Unified Performance Analysis of Blind Feedforward Timing Estimation

Yan Wang and Erchin Serpedin

Dept. of Electrical Engineering
Texas A&M University
College Station, TX 77843, USA

Philippe Ciblat

Ecole Nationale Supérieure des Télécomm.

Paris, France

Abstract

By exploiting an unified cyclostationary (CS) statistics based framework, this paper develops a rigorous and thorough performance analysis of blind feedforward timing estimators for linear modulations. Within the introduced CS-framework, it is shown that several estimators proposed in the literature can be asymptotically interpreted as Maximum Likelihood (ML) estimators, and the timing estimator proposed by Oerder and Meyr in [6] achieves the lowest asymptotic (large sample) variance in the class of estimators which exploit all the second-order statistics of the received signal, and its performance is insensitive to the oversampling rate P as soon as $P \geq 3$. Further, based on the presented theoretical analysis, we derive an Asymptotically Best Consistent (ABC) estimator, which exhibits asymptotically the best performance among all the existing blind feedforward timing estimators, especially when dealing with strongly bandlimited signaling.

1 Introduction and Channel Model

Figure 1: Common structure of blind feedforward timing estimators

Timing recovery is a challenging but very important task for reliable detection in synchronous receivers. For bandwidth efficiency reasons, non-data aided or blind feedforward timing estimation architectures for linear modulations have received much attention during the last decade [3]–[7].

Consider the following standard baseband channel model[1] (see e.g., [3]):

$$x_c^{(tr)}(t) = \sum_l w(l) h_c^{(tr)}(t - \epsilon T - lT) + v_c(t) \,, \qquad (1)$$

[1]The subscript $_c$ is used to denote a continuous-time signal.

where $\{w(l)\}$ is the zero-mean independently and identically distributed sequence of transmitted symbols, $h_c^{(tr)}(t)$ denotes the transmitter's signaling pulse, $v_c(t)$ is the complex-valued additive white Gaussian noise with power spectral density N_0, T is the symbol period, and ϵ stands for the (normalized) symbol timing epoch. After matched filtering with $h_c^{(rec)}$, the resulting signal $x_c(t)$ is oversampled with the sampling period $T_s := T/P$ ($P \geq 3$). The following discrete-time system model is obtained:

$$x(n) = \sum_l w(l) h_\epsilon(n - lP) + v(n) \,, \qquad (2)$$

where $x(n) := (x_c^{(tr)}(t) * h_c^{(rec)}(t))|_{t=nT_s}$ ($*$ denotes linear convolution), $v(n) := (v_c(t) * h_c^{(rec)}(t))|_{t=nT_s}$, and $h_\epsilon(n) := h_c(t)|_{t=nT_s - \epsilon T}$ with $h_c(t) := h_c^{(tr)}(t) * h_c^{(rec)}(t)$. $h_c(t)$ is assumed to be a raised cosine pulse of bandwidth $[-(1+\rho)/2T, (1+\rho)/2T]$, where the parameter ρ represents the rolloff factor ($0 < \rho \leq 1$).

As depicted by Eq. (2), the problem that we pose is to estimate the unknown timing epoch ϵ in a blind feedforward manner, assuming knowledge only of the received samples $\{x(n)\}_{n=0}^{N-1}$. Fig. 1 illustrates the common structure of the popular blind feedforward timing estimators proposed in the literature, which consists of filtering the received samples $\{x(n)\}_{n=0}^{N-1}$ through a nonlinearity that removes the modulation effects introduced by $w(l)$ and generates a data sequence $y(n)$ that contains spectral components whose phase information is exploited to recover the unknown timing epoch ϵ. The common-used nonlinearities are second-law (SLN) [3], [6], fourth-law (FLN) [7], absolute value (AVN) [1], [7], and logarithmic nonlinearity (LOGN) [5].

Irrespective of the nonlinearity function used, one of the common features of all the above mentioned timing estimators is the exploitation of the cyclosta-

0-7803-7576-9/02$17.00 © 2002 IEEE

tionary (CS) statistics induced by oversampling the received signal. The role of cyclostationarity in synchronization was clearly acknowledged in [1]. Our goal herein is to exploit optimally the CS-statistics of the received signal in order to develop efficient estimators, and rigorous and thorough performance analysis setups for the existing blind timing estimators.

In the next section, first we briefly introduce the blind feedforward SLN timing estimators proposed in [3] and [6], and then propose a unifying ML-framework that will enable to establish some interesting links with some of the existing estimators and to analyze their asymptotic performance.

2 Second Order Timing Estimators

2.1 The SLN Timing Estimators

By exploiting Eq. (2), straightforward calculations show that the time-varying correlation of the nonstationary process $x(n)$ is given as

$$r_{2x}(n;\tau) := \mathrm{E}\{x^*(n)x(n+\tau)\} = r_{2x}(n+P;\tau)\ , \quad \forall n, \tau,$$

where the superscript * stands for complex conjugation. Being periodic, $r_{2x}(n;\tau)$ admits a Fourier Series expansion, whose coefficients, also termed cyclic correlations, are given for $k = 0, \ldots, P-1$ by [3], [9]:

$$\begin{aligned} R_{2x}(k;\tau) &:= \frac{1}{P}\sum_{n=0}^{P-1} r_{2x}(n;\tau)e^{-2i\pi\frac{kn}{P}} \\ &= \frac{\sigma_w^2}{P}e^{i\pi\frac{k\tau}{P}}e^{-2i\pi k\epsilon}G(k;\tau) + h_{rc}(\tau)\delta(k)\ , \end{aligned} \tag{3}$$

where $h_{rc}(n) := h_c(t)|_{t=nT_s}$ and

$$G(k;\tau) := \frac{P}{T}\int_{-P/2T}^{P/2T} H_c(F - \frac{k}{2T})H_c(F + \frac{k}{2T})e^{2i\pi\frac{\tau TF}{P}}dF,$$

and $H_c(F)$ stands for the Fourier transform (FT) of $h_c(t)$. Since $H_c(F)$ is a real-valued even function, it is easy to check that $G(k;\tau)$ is a real-valued function, too. Moreover, due to the band-limited property of the filter $h_c(t)$, $G(k;\tau)$ and $R_{2x}(k;\tau)$ are nonzero only for cycles $k = 0, \pm 1$. Since

$$R_{2x}(k;\tau) = e^{\frac{2i\pi k\tau}{P}}R_{2x}^*(-k;-\tau)\ , \tag{4}$$

based on (3), it follows that only the subset $\{R_{2x}(1;\tau)\}$, $\forall\tau$, represents all the second-order statistics that may be used for estimating ϵ.

In practice, the cyclic correlations $R_{2x}(k;\tau)$ have to be estimated from a finite number of samples N, and the standard sample estimate of $R_{2x}(k;\tau)$ is given for $\tau \geq 0$ by (see e.g., [2], [3]):

$$\hat{R}_{2x}(k;\tau) = \frac{1}{N}\sum_{n=0}^{N-\tau-1} x^*(n)x(n+\tau)e^{-2i\pi\frac{kn}{P}}, \tag{5}$$

which is asymptotically unbiased and mean-square sense (m.s.s.) consistent [2].

Exploiting the second-order CS statistics of $x(n)$, the following general SLN timing estimator is proposed:

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\{\hat{R}_{2x}(1;\tau)e^{-i\pi\tau/P}\}\ . \tag{6}$$

Note that the second-order CS-based timing estimators choose different values for the timing lag τ in (6) ([3] ($\tau = 1$) and [6] ($\tau = 0$)).

2.2 The ML Estimator Links

According to [2], the sample cyclic correlation estimates $\{\hat{R}_{2x}(1;\tau)\}$, $\forall\tau$, are asymptotically jointly normal. To show that the second-order CS-based timing estimators are asymptotically ML estimators, let us first define the vector of cyclic correlations[2]: $\mathbf{R}_{2x} := [R_{2x}(1;-\tau_m), \ldots, R_{2x}(1;\tau_m)]^{\mathrm{T}}$, where τ_m is an arbitrary non-negative value. Denote the sample estimate of $\mathbf{R}_{2x}$ by $\hat{\mathbf{R}}_{2x}$. Thus, $\sqrt{N}[\hat{\mathbf{R}}_{2x} - \mathbf{R}_{2x}]$ is asymptotically jointly complex normal with zero-mean $\mathbf{0} := [0, \ldots, 0]^{\mathrm{T}}$, and its covariance and relation matrices are given by:

$$\begin{aligned} \mathbf{\Gamma} &:= \lim_{N\to\infty} N\mathrm{E}\{(\hat{\mathbf{R}}_{2x} - \mathbf{R}_{2x})(\hat{\mathbf{R}}_{2x} - \mathbf{R}_{2x})^{\mathrm{H}}\}\ , \\ \tilde{\mathbf{\Gamma}} &:= \lim_{N\to\infty} N\mathrm{E}\{(\hat{\mathbf{R}}_{2x} - \mathbf{R}_{2x})(\hat{\mathbf{R}}_{2x} - \mathbf{R}_{2x})^{\mathrm{T}}\}\ , \end{aligned}$$

respectively.

Next, we transform the complex Gaussian probability density function (pdf) $\mathcal{N}^c(\mathbf{0}, \mathbf{\Gamma}, \tilde{\mathbf{\Gamma}})$ into its equivalent algebraic form of the real Gaussian pdf $f_\epsilon(\hat{\mathbf{U}}_{2x})$ by defining the $(4\tau_m + 2)$-vectors: $\mathbf{U}_{2x} := [\mathrm{re}(\mathbf{R}_{2x})^{\mathrm{T}}\ \ \mathrm{im}(\mathbf{R}_{2x})^{\mathrm{T}}]^{\mathrm{T}}$ and $\hat{\mathbf{U}}_{2x} := [\mathrm{re}(\hat{\mathbf{R}}_{2x})^{\mathrm{T}}\ \ \mathrm{im}(\hat{\mathbf{R}}_{2x})^{\mathrm{T}}]^{\mathrm{T}}$, where the notations "re" and "im" stand for the real and imaginary part, respectively. Simple calculation shows that the covariance matrix of $\hat{\mathbf{U}}_{2x}$ is given by:

$$\begin{aligned} \mathbf{\Lambda}(\epsilon) &:= \lim_{N\to\infty} N\mathrm{E}\{(\hat{\mathbf{U}}_{2x} - \mathbf{U}_{2x})(\hat{\mathbf{U}}_{2x} - \mathbf{U}_{2x})^{\mathrm{T}}\} \\ &= \frac{1}{2}\begin{bmatrix} \mathrm{re}(\mathbf{\Gamma} + \tilde{\mathbf{\Gamma}}) & \mathrm{im}(\tilde{\mathbf{\Gamma}} - \mathbf{\Gamma}) \\ \mathrm{im}(\tilde{\mathbf{\Gamma}} + \mathbf{\Gamma}) & \mathrm{re}(\mathbf{\Gamma} - \tilde{\mathbf{\Gamma}}) \end{bmatrix}. \end{aligned}$$

Now define the error vector $\mathbf{e} := \hat{\mathbf{U}}_{2x} - \mathbf{U}_{2x}$ and consider the following nonlinear regression model:

$$\hat{\mathbf{U}}_{2x} = \mathbf{U}_{2x}(\epsilon) + \mathbf{e}\ , \tag{7}$$

where both $\hat{\mathbf{U}}_{2x}$ and $\mathbf{e}$ depend on the number of samples N, and $\mathbf{U}_{2x}$ is a function of the unknown timing

[2] In this paper, we use the superscripts $^{\mathrm{T}}$ and $^{\mathrm{H}}$ to denote transposition and conjugate transposition, respectively.

epoch ϵ. The ABC-estimator of ϵ for the above nonlinear regression problem is provided by the nonlinear least-squares estimator weighted by the inverse of the asymptotic covariance matrix of the error vector $\mathbf{e}$, and takes the following form [8, pp. 91–95]:

$$\hat{\epsilon} = \arg\min_{\dot{\epsilon}} J(\dot{\epsilon}) \ , \tag{8}$$

$$J(\dot{\epsilon}) = \frac{1}{2}[\hat{\mathbf{U}}_{2x} - \mathbf{U}_{2x}(\dot{\epsilon})]^{\mathrm{T}} \mathbf{\Lambda}(\dot{\epsilon})^{-1} [\hat{\mathbf{U}}_{2x} - \mathbf{U}_{2x}(\dot{\epsilon})] \ ,$$

and the notation $\dot{\epsilon}$ stands for the trial value of ϵ. As $\mathbf{e}$ is asymptotically normally distributed, one can observe that the ABC estimator (8) (thus (6)) is nothing else than the asymptotic ML estimator of $\hat{\epsilon}$ in terms of the observations contained in the vector $\hat{\mathbf{U}}_{2x}$.

2.3 Asymptotic Performance Analysis

The asymptotic variance of ABC-estimate $\hat{\epsilon}$ can be established in the following theorem (c.f., [8]):

Theorem 1: The asymptotic variance of the timing epoch estimator (8) is given by:

$$\operatorname{avar}(\hat{\epsilon}) := \lim_{N\to\infty} N\mathrm{E}\{(\hat{\epsilon}-\epsilon)^2\}$$
$$= \frac{\sin^2(4\pi\epsilon)}{16\pi^2}\left\{\frac{\Theta_{0,0}}{\cos^2(2\pi\epsilon)} + \frac{\Theta_{1,1}}{\sin^2(2\pi\epsilon)} - \frac{4\Theta_{0,1}}{\sin(4\pi\epsilon)}\right\}, \tag{9}$$

where

$$\mathbf{\Theta} := \begin{bmatrix}\Theta_{0,0} \, \Theta_{0,1} \\ \Theta_{0,1} \, \Theta_{1,1}\end{bmatrix} = (\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Lambda}^{-1}\mathbf{\Phi})^{-1} \ ,$$

$$\mathbf{\Phi} := \frac{\sigma_w^2}{P}\begin{bmatrix}\mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{\Phi}_2 & -\mathbf{\Phi}_1\end{bmatrix},$$

$$\mathbf{\Phi}_1 := \left[G(1;-\tau_m)\cos(-\frac{\pi\tau_m}{P})\ldots G(1;\tau_m)\cos(\frac{\pi\tau_m}{P})\right]^{\mathrm{T}},$$

$$\mathbf{\Phi}_2 := \left[G(1;-\tau_m)\sin(-\frac{\pi\tau_m}{P})\ldots G(1;\tau_m)\sin(\frac{\pi\tau_m}{P})\right]^{\mathrm{T}}.$$

It is easy to show that when $\tau_m = 0$, the general estimator (8) reduces to the O&M estimator [6]:

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\{\hat{R}_{2x}(1;0)\} \ , \tag{10}$$

and the asymptotic variance of O&M estimate can be established from (9) as:

$$\operatorname{avar}(\hat{\epsilon}) = \frac{P^2}{8\pi^2 G^2(1;0)}[\mathbf{\Gamma}_{0,0} - \operatorname{re}(e^{4i\pi\epsilon}\widetilde{\mathbf{\Gamma}}_{0,0})] \ . \tag{11}$$

Now it is of interest to ask whether the performance of the O&M estimator can be improved by exploiting additional cyclic correlations $\hat{\mathbf{R}}_{2x}(1;\tau)$ at lags $\tau \neq 0$. Surprisingly, from Fig. 2, the answer is no. In Fig. 2, we evaluate the theoretical mean square error $\mathrm{MSE}(\hat{\epsilon}) = \operatorname{avar}(\hat{\epsilon})/N$ of $\hat{\epsilon}$ for different values of τ_m in the case of $\rho = 0.9$ and the number of samples $N = 400$ (i.e., the observation length $L = 100$ symbols), assuming $P = 4$, $\epsilon = 0.3$ and QPSK input symbols. The modified CRB (MCRB) is adopted as a benchmark $\mathrm{MCRB}(\hat{\epsilon}) = 1/(8\pi^2 L\xi\mathrm{SNR})$, where the parameter ξ is given by [4, p. 65]: $\xi = (1/12) + \rho^2(0.25 - 2/\pi^2)$. Thus, asymptotically the O&M estimator achieves the best performance in the class of SLN estimators.

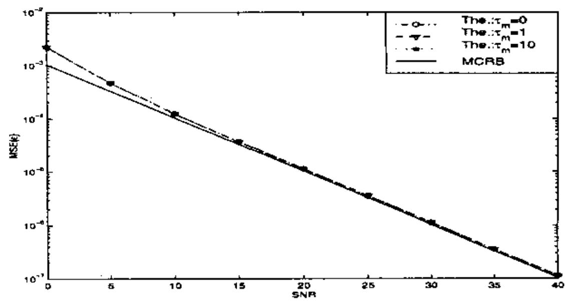

Figure 2: $\mathrm{MSE}(\hat{\epsilon})$ for different values of τ_m

2.4 Influence of the Oversampling Rate P

In this subsection, we analyze the effect of the oversampling rate P on the SLN timing estimators. To properly inspect the influence of P, we need to evaluate MSE. Then, by exploiting (3) and (11), it yields that:

$$\mathrm{MSE}(\hat{\epsilon}) = \frac{8(2N_0\mathcal{H}_{2,1} + N_0^2\mathcal{H}_{1,1})}{\pi^2\rho^2 L} \ , \tag{12}$$

where for $l, k = 1, 2, \ldots$

$$\mathcal{H}_{l,k} := \frac{1}{T^{l+k-1}}\int_{-\frac{\rho}{2T}}^{\frac{\rho}{2T}} H_c^l(F + \frac{1}{2T})H_c^k(F - \frac{1}{2T})\mathrm{d}F \ .$$

Since $\mathcal{H}_{l,k}$ does not depend on P, $\mathrm{MSE}(\hat{\epsilon})$ is independent of P whenever $P \geq 3$. One can observe that in the noiseless case ($N_0 \to 0$), the asymptotic variance of the O&M estimate is equal to 0, which means that asymptotically in SNR and N, the variance of the O&M estimate converges to zero faster than $O(1/N)$. Next, we will show further that this rate of convergence is even faster than $O(1/N^2)$ in the absence of additive noise.

2.5 Further Results on the Convergence Rate of the O&M Estimator

Define the following stochastic processes:

$$e_2(n) := x^*(n)x(n) - r_{2x}(n;0) \ ,$$

and let $r_{3e_2}(n;\tau_1,\tau_2) := \mathrm{E}\{e_2^*(n)e_2(n+\tau_1)e_2(n+\tau_2)\}$ denotes the third-order time-varying correlation of $e_2(n)$. Based on (10), following a procedure similar to the one presented in [9, Appendix A], one can obtain the following expression for the asymptotic variance of O&M estimate normalized by N^2 in noiseless case as:

$$\mathrm{avar}_2(\hat{\epsilon}) := \lim_{N\to\infty} N^2 \mathrm{E}\{(\hat{\epsilon}-\epsilon)^2\}$$
$$= \frac{P^3 \mathrm{re}\Big\{e^{6i\pi\epsilon} S_{3e_2}\Big(3;\frac{1}{P},\frac{1}{P}\Big) - e^{2i\pi\epsilon} S_{3e_2}\Big(1;\frac{1}{P},\frac{1}{P}\Big)\Big\}}{8\pi^2 G^3(1;0)},$$

where $S_{3e_2}(k; f_1, f_2)$ denotes the third-order cyclic spectrum of $e_2(n)$.

After some lengthy and tedious manipulations, we can obtain the following expressions for $k = 1, 3$:

$$S_{3e_2}(k; 1/P, 1/P)$$
$$= (P^2\kappa_6\mathcal{H}_{1,1}^3 + 6P^2\kappa\mathcal{H}_{1,1}\mathcal{H}_{2,2} + 2P^2\mathcal{H}_{3,3}) \cdot e^{-2i\pi k\epsilon},$$

where $\kappa_6 := \mathrm{cum}(w(l), w(l), w(l), w^*(l), w^*(l), w^*(l))$ and κ stands for the kurtosis of $w(l)$.

Therefore, it turns out that $\mathrm{avar}_2(\hat{\epsilon})$ is also equal to 0, which means that the O&M estimate exhibits a rate of convergence faster than $O(1/N^2)$ when the number of samples N and SNR are large enough. Finding the exact convergence rate of the O&M estimator in the absence of additive noise appears computationally very tedious and remains open.

3 Joint Second Order and Fourth Order CS-based Timing Estimator

As we know, SLN timing epoch estimators exhibit bad performance with small rolloffs due to the lack of CS-information and their large self-induced noise effects, especially in high SNR range [4]. Therefore, when dealing with strongly bandlimited pulses, nonlinearities other than the SLN must be considered. The most common used one is the FLN nonlinearity and the timing estimator with FLN nonlinearity takes a similar expression to (10) as:

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\{\hat{R}_{4x}(1;0,0,0)\}, \qquad (13)$$

whose asymptotic variance can be established in a similar form to (11).

Although the FLN estimator has a better performance than SLN in medium and high ranges with small rolloffs, it is inferior to the latter at low SNRs. Estimators (10) and (13) suggest designing a new optimal timing estimator (OPT) of the following form:

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\{\hat{R}_{2x}(1;0) + \alpha\hat{R}_{4x}(1;0,0,0)\}, \quad (14)$$

to improve the performance of both SLN and FLN estimators. The real-valued parameter α is to be chosen so that the asymptotic variance is minimized. By adopting the derivation presented in the previous section, we can obtain a closed-form expression for the optimal value of α. However, it turns out that this $\hat{\alpha}^{(\mathrm{OPT})}$ requires the knowledge of the operating SNR and the true value of timing epoch ϵ, which makes the OPT estimator impractical. Fortunately, extensive simulation experiments suggest that for most applications of interest, we can always fix α to a value in the range $[-0.17, \ -0.13]$ for implementing the estimator (14) without incurring any performance loss.

4 Simulations

In this section, we conduct computer simulations to confirm the analysis presented in the previous sections and to illustrate the performance of the proposed OPT estimator. All the experimental results are obtained by performing a number of 10^6 Monte-Carlo trials assuming the normalized timing epoch $\epsilon = 0.3$ and QPSK constellation. Unless otherwise noted, the oversampling rate $P = 4$.

Experiment 1-Performance of the O&M estimator versus P: By varying the oversampling rate P, we compare the experimental (Exp.) MSE of O&M estimator with its theoretical variance. The number of symbols is set to $L = 200$, $\rho = 0.5$, and SNR=10dB. The result is depicted in Fig. 3. It turns out that increasing P does not improve the performance of the O&M estimator as long as $P \geq 3$. This result may be also predicted using Shannon's interpolation theorem, since any $P \geq 3$ guarantees the set of obtained statistics to be sufficient within the class of second-order statistics.

Experiment 2-Comparison of theoretical variances of estimators (10), (13) and (14) w.r.t. the MCRB: Fig. 4 depicts the theoretical MSEs of the SLN (10), FLN (13), and OPT estimators (14), and MCRB, in a strongly bandlimited pulse shape $\rho = 0.1$. The performance of a practical implementation of (14) with fixed $\alpha = -0.165$, which is just an approximation of the OPT estimator, therefore termed APP, is also illustrated. It can be seen that the OPT estimator outperforms both SLN and FLN estimators, and is closer to MCRB. As expected, APP is a satisfying realizable alternative to OPT except at very low SNRs.

Experiment 3-Comparison of the MSEs of estimators versus SNR: In Fig. 5, the experimental MSE of the proposed APP estimator is compared with those of the existing methods (SLN ($\tau_m = 0$), FLN, AVN [7] and LOGN [5]), assuming $\rho = 0.1$, $L = 400$. This figure corroborates the results of Experiment 2 and shows again the merit of the proposed APP estimator.

5 Conclusions

In this paper, we have established a rigorous CS statistics-based ML-framework to design and analyze a class of blind feedforward timing estimators. We have shown that these estimators can be asymptotically interpreted as ML estimators and the O&M estimator achieves asymptotically the best performance in the class of SLN estimators, whose performance is insensitive to the oversampling rate P as long as $P \geq 3$. In the noiseless case, it has been shown that its rate of convergence is faster than $O(1/N^2)$. The proposed analysis framework of timing estimators can be extended straightforwardly to the case of correlated symbol streams and time-selective flat-fading channels, and provides a systematic method to design optimal ML timing recovery schemes. Moreover, in this paper, based on the proposed performance analysis, we have introduced an efficient estimator (OPT), which fully exploits the second and the fourth-order CS statistics of the received signal, and improves significantly the performance of the existing methods, when dealing with narrowband signaling pulses.

Acknowledgments

This work was supported by the NSF Career Award No. CCR-0092901.

References

[1] W. R. Bennett, "Statistics of regenerative digital transmission," *Bell Syst. Tech. J.*, vol. 37, no. 6, pp. 1501-1542, Nov. 1958.

[2] A. V. Dandawaté and G. B. Giannakis, "Asymptotic theory of mixed time average and kth-order cyclic-moment and cumulánt statistics," *IEEE Trans. on Information Theory*, vol. 41, no. 1, pp. 216-232, Jan. 1995.

[3] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: a cyclostationary approach," *IEEE Trans. on Commun.*, vol. 46, no. 3, pp. 400-411, March 1998.

[4] U. Mengali and A. N. D' Andrea, *Synchronization Techniques for Digital Receivers*, Plenum Press, NY, 1997.

[5] M. Morelli, A. N. D' Andrea and U. Mengali, "Feedforward ML-based timing estimation with PSK signals," *IEEE Commun. Letters*, vol. 1, no. 3, pp. 80-82, May 1997.

[6] M. Oerder and H. Meyr, "Digital filter and square timing recovery," *IEEE Trans. on Commun.*, vol. 36, no. 5, pp. 605-612, May 1988.

[7] E. Panayirci and E. Y. Bar-Ness, "A new approach for evaluating the performance of a symbol timing recovery system employing a general type of nonlinearity," *IEEE Trans. Commun.*, vol. 44, no. 1, pp. 29-33, Jan. 1996.

[8] T. Söderström and P. Stoica, *System Identification*, Prentice-Hall, 1989.

[9] Y. Wang, P. Ciblat, E. Serpedin and P. Loubaton, "Performance analysis of a class of non-data aided frequency offset and symbol timing estimators for flat-fading channels," *IEEE Trans. on Signal Processing*, vol. 50, no. 9, pp. 2295-2305, Sept. 2002.

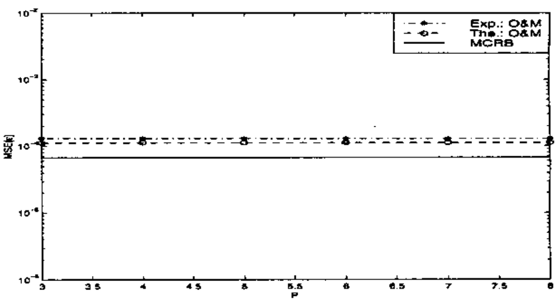

Figure 3: MSE of the O&M estimator versus P

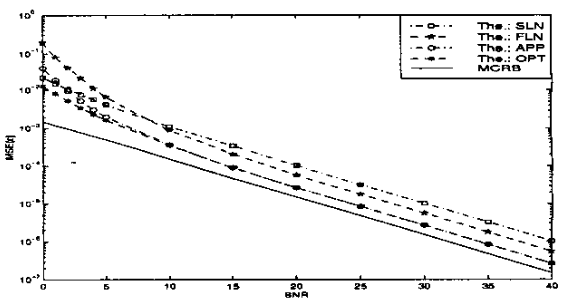

Figure 4: Comparison of MSE($\hat{\epsilon}$) versus SNR

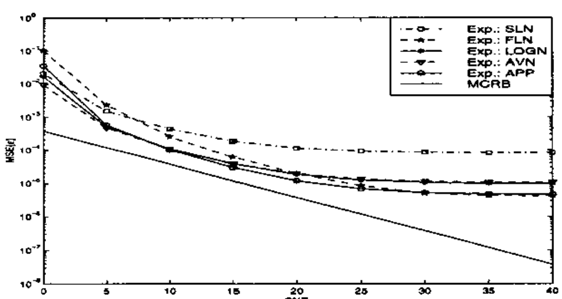

Figure 5: Comparison of MSEs