in the case of AWGN channels. A similar observation was made on the capacity of multipath wideband channels [7], i.e., the capacity of the channel becomes similar to that of the AWGN channel when the number of channel taps is very large.

C. Modified CRB

Here, we define the modified CRB (MCRB) as the inverse of the averaged (over the channel) conditional Fisher information matrix. The MCRB is given by

$$\mathrm{MCRB}_{\mathrm{RS}}(\nu) = \frac{J+1}{2(2J-1)}f(J)\mathrm{CRB}_{\mathrm{AWGN}}(\nu). \quad (16)$$

Using Jenssen's inequality, one can easily prove that the MCRB is a lower bound on the ACRB. The MCRB can be evaluated analytically, unlike the ACRB. The latter is however tighter.

V. Repetitive-Slot Pilot Design

For a random channel, we can intuitively explain why an optimal value of J should exist as follows. First, using the harmonic retrieval model of eq. (2), we can state that the multipath diversity order for CFO estimation when activating K subcarriers is given by $\min(K,L)$ provided that the channel taps are not fully correlated. Indeed, an (L)-tap channel has

[3]The mean of an inverse-chi-square random variable with n $(n>2)$ degrees of freedom is equal to $1/(n-2)$

$L-1$ zeros; so if $K < L$, the channel zeros are on the unit circle and their phases coincide with the activated subcarrier frequencies, the received signal would be identically zero. Thus, since K decreases with J, a large value of J may undermine the multipath diversity order. To capture the maximum multipath diversity gain when estimating the CFO, one has to set $K \geq L$. On the other hand, the number of unknown parameters $\{z(m), m = 0, \cdots, M-1\}$ in the RS signal model in eq. (3) increases when J decreases. This increase in the number of unknown parameters, that we here refer to as amplitude uncertainty, deteriorates the performance of CFO estimation[4]. Therefore, the optimal value of J should provide a trade-off between the multipath diversity gain and the amplitude uncertainty. Indeed, if $J = 1$, multipath diversity gain is maximum but so is the amplitude uncertainty; in this case the complex envelope of the signal is an arbitrary $(N \times 1)$ vector. If $J = N/2$, only the zero-frequency subcarrier is activated, thus the multipath diversity order is minimum (equal to one) but so is the amplitude uncertainty.

A. CFO Identifiability

Using the RS-based method, identifiability of the CFO in the acquisition range $[-J/2, J/2]$ is guaranteed if $\gamma_h \neq 0$. This implies that identifiability is lost if $H_k = 0, \forall k \in \mathbb{K}$. This condition is channel-dependent. A necessary and sufficient condition for identifiability *regardless* of the channel realization is, provided $\mathbf{h}$ is not the null vector, given by

$$J \geq 2 \ \text{ and } \ K \geq L$$

The second part of the above condition guarantees that even when all $L-1$ channel zeros coincide with activated subcarriers, $\gamma_h \neq 0$ would hold. For example, If $L = N/4 - 1$ and in the absence of VSC, strict identifiability of the CFO is guaranteed only for $J = 2$ and $J = 4$. However, since the channel impulse response $\mathbf{h}$ is a continuous-valued random vector, the probability of identifiability loss when $K \leq L-1$ is zero. Therefore, we only focus on the estimation performance when deriving the optimal value of J.

B. Power loading design

Here, the number of activated subcarriers and their positions are fixed, i.e., $\mathbb{K}$ (therefore J) is fixed. In the literature, the symbols $\{s_k, \ k \in \mathbb{K}\}$ are always set to have the same magnitude. To the best of our knowledge, no proof of optimality of such a choice is available in the literature. Further, in the case of correlated scattering, simulations results (presented later) show that this choice is not optimal.

First, note that the sequence $\{|s_k|^2, \ k \in \mathbb{K}\}$ that minimizes the CCRB of ν under the constraint of constant transmit power, $\sum_{k\in\mathbb{K}} |s_k|^2 = K$, is channel dependent. Further, no general closed-form expression is available for the ACRB. Although the ACRB can be estimated empirically, its numerical minimization with respect to the K-dimensional parameter set $\{|s_k|^2, \ k \in \mathbb{K}\}$ is prohibitive since for each parameter vector candidate, a large number of Monte-Carlo simulations are required to accurately estimate the ACRB. Optimizing $\{|s_k|^2, \ k \in \mathbb{K}\}$ using the MCRB leads to a somewhat similar conclusion as in the case when the channel is known at the transmitter, i.e., assigning all the transmit power to the subcarrier at which $\sigma_H^2(k)$ is maximum, which also maximizes the average SNR, $\overline{\gamma}$. However, this strategy lacks multipath diversity since only one subcarrier is activated.

To achieve a compromise between the average SNR and multipath diversity gain, we propose to use the following power loading

$$|s_k|^2 = \frac{K\sigma_H^2(k)}{\sum_{k\in\mathbb{K}} \sigma_H^2(k)} \tag{17}$$

where we have used a normalization factor to ensure that $\sum_{k\in\mathbb{K}} |s_k|^2 = K$. Notice that in the case of uncorrelated scattering, the $\sigma_H^2(k)$'s are equal to each other and therefore the $|s_k|^2$'s should all be equal to unity. This however does not hold in the case of correlated scattering.

We do not claim that the power distribution in eq. (17) is optimum in any sense. It simply tries to find a trade-off between maximizing the average receive SNR and multipath diversity gain. As mentioned above, a rigorous optimization of the ACRB wrt the $|s_k|^2$ seems intractable. Finally, if the channel statistics are unknown, then distributing the transmit power uniformly across all active subcarriers seems adequate.

C. Design of J

Because the channel is random, with $J = N$ (i.e., $K = 1$) the multipath diversity is of order one. Thus, a deep fade at single active subcarrier would cause a very low SNR, thus making CFO estimation very difficult. Simultaneous deep fades at several subcarriers are less likely than a fade at one subcarrier. Setting $J = 1$ (i.e. all subcarriers are modulated) offers maximum multipath diversity gain; but this also maximizes the amplitude uncertainty and thus makes the CFO unidentifiable because there would be more unknown parameters to estimate than equations[5]. Therefore, there must be a trade-off between these two phenomena.

The MCRB is useless when choosing the value of J. Indeed, the information about the multipath diversity is lost in the MCRB expression. This explain why the MCRB is a monotonically decreasing function of J. The CCRB leads to a channel-dependent optimal value of J, and is therefore not useful because the channel is unknown at the transmitter. Hence we resort to the ACRB. Since the ACRB cannot be analyzed analytically, we estimate it using Monte-Carlo simulations. The value of J that minimizes the ACRB estimate is selected to be the ‘optimal’ number of repetitions.

[4]This is valid because the noise-free received signal is linear in the $z(m)$'s. This would not necessarily hold true if the noise-free received signal is nonlinear in the $z(m)$'s.

[5]Since the transmitted symbols are known and the H_k's are parameterized by L coefficients only, the CFO can still be identifiable even when all subcarriers are modulated ($J = 1$). However, this will require joint channel and CFO estimation which complicates the CFO estimation algorithm.

VI. Simulation Results

We consider an OFDM pilot symbol with a total of 64 subcarriers and no virtual subcarriers. A cyclic prefix of length 15 is inserted. The discrete-time channel has 16 taps (i.e. $L = 16$). The channel coefficients are assumed to be Rayleigh with exponential power delay profile, with decay parameter 0.2, i.e., $\sigma_{h_\ell}^2 := \mathbb{E}[|h_\ell|^2] = \alpha e^{-0.2\ell}$, and covariance matrix given by

$$[\mathbf{R}_h]_{i,j} = \sigma_{h_i}\sigma_{h_j}\rho^{|i-j|}$$

where $\rho \in [0, 1)$ is the correlation factor. The scaling factor α is chosen such that $\mathrm{Tr}\{\mathbf{R}_h\} = 1$. We assume that the channel is static over an OFDM symbol. The empirical mean square error (MSE) of CFO estimates are calculated using 10,000 Monte-Carlo runs.

Figure 1 illustrates the MSE of the RS-ML CFO estimate and the corresponding ACRB for the uniform and proposed power loading schemes versus ρ, the correlation parameter, with $J = 4$ and SNR=10dB where SNR is defined as $1/\sigma^2$. We first observe that the MSE associated with uniform power loading increases with ρ because the multipath diversity gain of the channel decreases with ρ. Recall that when $\rho = 1$, the taps are fully correlated and the ACRB and the average MSE thus become infinite. Figure 1 also shows that the proposed power loading significantly outperforms the uniform power loading when the channel taps are highly correlated. Moreover when channel statistics are used at the transmitter, as it is the case for the proposed power distribution, channel correlation becomes a benefit for CFO estimation, as long as $\mathbf{R}_h$ remains full-rank. A similar benefit was obtained when calculating the capacity of multipath channels [8].

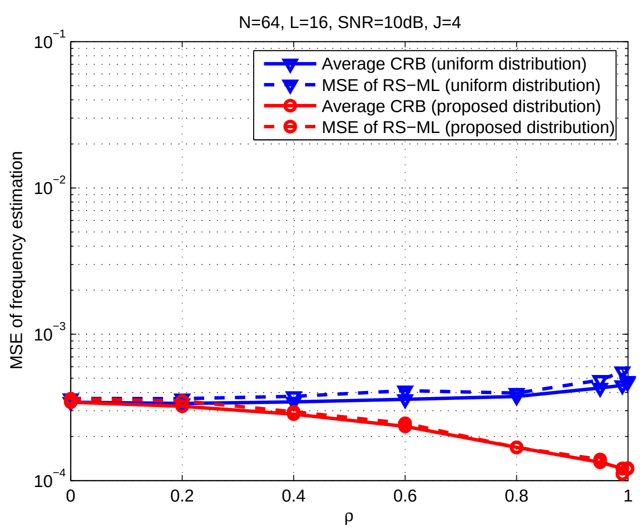

Fig. 1. MSE of RS-ML and ACRB on CFO estimation vs ρ

Using the proposed power loading scheme, Figure 2 shows the MSE of the RS-ML estimate and the ACRB versus J for different values of ρ, when SNR=10dB. Note that the optimal values of J predicted by the ACRB is in agreement with the value of J which minimizes the empirical MSE. Notice also the presence of outliers [9], particularly for large values of ρ and/or J.

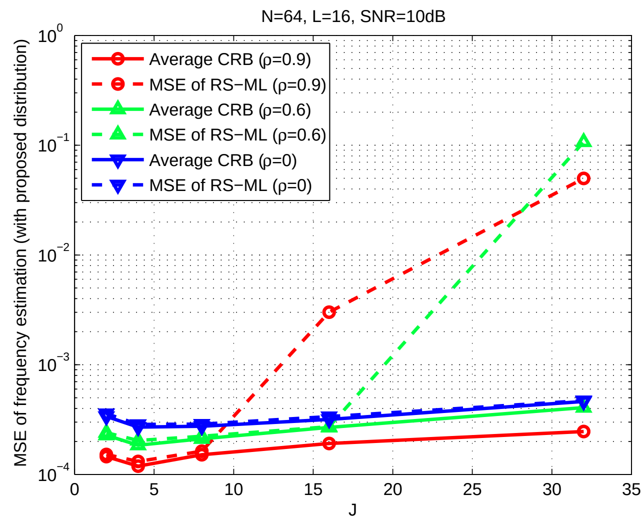

Fig. 2. MSE of RS-ML and ACRB on CFO estimation vs J

VII. Conclusions

We have analyzed the performance of CFO estimators based on a single repetitive-slot pilot symbol in the context of OFDM systems. We have provided insights into how this preamble should be designed. In the case of correlated Rayleigh fading channels with known statistics at the receiver, a new power loading scheme was proposed and shown through simulations to outperform the conventional uniform power loading scheme.

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