Estimation of the Ricean K-Factor from Noisy Complex Channel Coefficients

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***Abstract*—The estimation of the Ricean K factor in case of noisy complex channel coefficients is addressed. A new deterministic estimator is designed, and the relevant deterministic Cramer-Rao Lower Bound (CRLB) is derived. It is shown by simulation that the new estimator outperforms the existing ones in term of both bias and Normalized Mean Square Error (NMSE), and is close to the CRLB. We also design two Bayesian estimators, which outperform the deterministic ones and are robust to small sample size (≤ 30 samples), but as a drawback are more complex.**

I. Introduction

In wireless communications, the statistical behavior of the envelope of the fading component is well represented by the Ricean distribution [1]. The Ricean K factor is known to be an important indicator of the link quality. An accurate estimation of the K parameter is therefore of importance. This estimation problem is addressed in this paper. Especially, we focus on the case when the complex channel coefficients are estimated using known pilots symbols, and as a consequence are noisy, which is a realistic case in practice.

The estimators available in the literature are the following. In [2]–[8], different estimators that use noiseless channel coefficients magnitude are proposed and compared. The estimators developed in [5], [9]–[13] use the noiseless complex channel coefficients. It is shown that using complex coefficients allows better estimation than using magnitude only. All the estimators mentioned until now consider noiseless coefficients, which means that the channel is perfectly known. In [14] and [15], estimators based on noisy coefficients magnitude are proposed. To our best knowledge, the only estimators which consider noisy complex channel coefficients estimated with pilots are given in [16]. However, the estimators from [16] are valid only when the channel coefficients are correlated according to the Clark's model. *Our first contribution in this paper is the derivation of a new Maximum Likelihood (ML)-like estimator that uses independent noisy complex channel coefficients. We show by simulation that this estimator outperforms the ones from the literature and the ML one in term of both bias and Normalized Mean Square Error (NMSE), especially for realistic sample size (≤ 100 samples).*

The deterministic Cramer Rao Lower Bound (CRLB) of the K factor is derived in [5] for both magnitude-based estimators and complex coefficients-based estimators in the noiseless case. The deterministic CRLB for the K factor in case of noisy coefficients magnitude is obtained numerically in [14]. The authors of [17] proposed a deterministic CRLB for the complex coefficients estimation of K in the noiseless case. Finally, a stochastic CRLB is derived in [18] when the signal is unknown and stochastic. *Our second contribution is the derivation in closed form of the deterministic CRLB for the complex coefficients based estimation of K in the noisy case, which is not available in the literature to the best of our knowledge.*

When the sample size is small (less than 30 samples), the deterministic estimators may provide inaccurate estimation of K. *Our third contribution is the design of two new Bayesian estimators, the mean a posteriori and the maximum a posteriori, for the complex coefficients-based estimation of K in the noisy case*. As the prior distribution for K, we use the log-normal distribution, which has been shown through measurement campaigns to represent the real distribution of K in different scenarios [19]. These two Bayesian estimators are shown to have smaller bias and NMSE than the deterministic ones and to be more robust to small sample size. Their drawback is that they have a higher complexity than the deterministic estimators since the maximum a posteriori requires to solve a non linear equation of K and the mean a posteriori requires a numerical integration. The design of the two Bayesian estimators will be given in the final version of the paper. Their performance are shown in section V.

The paper is organized as follows. The system model is described in section II. The proposed estimator is given in section III, and the expression of the deterministic CRLB is in section IV. Finally, section V is devoted to numerical results.

II. System model

The narrow-band discrete complex impulse response of the Ricean channel is a single coefficient modeled by a complex Gaussian random process with variance $2\sigma_h^2 = \Omega/(K+1)$ and mean $\mu = e^{j\phi_0}\sqrt{K\Omega/(K+1)}$ [10]. Our objective is to estimate the parameter K from N estimated channel coefficients, denoted by $\hat{h}[i] = h[i] + n[i], i = 1, ..., N$ where $h[i], i = 1, ...N$ represent N independent realizations of the Ricean coefficient and $n[i]$ is a complex white zero-mean circularly symmetric Gaussian noise with known variance $2\sigma_n^2$.

III. Proposed Estimator

The ML complex coefficient-based estimator of the Ricean K factor in the noiseless case is $\hat{K}_{ML} = |\hat{\mu}|^2/2\hat{\sigma}_h^2$, where $\hat{\mu} = N^{-1}\sum_{i=1}^{N} h[i]$ and $2\hat{\sigma}_h^2 = N^{-1}\sum_{i=1}^{N}|h[i] - \hat{\mu}|^2$. It is proven in [11] that $\hat{K}_{ML}$ is biased, and another estimator, given by $\hat{K}_{MML} = N^{-1}((N-2)\hat{K}_{ML} - 1)$, is proposed to correct this bias. It is possible to derive the ML complex coefficient-based estimator of K in the noisy case as a direct extension of $\hat{K}_{ML}$, replacing the ML estimator of σ_h^2 in the noiseless case by its ML estimator in the noisy case. The ML estimator in the noisy case is therefor $\hat{K}_{ML}^n = |\tilde{\mu}|^2/(2\hat{\sigma}^2 - 2\sigma_b^2)$, where $\tilde{\mu} = N^{-1}\sum_{i=1}^{N}\hat{h}[i]$ and $2\hat{\sigma}^2 = N^{-1}\sum_{i=1}^{N}|\hat{h}[i] - \tilde{\mu}|^2$. However, as in the noiseless case, we can prove that $\hat{K}_{ML}^n$ is biased.

To find an unbiased estimator of K in the noisy case, we studied the bias of $\hat{K}_{ML}$ when the channel coefficients are noisy. After some derivations which will be reported in the final version of this paper, we obtain the following unbiased estimator

$$\hat{K}_{Prop}^n = \frac{1}{\alpha}\hat{K}_{MML}^n, \quad (1)$$

where $\alpha = \sigma_h^2/(\sigma_h^2+\sigma_n^2)$ and $\hat{K}_{MML}^n = N^{-1}((N-2)|\tilde{\mu}|^2/2\hat{\sigma}^2 - 1)$. The variance of $\hat{K}_{Prop}^n$ is derived and is given by $Var[\hat{K}_{Prop}^n] = (N-2)(2\alpha^2N^2(N-3))^{-1}((2+2NK\alpha)^2/(2N-4)+4NK\alpha+2)$. Unfortunately, α can not be computed in practice since σ_h^2 (unknown) appears in its expression. However, we succeeded to derive an

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unbiased estimator of α: $\hat{\alpha} = 1 + 2\sigma_n^2 N^{-1}(2-N)/(2\hat{\sigma}^2)$, and we propose to replace α by $\hat{\alpha}$ in (1). Although the resulting estimator of K is then biased, it will be shown that both its bias and NMSE are the smallest among all the deterministic estimators considered in this paper.

IV. Cramer-Rao lower bound

The CRLB for the estimation of the K factor when noisy complex coefficients are available has been derived, and is given by

$$\begin{aligned} \mathrm{CRLB}(K) = & \frac{2K}{N}\left(1 + 2(K+1)\frac{\sigma_n^2}{\Omega}\right) + \\ & \frac{K^2}{N}\left(1 + 2(K+1)\frac{\sigma_n^2}{\Omega}\right)^2 . \end{aligned} \quad (2)$$

Calculation leading to this CRLB will be presented in the final version of this paper.

V. Numerical results

The performance of the new estimators are compared with the one from [11] and the best moment-based estimators from [14]. The estimator from [11] is denoted by K_{IQ} and the one from [14] by K_{MB}. Finally, the maximum a posteriori is denoted by $K_{MaxPost}$ and the mean a posteriori by $K_{MeanPost}$. The accuracy of the four estimators is compared in term of both bias magnitude and NMSE, defined for a given estimator $\hat{K}$ as NMSE $= \mathbb{E}((\hat{K}-K)^2)/K^2$. Fig. 1 and 2 represents the bias magnitude of the estimators and their NMSE, respectively. K varies from 0 to 10 by step 1, which represents realistic values for this parameter. For every value of K, $50,000$ trials have been performed. Notice that the theoretical bias and variance of K_{IQ} are shown; their calculation will be presented in the final version of the paper.

The advantage of the proposed estimators is clear since both their bias magnitude and NMSE are smaller than the one of the other considered estimators. The mean a posteriori has the smallest biais among the proposed estimators. The maximum a posteriori has a bias similar to the ML, and it has the smallest NMSE among the proposed estimators. Finally, the proposed deterministic estimator has the smallest bias and NMSE among all the existing deterministic estimators. The NMSE of the Bayesian estimators is smaller than the CRLB, which can be explained by the prior knowledge on K introduced by the prior distribution.

In the final version of the paper, it will be shown that the two bayesian estimators are more robust than the deterministic ones to sample size as small as 30.

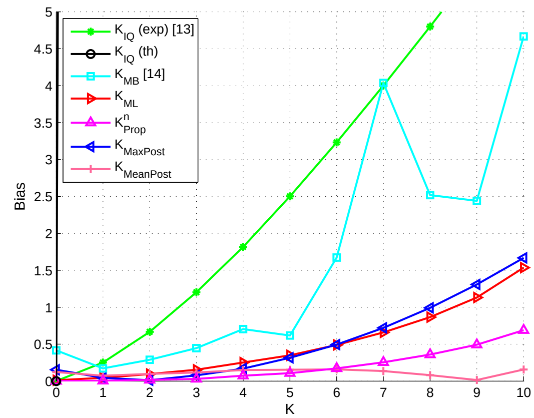

Figure 1. Bias magnitude of the different estimators, $N = 100$, $2\sigma_n^2 = 0.5$.

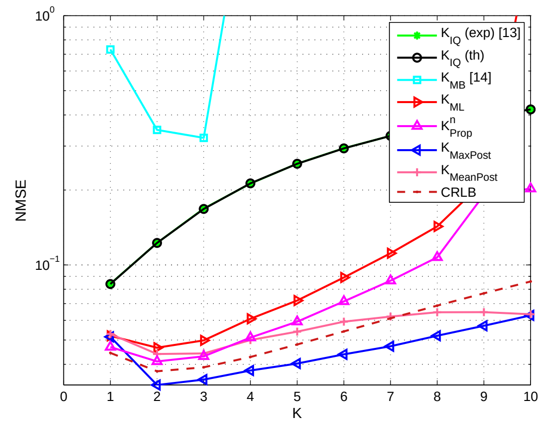

Figure 2. NMSE of the different estimators, $N = 100$, $2\sigma_n^2 = 0.5$.

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