Fifth-order Volterra series based Nonlinear Equalizer for long-haul high data rate optical fiber communications

Abdelkerim Amari, Philippe Ciblat, and Yves Jaouën

***Abstract*—We propose a *fifth-order* Inverse Volterra Series Transfer Function based nonlinearities compensation for ultra high data rate optical fiber communications using OFDM. Compared to the *third-order* case, we significantly improve the performance in terms of BER and/or transmission distance.**

I. Introduction

Concerning ultra high data rate optical communications for core and metroplitan networks, coherent detection along with digital signal processing has recently received a lot of attention due to its high ability to compensate for linear distorsions and to handle multi-level modulation formats (16QAM, 64QAM, etc). More precisely, next generation long haul wavelength-division multiplexing (WDM) transmission system has to reach from 400 Gb/s to 1 Tb/s as target data rates. Such data rates can be only reached by using multi-level modulation formats. However, such formats require high Optical Signal-to-Noise Ratio (OSNR) and so high input powers. Unfortunately, such input powers give rise to nonlinear impairments through the fiber transmission which reduce significantly the transmission distance. Therefore, mitigating nonlinear impairments is a hot topic for long-haul high data rate optical communications.

Several approaches have been proposed for mitigating these nonlinearites: Digital Back Propagation [1], Adaptive Loading Algorithm [2], and Inverse Volterra Series Transfer Function (IVSTF) based Nonlinear Equalizer (NLE) [3]. The main interest of IVSTF-NLE compared to other techniques is its computational load since parallel processing can be done. So far, only *third-order* IVSTF-NLE has been proposed.

Our contribution in this paper is to propose the *fifth-order* IVSTF-NLE. Firstly, we derive the fifth-order Volterra kernels of the receiver. Secondly, we show that such a IVSTF-NLE can be implemented. Finally, through simulations, we observe a significant gain in BER and transmission distance compared to its third-order version. In this summary, we focus on single-polarization multi-band OFDM systems with multi-span fibers. Due to lack of space, the dual-polarization case will be only treated in the final version.

II. VSTF based optical fiber model

Like [3], we consider a communication system with N spans: each of them is of length L and is built with a single mode fiber (SMF) of attenuation coefficient α, second-order dispersion parameter β_2, and nonlinear coefficient γ. In [4], it is shown that the Volterra Series Transfer Function (VSTF) is a powerful tool for solving the so-called NonLinear Schrödinger Equation (NLSE) that governs the propagation within a span and shows that the link, in the frequency domain, between the output $Y(\omega)$ of the N-th span and the input $X(\omega)$ of the first span can be modelled as follows

$$Y = H_1[X] + H_3[X] \quad (1)$$

The authors are with Comelec Dpt, Telecom ParisTech, France.
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Contact: abdelkerim.amari@telecom-paristech.fr.

where H_1 and H_3 are the first-order and third-order VSTF operators respectively given by

$$\begin{aligned} Y_1 = H_1[X] \quad &\Leftrightarrow \quad Y_1(\omega) = h_1(\omega)X(\omega) \\ Y_3 = H_3[X] \quad &\Leftrightarrow \quad Y_3(\omega) = \iint h_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2) \\ &\qquad .X(\omega_1)X^*(\omega_2)X(\omega-\omega_1+\omega_2)d\omega_1 d\omega_2. \end{aligned}$$

The kernels h_1 and h_3 are available in [3] and equal to

$$h_1(\omega) = e^{-j\omega^2\beta_2 NL/2} \quad (2)$$

$$h_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2) = \frac{-jch_1(\omega)}{4\pi^2}\sum_{k=0}^{N-1} e^{-jk\beta_2\Delta\Omega L} \quad (3)$$

with $\Delta\Omega = (\omega_1-\omega)(\omega_1-\omega_2)$ and $c = \gamma(1-e^{-\alpha L})/\alpha$.

III. Fifth-order IVSTF based nonlinear equalizer

We propose to make the decision on Z obtained as follows

$$Z = K_1[Y] + K_3[Y] + K_5[Y] \quad (4)$$

where K_1, K_3, and K_5 are the so-called Inverse VSTF operators (up to fifth-order) associated with the system (H_1, H_3) given by Eq. (1). Note that we can omit the even-order terms for the IVSTF since the even-order terms in Eq. (1) vanish. According to [5], [6], we know that

$$K_1 = H_1^{-1} \quad (5)$$

$$K_3 = -K_1H_3K_1 \quad (6)$$

$$\begin{aligned} K_5 &= K_1[-H_3[K_1+K_1H_3K_1] - 3H_3K_1] \quad (7)\\ &+ K_1[0.5H_3K_1H_3K_1 + 0.5H_3[2K_1+K_1H_3K_1]]. \end{aligned}$$

In [3], explicit expressions for $Z_1 = K_1[Y]$ and $Z_3 = K_3[Y]$ have been developed when H_1 and H_3 are given by Eqs. (2)-(3). Our main contribution is to exhibit a closed-form expression for the operator K_5 for which the input is Y and the output is denoted Z_5 when H_1 and H_3 follow Eqs. (2)-(3). For any H_1 and H_3, we first obtain after simple derivations that

$$\begin{aligned} \frac{Z_5(\omega)}{k_1(\omega)} = &-\iiiint h_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2)k_1(\omega_1)k_1^*(\omega_2) \\ &\times\ k_3(\omega_3,\omega_4,\omega-\omega_1+\omega_2-\omega_3+\omega_4)y(\omega_1)y^*(\omega_2) \\ &\times\ y(\omega_3)y^*(\omega_4)y(\omega-\omega_1+\omega_2-\omega_3+\omega_4)d\boldsymbol{\omega} \\ &-\iiiint h_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2)k_1(\omega-\omega_1+\omega_2) \\ &\times\ k_1(\omega_1)k_3^*(\omega_3,\omega_4,\omega_2-\omega_3+\omega_4)y(\omega_1)y^*(\omega_3) \\ &\times\ y(\omega_4)y(\omega-\omega_1+\omega_2)y^*(\omega_2-\omega_3+\omega_4)d\boldsymbol{\omega} \\ &-\iiiint h_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2)k_1(\omega-\omega_1+\omega_2) \\ &\times\ k_1^*(\omega_2)k_3(\omega_3,\omega_4,\omega_1-\omega_3+\omega_4)y^*(\omega_2) \\ &\times\ y(\omega-\omega_1+\omega_2)y(\omega_3)y^*(\omega_4)y(\omega_1-\omega_3+\omega_4)d\boldsymbol{\omega} \end{aligned}$$

with $\boldsymbol{\omega} = [\omega_1,\omega_2,\omega_3,\omega_4]$. Note that k_1 and k_3 are the kernels of K_1 and K_3 respectively.

Using Eqs. (2)-(3), we obtain after tedious algebraic manipulations that

$$Z_5(\omega) = \sum_{k,\ell=1}^{N} \left(S_1^{(k,\ell)}(\omega) + S_2^{(k,\ell)}(\omega) \right)$$

with

$$\begin{aligned} S_1^{(k,\ell)}(\omega) &= -\frac{2c^2}{(2\pi)^4} h_{cd}^N(\omega) \iiiint e^{j\beta_2 L(k\Delta\Omega + \ell\Delta\Omega_1)} Y^*(\omega_2) \\ &\times \ Y(\omega - \omega_1 + \omega_2) Y(\omega_3) Y^*(\omega_4) Y(\omega_1 - \omega_3 + \omega_4) d\boldsymbol{\omega} \end{aligned}$$

and

$$\begin{aligned} S_2^{(k,\ell)}(\omega) &= \frac{c^2}{(2\pi)^4} h_{cd}^N(\omega) \iiiint e^{j\beta_2 L(k\Delta\Omega - \ell\Delta\Omega_2)} Y(\omega_1) \\ &\times \ Y(\omega - \omega_1 + \omega_2) Y^*(\omega_3) Y(\omega_4) Y^*(\omega_2 - \omega_3 + \omega_4) d\boldsymbol{\omega} \end{aligned}$$

with $\Delta\Omega_1 = (\omega_3 - \omega_1)(\omega_3 - \omega_4)$, $\Delta\Omega_2 = (\omega_3 - \omega_2)(\omega_3 - \omega_4)$, and $h_{cd}(\omega) = e^{j\beta_2 \omega^2 L/2}$.

The above fourth-order multiple integrals seem to be impossible to implement in practice. Actually, it is possible to remove all integrals by working in the frequency and time domains successively. Let us first focus on $S_1^{(k,\ell)}(\omega)$. Let $y_k(t)$ be the Inverse Fourier Transform of $Y_k(\omega) = h_{cd}^k(\omega) Y(\omega)$. By replacing h_{cd} with its expression, after tedious derivations, we obtain

$$S_1^{(k,\ell)}(\omega) = -h_{cd}^{N-k}(\omega) U_1^{(k,l)}(\omega)$$

where the Inverse Fourier Transform of $U_1^{(k,l)}(\omega)$, denoted by $u_1^{(k,l)}(t)$, writes as follows

$$u_1^{(k,l)}(t) = \left(\sqrt{2}c|y_k(t)|^2\right) . \left(\tilde{h}_{k,\ell}(t) \star (\sqrt{2}c|y_\ell(t)|^2 y_\ell(t))\right)$$

where $\star$ denotes the convolution product and $\tilde{h}_{k,\ell}(t)$ is the Inverse Fourier Transform of $h_{cd}^{k-\ell}(\omega)$. So instead of doing four integrals, we just have to multiply signals in time domain and apply some Fourier Transform. Obviously, similar expressions can be found for $S_2^{(k,\ell)}(\omega)$. This leads to the practical implementation scheme given by Fig. 1.

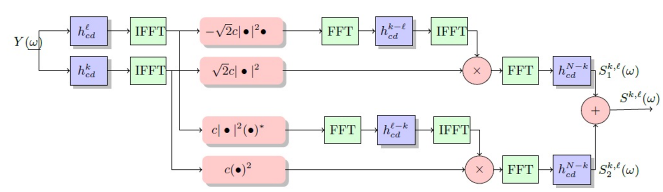

Figure 1. Fifth-order IVSTF-NLE scheme for single-polarization

In Fig. 1, we only report the computation of fifth-order terms. The first-order and third-order terms can be found in [3] and computed in parallel due to the additive structure of Eq. (4).

IV. Numerical results

In this simulation part, the setup is inspired from the SASER European project whose the goal is to design a 400Gb/s system for long-haul communications. As we consider here one polarization, the data rate is only 200Gb/s. We have four bands of bandwidth 20GHz each and spaced by a 10GHz interval guard. On each band, we consider a 16-QAM OFDM with 512 subcarriers. The communication is done over 20 spans of 100km each. Each span is a standard SMF with $\alpha = 0.2$dB.km^{-1}, $\beta_2 = 17$ps.nm^{-1}.km^{-1}, and $\gamma = 0.0014$m^{-1}.W^{-1}. We use Erbium-Doped Fiber Amplifier (EDFA) with a 5.5dB noise figure and a 22dB gain at each span.

In Fig. 2, we plot BER vs. input power ($P_{\rm in}$) when only one band is active. The fifth-order IVSTF-NLE outperforms the third-order one and the smallest BER is really improved.

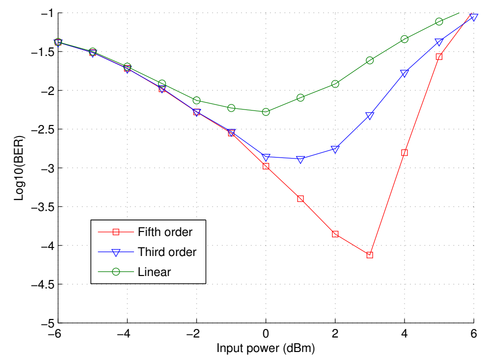

Figure 2. BER vs. $P_{\rm in}$ for single-band transmission

In Fig. 3, we plot the reached transmission distance for a target BER@10^{-2} when the four bands are active. Compared to the third-order IVSTF-NLE, the gain is around 100km.

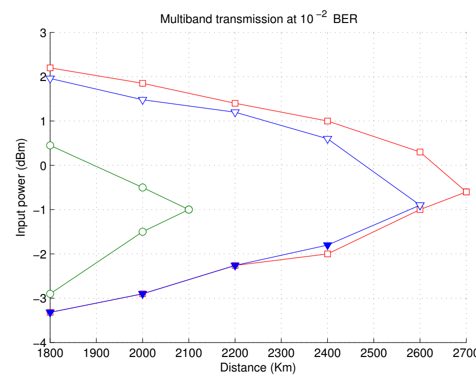

Figure 3. $P_{\rm in}$ vs. transmission distance for multi-band transmission

The fifth-order IVSTF-NLE is rather efficient for combating the intra-band nonlinear effect (see single-band case) than the inter-band effect (see multi-band effect).

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