A fractionally-sampling based frequency offset enhanced blind estimator for non-circular transmissions

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Abstract

This paper deals with the problem of non-data-aided carrier frequency offset estimation of non-circular modulations transmitted through unknown frequency-selective channels. By exploiting the unconjugated cyclostationary statistics induced by oversampling the received waveform, an optimized carrier frequency offset estimator is proposed and its asymptotic (large sample) performance analyzed. In order for the proposed frequency estimator to achieve the minimum mean-square estimation error, it is shown that the oversampling rate need not be larger than twice the symbol rate. It is also shown that the proposed synchronizer is asymptotically jitter-free, admits a feedforward structure that may be implemented in digital form and is suitable for burst transmissions.

1 Introduction

The growth of mobile wireless communications systems has prompted an increased interest in designing digital receivers operating on samples of the fractionally-sampled (oversampled) received signal. The main reason is that oversampling gives rise to cyclostationarity (CS), which implies that more statistical information can be used for designing digital receivers with improved performance.

Several carrier frequency-offset estimators for flat-fading channels that exploit the conjugated second-order cyclostationary statistics of the oversampled received signal were proposed in [1, 4, 6]. A high performance blind Maximum Likelihood (ML) framework to estimate the Doppler shift in the presence of an unknown frequency selective channel was proposed in [7]. However, the proposed ML-solution relies on an iterative Baum-Welch type algorithm, whose large numerical complexity and possible lack of convergence prohibit its use for many practical applications

[7]. Reference [8] proposed a different solution for blind joint detection and carrier recovery in the presence of an unknown multipath channel using a Viterbi-like algorithm. Since this solution relies on a critical channel identifiability condition, which may not be always satisfied, the applicability of this algorithm is also limited [8].

The above mentioned considerations hold irrespective of the nature of the transmit signal constellation: circular or non-circular. This paper shows that the estimation of the carrier frequency offset estimation can be significantly simplified when the constellation is non-circular (e.g., real-valued constellation such as BPSK). Under this assumption, this paper proposes a blind frequency offset estimator that does not require knowledge of the multipath channel and transmitted data sequence. The proposed synchronizer admits a feedforward structure that may be easily implemented in digital form, does not present high computational complexity, exhibits much faster convergence rates ($O(1/N^3)$, N denotes the number of available data samples) than the algorithms [1, 4, 6], whose convergence rates are $O(1/N)$, and its convergence and consistency are guaranteed even in the presence of unknown multipath effects.

In this paper, the frequency offset estimator is designed in an optimized manner by exploiting efficiently all the unconjugated second-order statistics that are present in the received waveform. By performing a rigorous performance analysis, the mean-square error (MSE) of the resulting carrier estimator is shown to be invariant with respect to (wrt) the oversampling factor P as soon as $P \geq 2$ and its minimum MSE is achieved when $P = 2$. Thus, selecting a low oversampling factor (e.g., $P = 2$) leads to an optimal carrier recovery scheme that requires a low complexity receiver, and without being necessary to adopt large oversampling factors. From this perspective, the present carrier synchronizer represents a generalization of the estimators reported earlier by the authors in [2] and [10], and which exploit only

a subset of the received signal's unconjugated second-order CS statistics.

We remark also that [9] represents a different extension of the results reported in [2]. Reference [9] proposes a unifying carrier frequency estimation framework that can be used for systems employing linear block precoders at the transmitter. However, only CDMA and OFDM modulation schemes are analyzed within this unifying transmitter precoding set-up [9].

Since the operation of oversampling the received waveform may be interpreted as a transmitter precoding scheme, the theoretical asymptotic performance analysis framework in [2, 3] and [9] is exploited herein to design optimized frequency recovery schemes that operate on asynchronous signal samples taken at a rate faster than the symbol rate (T_s). Thus far, to the best knowledge of the authors, such an analysis has not been reported in the literature and our goal herein is to complete this analysis by choosing the optimal set of parameters (in particular, number of cyclic correlation lags and oversampling rate) that have to be considered in the frequency offset estimator so that its asymptotic variance is minimized.

2 Proposed Estimator

We focus on single-carrier and single-user wireless communications channels. The continuous-time base-band received signal $y_a(t)$ can be expressed as follows :

$$y_a(t) = (\sum_{k \in \mathbb{Z}} s_k h_a(t - kT_s)) e^{2i\pi f_a t} + w_a(t),$$

where the symbol sequence $\{s_k\}$ transmitted at the baud rate $1/T_s$ is assumed to be non-circular (i.e., $\mathbb{E}[s_k^2] \neq 0$), independently and identically distributed (i.i.d.) with zero-mean and unit-variance. The filter $h_a(t)$, which is assumed to be time-limited and causal, and results from the convolution of the multipath propagation channel and the shaping filter, supposed of bandwidth $[-(1+\rho)/2T_s, (1+\rho)/2T_s]$, with the roll-off factor $\rho \in [0, 1]$. The additive noise $w_a(t)$ is assumed to be white circularly and normally distributed with power spectral density $2N_0$. Finally, f_a stands for the analog carrier frequency offset, which may be induced by the local oscillator drifts and Doppler effects.

In general, the parameters f_a and $h_a(t)$ are unknown and have to be estimated in order to detect the transmitted data s_k. This paper proposes an optimized frequency-offset compensator that does not require knowledge of the channel impulse response or training sequence. The proposed frequency estimator is implemented digitally and assumes at the front end of the receiver an anti-aliasing filter (AAF) $g_a(t)$, whose output is sampled at the rate $1/T = P/T_s$, where the oversampling factor P is an integer (see e.g., [5,

p. 139]). The AAF is assumed to be an ideal low-pass filter (although less stringent conditions on the frequency response of the AAF may be adopted) with bandwidth B sufficiently large in order to preserve all the signal components at the filter output. The output of the AAF, denoted by $z_a(t)$, is sampled so that Nyquist's condition is satisfied $1/T = 2B$ (i.e., $B = P/2T$). Since $z_a(t) := g_a(t) * y_a(t)$, where $*$ stands for the convolution operation, the following discrete-time channel model is obtained:

$$z(k) := z_a(kT) = \left(\sum_{m=0}^{L} l_m v_{k-m} \right) e^{2i\pi f_d k} + n(k), \quad (1)$$

where $\{v_k\}$ stands for the sequence obtained by padding $P - 1$ zeros between any two consecutive symbols $\{s_k\}$. Define also: $l(z) := \sum_{m=0}^{L} l_m z^{-m}$, with $l_m := (g_a(t) * h_a(t))|_{t=mT}$, the digital frequency offset $f_d := f_a T$, and the discrete-time white noise sequence $n(k) := (g_a(t) * w_a(t))|_{t=kT}$ of variance $\sigma^2 := \mathbb{E}[|n(k)|^2] = 2N_0/T$. Our goal is to build an optimal frequency offset estimator based on the sole knowledge of the observation samples $\{z(k)\}_{k=0}^{N-1}$. In essence, we propose an estimator that optimally exploits all the unconjugated second-order cyclic statistics of the received data and show that its asymptotic variance is invariant with respect to the oversampling factor P as soon as $P \geq 2$.

Since the channel model (1) works as if one has transmitted the block sequence $[v_{kP}, \cdots, v_{kP+P-1}] = \mathbf{k}^T s_n$, with $\mathbf{k} = [1, \mathbf{0}_{1,P-1}]^T$, (1) can be interpreted as a linearly precoded system. Consequently, we can use the frequency offset estimator [9] that holds for general linear precoders, described by a tall matrix $\mathbf{K}$. Let $r_c(n, \tau) := \mathbb{E}[z(n+\tau)z(n)]$ denote the unconjugated correlation at lag τ of $z(n)$, and define $\alpha_0 := (2f_d \text{ modulo } 1)$. Considering a Fourier series expansion of $r_c(n, \tau)$, we obtain:

$$r_c(n, \tau) = \sum_{p=0}^{P-1} r_c^{(\alpha_0 + p/P)}(\tau) e^{2i\pi(\alpha_0 + p/P)n}, \quad (2)$$

where $r_c^{(\alpha)}(\tau)$ stands for the unconjugated cyclo-correlation at lag τ and cyclic frequency α. Let $\mathcal{A}_0$ be a compact set included in $(0, \min(1/2, 1/P))$. According to (2), we get

$$\forall \alpha \in \mathcal{A}_0, \alpha \neq \alpha_0, \forall \tau, \forall p, \quad r_c^{(\alpha+p/P)}(\tau) = 0.$$

Then, an asymptotically unbiased and consistent estimate $\hat{\alpha}_N$ of α_0 is obtained as follows:

$$\hat{\alpha}_N := \arg \max_{\alpha \in \mathcal{A}_0} J_N(\alpha),$$

with

$$J_N(\alpha) := \sum_{p=0}^{P-1} \left|\left| \hat{\mathbf{r}}_{c,N}^{(\alpha+p/P)} \right|\right|_{\mathbf{W}_p}^2$$

where $\hat{\mathbf{r}}_{c,N}^{(\alpha)} := [\hat{r}_{c,N}^{(\alpha)}(-M), \cdots, \hat{r}_{c,N}^{(\alpha)}(M)]^{\mathrm{T}}$, M denotes a positive integer ($M \geq L$), and $\{\mathbf{W}_p\}_{p=0}^{P-1}$ is a sequence of positive-definite Hermitian matrices[1]. The term

$$\hat{r}_{c,N}^{(\alpha)}(\tau) := (1/N) \sum_{n=0}^{N-1} z(n)z(n+\tau)e^{-2i\pi\alpha n}$$

denotes a sample estimate of $r_c^{(\alpha)}(\tau)$, assuming N observations available.

A few observations are now in order. The introduced estimate is an extension of the estimator [10], which exploits only one cyclic correlation lag ($M = 0$). Second, the introduced estimator may be interpreted as a special case of the estimators proposed in [9], which are associated with different precoding schemes. Nevertheless, the design and analysis of the new estimator reported herein remain of interest and have not been reported in the literature.

By exploiting the results of [9], the consistency and asymptotic normality of $\hat{\alpha}_N$ can be established. In addition, a closed-form expression for the asymptotic covariance defined as:

$$\gamma := \lim_{N \to \infty} N^3 \mathbb{E}[(\hat{\alpha}_N - \alpha_0)^2] , \qquad (3)$$

may be obtained. Furthermore, in order to minimize the asymptotic variance γ, it can be shown that it is optimal to consider $M = L$ and $\mathbf{W}_p = \delta_{0,p}\mathbf{I}_{2L+1}$, where δ and $\mathbf{I}_{2L+1}$ denote the Kronecker index and the $(2L+1) \times (2L+1)$-identity matrix, respectively [9]. Simply stated, the extraction of the frequency offset should be performed based solely on the harmonic $\alpha_0 + p/P$ with $p = 0$, and all the cyclic correlation lags have to be taken into account. These results may be derived using similar techniques as the ones presented in [2] and [9], and will not be detailed herein. In the sequel, we focus on the estimate associated with such an optimal design setting and analyze its asymptotic performance.

3 Influence of the oversampling factor

As $M = L$ and $\mathbf{W}_p = \delta_{0,p}\mathbf{I}_{2L+1}$, the closed-form expression of γ given in [9] can be reduced to

$$\gamma = \frac{3P\sigma^2}{\pi^2 a^2}(P\sigma^2 a + 2b) ,$$

with:

$$a := \int_{-1/2}^{1/2} |l(e^{2i\pi f})|^2 |l(e^{-2i\pi f})|^2 df,$$

and

$$b := \int_{-1/2}^{1/2} |l(e^{2i\pi f})|^4 |l(e^{-2i\pi f})|^2 df.$$

To properly study the influence of the oversampling factor P on the performance of frequency estimator $\hat{\alpha}_N$, we evaluate the following term:

$$E_P = \mathbb{E}[(f_a T_s - \widehat{f_a T_s}|_{P,N_s})^2] , \qquad (4)$$

with $\widehat{f_a T_s}|_{P,N_s} := \hat{\alpha}_N P/2$ and $N_s := N/P$. In fact, E_P represents the theoretical mean-square error of the analog frequency offset estimate normalized with the symbol duration, assuming that the duration of the observation window is $N_s T_s$. From (3) and (4), it turns out that:

$$E_P = \frac{\gamma}{4PN_s^3} . \qquad (5)$$

After some quite straightforward but very long calculations based on Poisson's formulae, (5) can be expressed as:

$$E_P = \frac{3N_0}{\pi^2 N_s^3 \zeta_P^{(1)2}}(N_0 T_s \zeta_P^{(1)} + \zeta_P^{(2)}) ,$$

with:

$$\begin{cases} \zeta_1^{(1)} := \int_{-1/2T_s}^{1/2T_s} |H_a(f)|^2 |H_a(-f)|^2 df \\ \zeta_1^{(2)} := \int_{-1/2T_s}^{1/2T_s} |H_a(f)|^4 |H_a(-f)|^2 df \end{cases} \quad \text{for} \quad P = 1,$$

and

$$\begin{cases} \zeta_P^{(1)} := \int_{-1/T_s}^{1/T_s} |H_a(f)|^2 |H_a(-f)|^2 df \\ \zeta_P^{(2)} := \int_{-1/T_s}^{1/T_s} |H_a(f)|^4 |H_a(-f)|^2 df \end{cases} \quad \text{for} \quad P \geq 2.$$

As soon as $P \geq 2$, $\zeta_P^{(1)}$ and $\zeta_P^{(2)}$ do not depend on P. Thus, the theoretical mean-square error is independent of P, a result which intuitively might be predicted based on Shannon's interpolation theorem, and is equal to zero in the noiseless case ($N_0 = 0$) (i.e., an asymptotically jitter-free timing recovery scheme).

4 Simulations

The Signal-to-Noise Ratio (SNR) is expressed regardless of the oversampling factor as $\mathrm{SNR} := \int_{\mathbb{R}} |H_a(f)|^2 df / 2N_0$. We fix also $\rho = 0.2$, $f_d = 0.05$, $T_s = 3\mu s$, and the circularly distributed noise $n(k)$ is assumed white and Gaussian. The theoretical and experimental MSE of the frequency estimator are obtained by averaging E_P and $||f_a T_s - \widehat{f_a T_s}|_{P,N_s}||^2$ over $MC = 100$ Monte-Carlo trials, respectively. At each trial, a (slow Rayleigh) fading multipath propagation channel with three paths is adopted. The complex amplitudes of the paths are normally distributed and the timing delays assume uniform distributions

in $[0, \; 3T_s]$. As it is usually performed, we proceed in two steps to obtain the frequency offset estimate: first, a *coarse* search step is performed to maximize the criterion $J_N(\alpha)$ via a Fast Fourier Transform (FFT). Then, a *fine* search step is performed based on a gradient algorithm, initialized with the estimate provided by the coarse search step. Since the asymptotic analysis studies the behavior of the criterion around the true point α_0, this analysis does not provide any relevant information on the performance of the first step, which optimizes the criterion over an FFT grid of frequencies spread on the entire interval $[0, 1/P]$. The performance of the first step can be relevantly evaluated by means of the occurrence probability of a wrong detection of the peak, which normally should occur around α_0. Figure 1 depicts the number of samples N_s with respect to SNR for which the probability of failure is less than 1%.
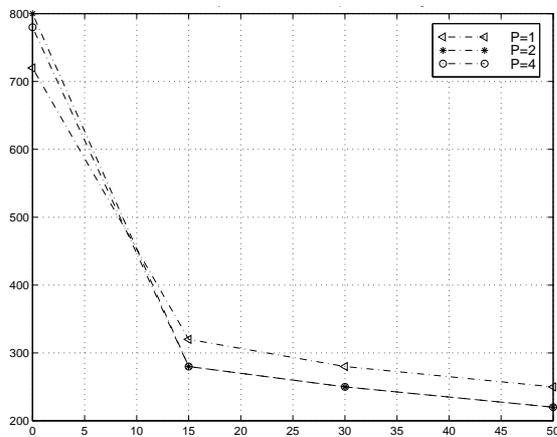

Figure 1: Lower bound for N_s versus SNR

In fact, the amplitude of the spectral line localized at frequency α_0 depends on the terms $\zeta_P^{(k)}$, $k = 1, 2$. We have observed that the false detection occurs whenever the terms $\zeta_P^{(k)}$, $k = 1, 2$, are numerically weak, in general smaller than the noise variance.

In the sequel, we only consider the trials which succeeded to detect the right peak. Figure 2 plots the MSE versus N_s.

We observe that the MSE is proportional to $O(1/N_s^3)$. For the remaining simulations, we fix $N_s = 500$. Figure 3 plots the MSE versus SNR, and shows that the performance of the baud-rate estimator is worse than the performance of the oversampled estimator.

In Figure 4, we depict the MSE versus ρ.

For $P \geq 2$, the performance is quite the same with respect to ρ. On the contrary, for $P = 1$, the theoretical and experimental performances slightly degrade as ρ increases. Indeed, as the roll-off factor increases, the loss of information becomes more important. Judicious exploitation of the
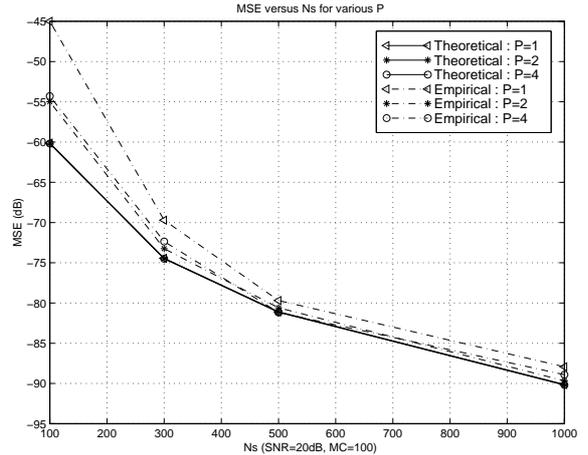

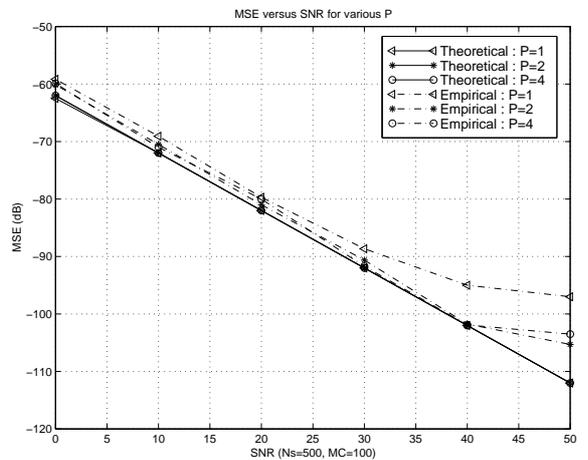Figure 2: Theoretical and experimental MSE versus N_s

Figure 3: Theoretical and experimental MSE versus SNR

entire statistical information requires to select $P > 1$. In all the figures, one can observe that the performance is independent of the oversampling factor P as soon as $P \geq 2$. In addition, the estimation performance corresponding to the oversampled case ($P \geq 2$) is always better than the baud-rate case ($P = 1$).

5 Conclusions

We have investigated the theoretical and experimental MSE-performances of a blind frequency offset estimator based on the unconjugated cyclostationary statistics of the oversampled received signal. In perfect agreement with Shannon's interpolation theorem, we deduced that for optimum performance the oversampling factor does not need to be larger
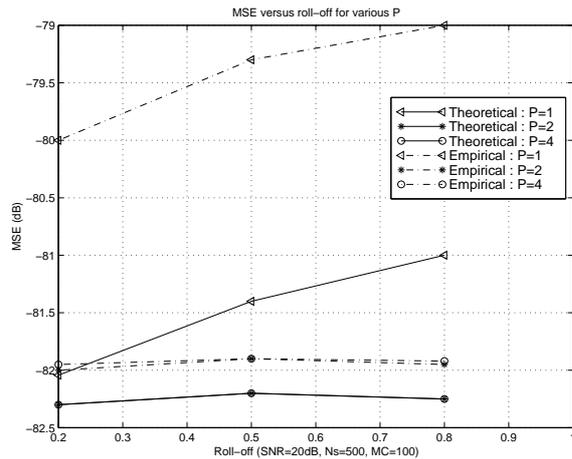
Figure 4: Theoretical and experimental MSE versus ρ

than two. Consequently, receivers with reduced sampling rates (complexity) may be designed without any loss in performance.

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