ASYMPTOTIC ANALYSIS OF BLIND CYCLIC CORRELATION BASED SYMBOL RATE ESTIMATION

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ABSTRACT

We consider symbol rate estimation of an unknown signal linearly modulated by a sequence of symbols. We rely on the received signal is cyclostationarity, and consider an existing estimator obtained by maximizing in the cyclic domain a (possibly weighted) sum of modulus squares of cyclic correlation estimates. Although widely used, this estimate seems not to have been studied rigorously when the number of samples N is large. In this paper, we study rigorously the asymptotic behavior of this estimate. We establish consistency and asymptotic normality of the estimate, prove that its convergence rate is $N^{3/2}$, and calculate in closed form its asymptotic variance. The obtained formula allows us to discuss in relevant way on the influence of the number of estimated cyclic correlation coefficients to take into account in the cost function to maximize.

1 INTRODUCTION

We assume available the complex envelope $y_a(t)$ of the continuous time signal transmitted by an *unknown* transmitter using a linear digital modulation scheme. Signal $y_a(t)$ can thus be written as

$$y_a(t) = \sum_{k \in \mathbb{Z}} s_k h_a(t - kT_s) + w_a(t)$$

where $\{s_n\}$ is a centered unit variance i.i.d. sequence of symbols, where $1/T_s$ is the baud rate of the transmitter and $h_a(t)$ is the convolution of the shaping filter with the unknown multi-path channel. $w_a(t)$ represents an additive white Gaussian noise. In certain applications such a passive listening or automatic classification of modulation, it can be interesting to characterize some parameters of the transmitter, e.g., the symbol period T_s and the symbol alphabet ([7]).

In this paper, we consider the estimation problem of T_s from a sampled version $y(n) = y_a(nT_e)$ of $y_a(t)$, where T_e is assumed to satisfy the condition $T_e < T_s/4$. As it is well known, $y_a(t)$ is a cyclostationary signal and its cyclic frequencies are integer multiples of $1/T_s$. In this paper, we assume as usual, that the bandwidth of $y_a(t)$ is an interval $[-\frac{1+\rho}{2T_s}, \frac{1+\rho}{2T_s}]$ for some parameter $0 < \rho < 1$ called the excess bandwidth. In this case, $1/T_s$ is the unique strictly positive cyclic frequency of $y_a(t)$. The discrete time signal $y(n)$ can be written as follows

$$y(n) = \sum_{k \in \mathbb{Z}} s_k h\left(n - \frac{k}{\alpha_0}\right) + w(n)$$

with $h(n) = h_a(nT_e)$ and $w(n) = w_a(nT_e)$. α_0 is given by $\alpha_0 = T_e/T_s$. $y(n)$ is thus cyclostationary with a unique strictly positive cyclic frequency α_0. Estimating T_s is thus equivalent to estimating α_0.

Let $r(n,\tau)$ be the autocorrelation function of $y(n)$ defined by $\mathbb{E}[y(n+\tau)y^*(n)]$. In our context, we obtain that

$$r(n,\tau) = \sum_{k=-1}^{1} r^{(k\alpha_0)}(\tau) e^{2i\pi k\alpha_0 n} \tag{1}$$

where $r^{(k\alpha_0)}(\tau)$ is the cyclic correlation of $y(n)$ at cyclic frequency $k\alpha_0$ and at delay τ. For each τ, $r^{(\alpha)}(\tau) = 0$ when α is different from $-\alpha_0$, 0 and α_0. As α_0 is the sole non-zero positive cyclic frequency, the autocorrelation function provides enough information to estimate α_0 as follows ([3]) :

$$\alpha_0 = arg \max_{\alpha \in \mathcal{I}} J_W(\alpha)$$

where

$$J_W(\alpha) = \mathbf{r}^{(\alpha)^*} W \mathbf{r}^{(\alpha)}$$

with

$$\mathbf{r}^{(\alpha)} = [r^{(\alpha)}(-\Upsilon), \cdots, r^{(\alpha)}(\Upsilon)]^T$$

and the superscript * stands for conjugate transposition. $2\Upsilon + 1$ is the number of cyclic correlation coefficients considered. Here, $\mathcal{I}$ is a closed interval included in $]0, \frac{1}{2}[$ and W a weighting positive Hermitian matrix. Of course, in practice, we only have a finite number N of samples. Therefore $\mathbf{r}^{(\alpha)}$ is unknown and has to be estimated by the classical empirical estimate $\hat{\mathbf{r}}_N^{(\alpha)}$ defined by

$$\hat{\mathbf{r}}_N^{(\alpha)} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{y}_2(n) e^{-2i\pi\alpha n}$$

where $\mathbf{y}_2(n) = [y(n-\Upsilon), \cdots, y(n+\Upsilon)]^T y^*(n)$. Cycle α_0 can thus be estimated by the element $\hat{\alpha}_{N,W}$ of $\mathcal{I}$ defined by

$$\hat{\alpha}_{N,W} = arg \max_{\alpha \in \mathcal{I}} J_{N,W}(\alpha)$$

where

$$J_{N,W}(\alpha) = \hat{\mathbf{r}}_N^{(\alpha)^*} W \hat{\mathbf{r}}_N^{(\alpha)}$$

Although widely used (at least for $W = I$), the behavior of this estimate seems not to have been studied rigorously in previous works. In this paper, we prove the consistency and the asymptotic normality of $\hat{\alpha}_{N,W}$. We also show that the rate of convergence of $\hat{\alpha}_{N,W}$ is $N^{-3/2}$ and calculate in closed form its asymptotic variance.

The starting point of our work is based on the observation, first formulated in [9], that certain cyclic frequency estimation problems can be formulated as frequency estimation problems of sinusoids corrupted by additive cyclostationary noise, and that the cost function $J_{N,I}(\alpha)$ is equivalent to a periodogram. The most standard approach to studying the asymptotic behavior of periodogram estimates is to introduce an auxiliary non linear least square problem ([4, 5, 1, 6, 9]). However, this approach cannot be used to analyze the behavior of $\hat{\alpha}_{N,W}$ for $W \neq I$. More important, calculating the variance of $\hat{\alpha}_{N,I}$ by this approach necessitates very complicated computations which do not lead to interpretable closed form expressions. In this paper, we show that the use of the auxiliary non linear least square criterion is not necessary, and we analyze successfully the asymptotic properties of $\hat{\alpha}_{N,W}$ by using an alternative approach.

This paper is organized as follows. In Section 2 we explain the connections between the estimation of α_0 and the estimation of the frequency of a sinusoid corrupted by additive noise. In Section 3, we state our main results and give the closed form expression of the asymptotic variance of $\hat{\alpha}_{N,W}$. We take benefit of this formula to show the importance of a relevant choice of parameter Υ. In Section 4, we finally illustrate the variance formula by some numerical evaluations.

2 HARMONIC RETRIEVAL LINKS

The connection between the estimation problem of α_0 and the estimation of sinusoid frequencies follows from the following simple observation ([9]) : let $\mathbf{e}(n)$ be the centered $(2\Upsilon+1)$–dimensional stochastic process defined by

$$\mathbf{e}(n) = \mathbf{y}_2(n) - \mathbb{E}[\mathbf{y}_2(n)]. \quad (2)$$

It follows readily from Equation (1) that

$$\mathbf{y}_2(n) = \sum_{k=-1}^{1} \mathbf{r}^{(k\alpha_0)} e^{2i\pi k\alpha_0 n} + \mathbf{e}(n).$$

$\mathbf{y}_2(n)$ can thus be interpreted as a sum of vector-valued sinusoids of frequencies 0, α_0 and $-\alpha_0$ corrupted by the additive noise $\mathbf{e}(n)$. Moreover, the criterion $J_{N,W}$ is a weighted periodogram because

$$J_{N,W}(\alpha) = \left\| \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{y}_2(n) e^{-2i\pi\alpha n} \right\|_W^2$$

where the norm $\|x\|_W^2$ is defined by $\|x\|_W^2 = x^* W x$ for every $(2\Upsilon+1)$–dimensional vector x. Several works have been devoted to sinusoid frequency estimation (e.g., [4, 5, 1, 6]). However, the differences between the present context and these works are threefold : i) $\mathbf{e}(n)$ is not stationary but cyclostationary, ii) $\mathbf{y}_2(n)$ is multivariate process and iii) the periodogram is weighted. [9] showed that most of the results of [4, 5, 1, 6] can be generalized when $\mathbf{e}(n)$ is cyclostationary (in [9], $\mathbf{e}(n)$ has a simpler structure because it is periodically correlated; however, the general approach developed in [4, 5, 1, 6] remains valid). However, [9] considered the case $\Upsilon = 0$ ($\mathbf{y}_2(n)$ is a scalar process). In this context, one can follow without problem the classical approach based on the introduction of the non linear least square estimation (NLSE) auxiliary problem :

$$[\hat{\theta}_N, \hat{\alpha}_N^{(K)}] = arg \min_{\alpha \in \mathcal{I}, \theta \in \mathbb{C}^{3(2\Upsilon+1)}} K_N(\theta, \alpha)$$

where $K_N(\theta,\alpha)$ is the cost function defined by

$$K_N(\theta,\alpha) = \frac{1}{N} \sum_{n=0}^{N-1} \left\| \mathbf{y}_2(n) - \sum_{k=-1}^{1} \theta_k e^{2i\pi k\alpha n} \right\|_I^2$$

with $\theta = [\theta_{-1}, \theta_0, \theta_1]^T$. Consistency and asymptotic normality of the NLSE estimate $\hat{\alpha}_N^{(K)}$ are rather easy to obtain. Moreover, it can be shown that the estimates $\hat{\alpha}_N^{(K)}$ and $\hat{\alpha}_{N,I}$ are equivalent, i.e., that they have the same asymptotic performance. In particular, the asymptotic variance of $\hat{\alpha}_{N,I}$ is equal to the asymptotic variance of $\hat{\alpha}_N^{(K)}$.

However, calculating of the variance of $\hat{\alpha}_N^{(K)}$ is quite difficult because it requires evaluation of the asymptotic covariance matrix of the vector-valued estimate $(\hat{\theta}_N, \hat{\alpha}_N^{(K)})$. Using this approach, it is quite difficult (not to say impossible) to obtain an interpretable closed form expression if the parameter $\Upsilon \neq 0$. Moreover, the present NLSE approach cannot be generalized to analyze the properties of the weighted estimates $\hat{\alpha}_{N,W}$ for $W \neq I$. Therefore, we have analyzed the properties of $\hat{\alpha}_{N,W}$ without introducing the auxiliary NLSE problem.

3 ASYMPTOTIC PERFORMANCE

Our results are valid under the following assumption :

Assumption 1 *Let $\mathbf{e}(n)$ be the multi-variate random process defined by (2). We denote $\mathbf{e}^{(0)}(n) = \mathbf{e}(n)$ and $\mathbf{e}^{(1)}(n) = \overline{\mathbf{e}(n)}$. The overbar stands for complex conjugation. We assume from now on that $\mathbf{e}(n)$ satisfies the following cumulants property :*

$$\forall L, \exists \mathcal{M}_L, \forall n_1, \nu_i = 0,1$$

$$\sum_{n_2,\ldots,n_L=-\infty}^{+\infty} \left\| cum_L\left(\mathbf{e}^{(\nu_1)}(n_1), \cdots, \mathbf{e}^{(\nu_L)}(n_L)\right) \right\| \leq \mathcal{M}_L$$

This assumption extends the classical mixing condition verified by statationary process ([4]) to cyclostationary process. This assumption, which is valid if the impulse response of the filter $h_a(t)$ is finite (a condition however uncompatible with the hypothesis that $y_a(t)$ has a band limited spectrum), is certainly not restrictive (it is however difficult to derive conditions on $h_a(t)$ guaranteeing assumption 1). Under Assumption 1, it is possible to prove the following lemma which generalizes the key result of the approach of [4, 5, 1, 6, 9].

Lemma 1 *Assume that $\mathbf{e}(n)$ satisfies assumption 1. Let*

$$\mathbf{s}_N^{(K)}(\alpha) = \frac{1}{N^{K+1}} \sum_{n=0}^{N-1} n^K \mathbf{e}(n) e^{2i\pi\alpha n}.$$

Then,

$$\forall K \in \mathbb{N}, \quad \sup_{\alpha \in [0,1]} \left\| \mathbf{s}_N^{(K)}(\alpha) \right\| \xrightarrow{a.s.} 0, \quad as \quad N \to \infty$$

It is possible to show the following theorem without introducing the auxiliary NLSE problem.

Theorem 1 *Assume that the positive matrix W satisfies $R^{(\alpha_0)*} W R^{(\alpha_0)} > 0$ (if $W > 0$, this condition is of course verified). Then, the cyclic frequency estimate $\hat{\alpha}_{N,W}$ converges, almost surely, as $N \to \infty$, as follows :*

$$(\hat{\alpha}_{N,W} - \alpha_0) \to 0 \quad \text{and} \quad N(\hat{\alpha}_{N,W} - \alpha_0) \to 0$$

Due to lack of space, we just sketch the proof. We put $\mathbf{r}(n) = \sum_{k=-1}^{1} \mathbf{r}^{(k\alpha_0)} e^{2i\pi k\alpha_0 n}$. $\mathbf{y}_2(n)$ can thus be written as $\mathbf{y}_2(n) = \mathbf{r}(n) + \mathbf{e}(n)$. According to this previous expression of $\mathbf{y}_2(n)$, one remarks that $J_{N,W}(\alpha)$ is a sum of 4 terms, the first of which being $T_{N,W}(\alpha) = \left\| 1/N \sum_{n=0}^{N-1} \mathbf{r}(n) e^{-2i\pi\alpha n} \right\|_W^2$. Using lemma 1 for $K = 1$, one can show that $J_{N,W}(\hat{\alpha}_{N,W})$ and $T_{N,W}(\hat{\alpha}_{N,W})$ have the same behaviour if $N \to \infty$. Using that $\hat{\alpha}_{N,W}$ is the argument of the maximum of $J_{N,W}(\alpha)$, one can prove that the sequence

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} e^{-2i\pi n(\alpha_0 - \hat{\alpha}_{N,W})}$$

converges to 1 as $N \to \infty$, a condition which in turn implies Theorem 1. 2

The asymptotic normality and the convergence rate of $\hat{\alpha}_{N,W}$ are next obtained by a second order Taylor expansion of $J_{N,W}$ around α_0. Setting $\left.\frac{\partial J_{N,W}(\alpha)}{\partial \alpha}\right|_{\alpha=\hat{\alpha}_{N,W}} = 0$, we obtain that

$$\delta\hat{\alpha}_{N,W} = -\left[\left.\frac{\partial^2 J_{N,W}(\alpha)}{(\partial\alpha)^2}\right|_{\alpha=\tilde{\alpha}_{N,W}} \right]^{-1} \left.\frac{\partial J_{N,W}(\alpha)}{\partial\alpha}\right|_{\alpha=\alpha_0} \quad (3)$$

where $\delta\hat{\alpha}_{N,W} = (\hat{\alpha}_{N,W} - \alpha_0)$ and $\tilde{\alpha}_{N,W}$ are respectively the estimation error and a scalar belonging to $[\alpha_0, \hat{\alpha}_{N,W}]$. We put

$$a_{N,W} = \frac{1}{N^2} \left.\frac{\partial^2 J_{N,W}(\alpha)}{(\partial\alpha)^2}\right|_{\alpha=\tilde{\alpha}_{N,W}} \quad (4)$$

$$b_{N,W} = \frac{1}{\sqrt{N}} \left.\frac{\partial J_{N,W}(\alpha)}{\partial\alpha}\right|_{\alpha=\alpha_0} \quad (5)$$

Plugging (4) and (5) back into (3), we obtain

$$N^{\frac{3}{2}} \delta\hat{\alpha}_{N,W} = -a_{N,W}^{-1} b_{N,W} \quad (6)$$

By using the mixing condition, some simple algebraic results and Lemma 1, we arise at the following result.

Theorem 2

$$a_{N,W} \xrightarrow{a.s.} \gamma_{\mathcal{A}}$$

$$b_{N,W} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \gamma_{\mathcal{B}})$$

where $\mathcal{N}(0,\gamma_{\mathcal{B}})$ is a normal distribution with zero mean and variance $\gamma_{\mathcal{B}} = \lim_{N\to\infty} \mathbb{E}[b_{N,W} b_{N,W}^*]$.

Using Theorem 2 and (6), we finally deduce the main theorem.

Theorem 3 *Under Assumption 1, we have*

$$N^{\frac{3}{2}} (\hat{\alpha}_{N,W} - \alpha_0) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \gamma)$$

where $\gamma = \lim_{N\to\infty} N^3 \mathbb{E}[\delta\hat{\alpha}_{N,W}\, \delta\hat{\alpha}_{N,W}^*] = \gamma_{\mathcal{A}}^{-1} \gamma_{\mathcal{B}} \gamma_{\mathcal{A}}^{-1}$.

The convergence rate of $\hat{\alpha}_{N,W}$ is thus $N^{\frac{3}{2}}$ as expected. One of the benefit of this approach over NLSE approach is that γ can be calculated quite easily. In effect, the criterion $J_{N,W}$ is a function of the single variable α while the NLSE depends on the $(3(2\Upsilon+1)+1)$–dimensional parameter vector (θ, α). Calculation of the asymptotic variance of the NLSE estimate necessitates computating the Taylor expansion of $K_N(\theta, \alpha)$, the calculation of the asymptotic covariance matrix $\Gamma_{\mathcal{B}}$ of its first derivative with respect to the parameter (θ, α) at (θ_0, α_0), the calculation of the limit $\Gamma_{\mathcal{A}}$ of its second derivative at (θ_0, α_0), and finally the computation of the last entry of the matrix $\Gamma_{\mathcal{A}}^{-1} \Gamma_{\mathcal{B}} \Gamma_{\mathcal{A}}^{-1}$. This approach leads to very tedious calculations. In contrast, using our scheme, we obtain easily that

$$\gamma = \frac{3}{\pi^2} \left(\mathbf{R}^{(\alpha_0)^*} \mathbf{W} \mathbf{R}^{(\alpha_0)} \right)^{-2} \mathbf{R}^{(\alpha_0)^*} \mathbf{W} \mathbf{G} \mathbf{W} \mathbf{R}^{(\alpha_0)} \quad (7)$$

where

$$\mathbf{R}^{(\alpha_0)} = \begin{bmatrix} \mathbf{r}^{(\alpha_0)} \\ \overline{\mathbf{r}^{(\alpha_0)}} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} W & 0 \\ 0 & \overline{W} \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} \Gamma & -\Gamma^{(c)} \\ -\overline{\Gamma^{(c)}} & \overline{\Gamma} \end{bmatrix}$$

with

$$\Gamma = \lim_{N\to\infty} N \mathbb{E}\left[\delta\hat{\mathbf{r}}_N^{(\alpha_0)} \delta\hat{\mathbf{r}}_N^{(\alpha_0)^*} \right]$$

$$\Gamma^{(c)} = \lim_{N\to\infty} N \mathbb{E}\left[\delta\hat{\mathbf{r}}_N^{(\alpha_0)} \delta\hat{\mathbf{r}}_N^{(\alpha_0)^T} \right]$$

and

$$\delta\hat{\mathbf{r}}_N^{(\alpha_0)} = \hat{\mathbf{r}}_N^{(\alpha_0)} - \mathbf{r}^{(\alpha_0)}$$

A natural question is that of choosing the weighting matrix W. As in [8], we can observe that for each W, the left hand side of (7) is less than $\frac{3}{\pi^2} \left(\mathbf{R}^{(\alpha_0)^*} \mathbf{G}^{\#} \mathbf{R}^{(\alpha_0)} \right)^{-1}$, which represents the value of (7) for $\mathbf{W} = \mathbf{G}^{\#}$. However, the matrix $\mathbf{G}^{\#}$ is not a block diagonal matrix. Therefore, there does not exist any matrix W for which (7) coincides with $\frac{3}{\pi^2} \left(\mathbf{R}^{(\alpha_0)^*} \mathbf{G}^{\#} \mathbf{R}^{(\alpha_0)} \right)^{-1}$. Hence, the determination of an optimal weighting matrix seems to be a difficult problem.

Actually, we now show that $W = I$ leads to a very low variance estimate if Υ is chosen large enough. For this, we assume for the sake of simplicity that the sequence s_n is Gaussian and circular: in this case, the expressions of Γ and Γ_c are simple, and the expression of γ for $W = I$ can be interpreted. Using that the spectrum of $y_a(t)$ is limited to the interval $[-\frac{1+\rho}{2T_s}, \frac{1+\rho}{2T_s}]$ for some parameter $0 < \rho < 1$, one can show that

$$\Gamma = \int_0^1 S^{(0)}(e^{2i\pi f}) \overline{S^{(0)}(e^{2i\pi(f-\alpha_0)})} \mathcal{D}_\Upsilon(e^{2i\pi f}) \mathcal{D}_\Upsilon^*(e^{2i\pi f}) df$$

$$\Gamma^{(c)} = \int_0^1 \left(S^{(\alpha_0)}(e^{2i\pi f}) \right)^2 \mathcal{D}_\Upsilon(e^{2i\pi f}) \mathcal{D}_\Upsilon^T(e^{2i\pi f}) df$$

where $S^{(\alpha)}(e^{2i\pi f})$ is the spectrum of $y(n)$ at cyclic frequency α and $\mathcal{D}_\Upsilon(e^{2i\pi f})$ is the vector $[e^{-2i\pi\Upsilon f}, \cdots, e^{2i\pi\Upsilon f}]^T$. As $T_e < T_s/4$, $S^{(\alpha)}(e^{2i\pi f})$ coincides (up to a scalar factor) for each α with the cyclic spectrum at cyclic frequency α/T_e of the continuous time signal $y_a(t)$. From this, we get immediately that $S^{(0)}(e^{2i\pi f}) = \frac{T_e}{T_s} |h(e^{2i\pi f})|^2 + \sigma^2$ and that $S^{(\alpha_0)}(e^{2i\pi f}) = \frac{T_e}{T_s} h(e^{2i\pi f}) h^*(e^{2i\pi(f-\alpha_0)})$. Here, $h(z)$ denotes the function $h(z) = \sum_k h_a(kT_e) z^{-k}$ and σ^2 is the variance of the white Gaussian noise. Let us consider the case $\sigma^2 = 0$. The product $S^{(0)}(e^{2i\pi f}) \overline{S^{(0)}(e^{2i\pi(f-\alpha_0)})}$ coincides with $|S^{(\alpha_0)}(e^{2i\pi f})|^2$. After some straightforward manipulations, we get that the asymptotic variance of the estimate $\hat{\alpha}_{N,I}$ is given by

$$\gamma = \frac{3\Phi(\Upsilon)}{2\pi^2 \left\| \mathbf{r}^{(\alpha_0)} \right\|^4} \quad (8)$$

where

$$\Phi(\Upsilon) = \int_0^1 |S^{(\alpha_0)}(e^{2i\pi f})|^2 \left| \sum_{\tau=-\Upsilon}^{\Upsilon} r^{(\alpha_0)}(\tau) e^{-2i\pi f\tau} \right|^2 df$$

$$-\text{Re}\left[\int_0^1 \overline{S^{(\alpha_0)}(e^{2i\pi f})}^2 (\sum_{\tau=-\Upsilon}^{\Upsilon} r^{(\alpha_0)}(\tau)e^{-2i\pi f\tau})^2 df\right].$$

The important point is to observe that if Υ is large enough,

$$\sum_{\tau=-\Upsilon}^{\Upsilon} r^{(\alpha_0)}(\tau)e^{-2i\pi f\tau} \simeq S^{(\alpha_0)}(e^{2i\pi f}),$$

and that $\Phi(\Upsilon) \simeq 0$. It turns out that, in the noiseless case, the asymptotic variance of $\hat{\alpha}_{N,I}$ tends to 0 as Υ increases. We note that this result holds because we have chosen $W = I$. In particular, it seems that if $W \neq I$, the estimate $\hat{\alpha}_{N,W}$ does not have the above property. Therefore, the choice $W = I$ seems the most reasonable, at least if the signal to noise ratio is high enough.

4 NUMERICAL RESULTS

We now illustrate the above results by some numerical evaluations of the asymptotic variances of $\hat{\alpha}_{N,W}$. We assume that the shaping filter used by the transmitter is a square root raised cosine with excess bandwidth $\rho = 0.2$, and that the propagation channel is a multi-path channel. The amplitude, phases and time delays of the channel are random variables, and each curve is obtained by averaging the variances of our estimate over 100 different realizations of the propagation channel. α_0 is equal to $1/4$. In order to compare a weighted estimate with $\hat{\alpha}_{N,I}$, we consider the particular matrix $W_0 = \Gamma^{-1}$ which is particularly relevant in the context of detection of cyclic frequencies ([2]). In [2], the authors propose to test that a frequency α is a cyclic frequency of $y(n)$ by comparing the statistics $\left\|\hat{\mathbf{r}}_N^{(\alpha)}\right\|_W^2$ to a threshold. Using inverse of the asymptotic covariance if $\hat{\mathbf{r}}_N^{(\alpha)}$ was shown to be convenient because the asymptotic distribution of $\left\|\hat{\mathbf{r}}_N^{(\alpha)}\right\|_W^2$ does not depend on α in the null hypothesis. W_0 thus coincides with this matrix for $\alpha = \alpha_0$. In order to obtain quasi band limited signals, we have used a degree $M = 60$ polynomial $h(z)$ in order to calculate matrices Γ and Γ_c. In Figure 1, SNR is equal to $20dB$. We compare the unweighted estimate and the weighted estimate. We notice that the number of lags Υ taken into account has a great influence on the variance. This curve confirms that the variance of $\hat{\alpha}_{N,I}$ decreases toward 0 when Υ increases. Moreover, the unweighted estimator has a lower variance for Υ large enough.

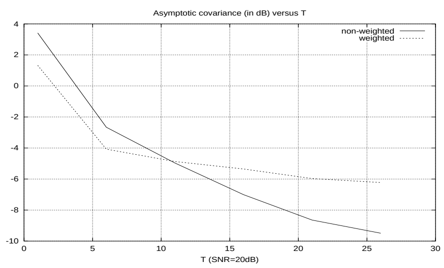

Figure 1: γ (in dB) versus Υ

We now study the covariance versus SNR. In Figure 2, for sake of clarity, we only plot the $\Upsilon = M$ case. The variance of the weighted estimator does not converge toward 0 when σ^2 decreases, while the curve of the unweighted estimator confirms our calculations.

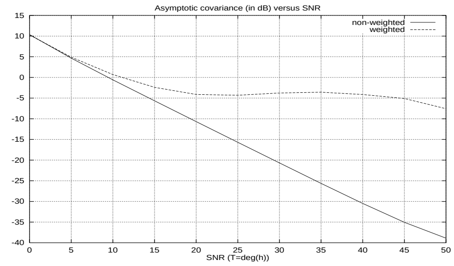

Figure 2: γ (in dB) versus SNR, for $\Upsilon = M$

5 CONCLUSION

In this paper, we have studied rigorously the asymptotic performance of a baud rate estimator. Using a new approach, we have shown that the estimator is consistent, asymptotically normal, and that its convergence speed is $N^{-3/2}$. Our approach also leads to an interpretable closed form of the asymptotic variance. We have taken advantage of our interpretable formula for choosing certain important parameters.

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