Resource Allocation for Type-I HARQ based Wireless Ad Hoc Networks

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***Abstract*—In a multi-user context, the paper deals with power and bandwidth allocation for systems using Orthogonal Frequency Division Multiple Access (OFDMA) and Type-I Hybrid Automatic Repeat reQuest (HARQ). Assuming only statistical channel state information at the resource allocator, we propose new algorithms minimizing the total transmitted power when minimum individual rate and packet error rate are required.**

I. Introduction

We consider a wireless clustered ad hoc network (CAHN) topology for which we propose a solution for the resource allocation when Type-I HARQ is used at the link layer along with an Orthogonal Frequency Division Multiple Access (OFDMA) scheme. The nodes in a CHAN are managed by a cluster head (CH) collecting the transmission nodes' requests and performing a centralized resource allocation accordingly.

OFDMA is assumed to allow simultaneous and interference-free peer to peer links in the cluster avoiding to concentrate all the traffic at the CH. We also consider HARQ at the link layer since it is a powerful mechanism enabling to accommodate the unknown channel variations efficiently by achieving a good trade-off between channel coding and retransmission [1].

To manage the CAHN, in addition to OFDMA, the transmissions also follow a Time Division Multiple Access (TDMA) scheme with specific slots reserved for signalling and data. An important consequence of such an organization lies in the fact that the CH will only be able to get statistical channel state information (CSI) for the different links in its cluster. Indeed, since the resource allocation is centralized at the CH, the time to initiate a specific link transmission and to transfer back the CSI may last several frame periods resulting in a CSI which is completely outdated when available at the CH. The only possibility offered to the CH is to draw statistics from the received CSI along time and use it for resource allocation. This makes a huge difference with modern cellular networks for which CSI feedback is very quick (thanks to FDD or channel reciprocity in TDD) and thus allows to use powerful allocation techniques based on CSI at the transmitter. Since most literature is concentrated on the later problem, very few literature is available on the addressed topic.

The considered criterion to optimize our resource allocation is the minimization of the total transmit power (summed up over all the transmitting nodes) subject to some statistical Quality of Service (QoS) constraints. This objective function allows to minimize the nodes' consumption and to reduce the frequency spatial footprint of the network, while maintaining link qualities. When HARQ mechanism is used, the data rate metric is well represented by the goodput which is proportional to the useful data rate (after the erroneous packets have been discarded). Although its mathematical expression depends on the packet error rate (PER) value, the goodput is not enough for characterizing a link performance. Indeed, as stated in [2], the PER has also to be kept below a certain threshold. Thus, our objective is to develop algorithms for finding the number of subcarriers and the transmit power per link in order to minimize the total transmit power subject to individual goodput and PER constraints, under statistical CSI.

To our best knowledge, this problem has never been addressed with these assumptions. Indeed, only a few works focused on the multi-user resource allocation when HARQ mechanism is used, and so when the goodput metric is taken into account (as either objective function or constraint) [2]–[5]. These works addressed the problem of bit-loading, power control and subcarrier assignment when perfect (or degraded deterministic) CSI at the resource allocator is available.

The paper is organized as follows. The system model is depicted in Section II. Section III is devoted to the optimization issue and proposed algorithms. Numerical results are provided in Section IV. Finally, conclusion is drawn in Section V.

II. System model and notations

Each link is modeled as a (time-varying) frequency-selective channel, hence OFDM is used to compensate for the frequency selectivity. It is assumed that the channel remains constant over one OFDM symbol but may change between two consecutive OFDM symbols. In the sequel, the superscript ^{T} stands for the transposition operator, and the multi-variate complex-valued circular Gaussian distribution with mean a and covariance matrix Σ is denoted $\mathcal{CN}(a, \Sigma)$. The inverse of any function f with respect to composition is denoted f^{-1}.

Let $\boldsymbol{h}_k(i) = [h_k(i,0), \ldots, h_k(i, M-1)]^T$ be the channel impulse response of user k associated with OFDM symbol i, where M is the number of taps. Let us denote by $\boldsymbol{H}_k(i) = [H_k(i,0), \ldots, H_k(i, N-1)]^T$ the Fourier Transform of $\boldsymbol{h}_k(i)$ where N denotes the number of subcarriers of the OFDM symbol. Assuming well-designed cyclic prefix, the received signal at OFDM symbol i and subcarrier n for user k is

$$Y_k(i,n) = H_k(i,n)X_k(i,n) + Z_k(i,n), \quad (1)$$

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where $X_k(i,n)$ is the symbol transmitted by user k at subcarrier n of OFDM symbol i, and the additive noise $Z_k(i,n) \sim \mathcal{CN}(0, N_0W/N)$ where N_0 is the noise power spectral density and W is the total bandwidth. It is assumed that each channel is an independent random process with possibly different variances for each tap, *i.e.* $\boldsymbol{h}_k(i) \sim \mathcal{CN}(0, \Sigma_k)$ with $\Sigma_k = \text{diag}_{M\times M}(\varsigma_{k,m}^2)$. Thus, the Fourier Transform vector is an independent random process $\boldsymbol{H}_k(i) \sim \mathcal{CN}(0, \varsigma_k^2 \mathbf{I}_N)$ with $\varsigma_k^2 = \text{Tr}(\Sigma_k)$. Therefore, the subcarriers of a given user are identically distributed.

Let $g_k(i,n) = |H_k(i,n)|^2/N_0$ be the instantaneous gain-to-noise ratio (GNR) for user k at subcarrier n and OFDM symbol i. Assuming a Rayleigh channel, $g_k(i,n)$ is exponentially distributed, with a mean given by

$$G_k = \mathbb{E}\left[g_k(i,n)\right] = \frac{\varsigma_k^2}{N_0}. \tag{2}$$

In this paper, we assume that the resource allocator only knows the terms G_k (*i.e.*, the average GNR instead of the instantaneous one) for each active link. Since G_k is independent of n, the resource allocation algorithm will not distinguish between the subcarriers for a given user. On the other hand, the users will obviously be treated differently since G_k depends on k. We assume that the behavior of G_k is driven by the so-called path-loss. Let D_k be the distance between user k and the corresponding receiver. Then $G_k = \ell(D_k)/N_0$, where $\ell(D_k)$ depends on the path-loss model.

We assume that K users are active and OFDMA is used to separate the users. We remind that the resource allocator does not have instantaneous CSI but only statistical CSI through G_k, for each user/link k. As G_k does not depend on the subcarrier index, the resource allocator cannot allocate which subcarriers user k will use, but only how many. Let n_k be the number of subcarriers assigned to user k. So the bandwidth proportion occupied by user k is equal to

$$\gamma_k = \frac{n_k}{N}$$

and corresponds to the bandwidth parameter to be optimized. Due to the independence of G_k with respect to the subcarrier index, it is natural for user k to use the same average power $P_k = \mathbb{E}\left[|X_k(i,n)|^2\right]$ on each subcarrier. Let $E_k = P_k/(W/N)$ and $\sigma_k^2 = N_0(W/N)$ be the energy consumed to send one symbol on each subcarrier and the corresponding noise variance, respectively. Then, on each subcarrier, user k undergoes an average signal-to-noise ratio (SNR) given by

$$\text{SNR}_k = \frac{\varsigma_k^2 P_k}{\sigma_k^2} = E_k G_k. \tag{3}$$

Finally, let Q_k be the average energy consumed to send the part of the OFDM symbol associated with user k, corresponding to the power parameter to be optimized. It can be easily shown that

$$Q_k = \frac{n_k P_k}{W} = \gamma_k E_k. \tag{4}$$

We assume that the modulation and coding scheme (MCS) for user k is based on a 2^{m_k}-QAM modulation scheme and a forward error correcting code with a coding rate R_k. Concerning the HARQ mechanism, the users use Type-I HARQ for which a single information packet can be sent at most L times. Extension to Type-II HARQ is not straightforward at all and so out of the scope of this contribution.

As a consequence, the PER of user k after applying the HARQ (so at the MAC layer) is given by

$$\Pi_k = \pi_k^L, \tag{5}$$

where π_k is the PER of the information packet (at the physical layer). The useful data rate ρ_k (in bits/s) is proportional to its HARQ goodput η_k (in bits/s/Hz), *i.e.*, $\rho_k = \eta_k W$. According to [1], the goodput is equal to

$$\eta_k = \gamma_k m_k R_k (1 - \pi_k). \tag{6}$$

A well-designed Frequency-Hopping (FH) pattern is assumed in order to recover at least the diversity offered by the channel, *i.e.*, M, which leads to a fast-fading channel model. In the uncoded case, assuming fast-fading channel and information packets of n_s symbols, the PER can be written with respect to SNR as [6]

$$\pi_k(\text{SNR}) \approx \frac{n_s a_{m_k}}{1 + \frac{g_{m_k}}{2^{m_k}-1}} \frac{1}{\text{SNR}}, \tag{7}$$

where a_{m_k} and g_{m_k} are constants related to the chosen constellation. In the coded case, Bit Interleaved Coded Modulation (BICM) is carried out in order to retrieve the entire diversity offered by the code. For a fast-fading channel, the PER can be written with respect to SNR as [7]

$$\pi_k(\text{SNR}) \approx \left(\frac{4}{d_h^2(m_k)}\right)^{d_f(R_k)} \frac{g_c(m_k, R_k)}{\text{SNR}^{d_f(R_k)}}, \tag{8}$$

where $d_f(R_k)$ is the minimum (Hamming or free) distance of the code of rate R_k, $d_h(m_k)$ is the harmonic distance related to the modulation, and $g_c(m_k, R_k)$ is a coding gain.

III. Power and bandwidth allocation

A. Optimization problem

As the objective is to minimize the total energy used for sending an OFDM symbol, we would like to minimize $\mathcal{Q}_T = \sum_{k=1}^K Q_k$ which is also equal to $\sum_{k=1}^K \gamma_k E_k$ through Eq. (4). To do that, we will adjust relevantly the user energy Q_k, the bandwidth γ_k, and the MCS (driven by m_k and R_k). Due to lack of space, the choice of the best MCS will not be discussed here, and can be selected as done in [3]. Thus, m_k and R_k are now fixed for each user.

As explained in Section I, two constraints per user must be satisfied: each user has to ensure minimum data rate and maximum PER, *i.e.*, there exists strictly positive constants $\rho_k^{(0)}$ and $\Pi_k^{(0)}$ such that $\rho_k \geq \rho_k^{(0)}$ and $\Pi_k \leq \Pi_k^{(0)}$, respectively. The data rate constraint can be translated into the goodput, $\eta_k \geq \eta_k^{(0)}$ with $\eta_k^{(0)} = \rho_k^{(0)}/W$. Given Eq. (5), the PER (at the MAC layer) leads to the following PER requirement at the physical layer, $\pi_k \leq \pi_k^{(0)}$ with $\pi_k^{(0)} = (\Pi_k^{(0)})^{1/L}$. So the optimization problem is summarized in Problem 1.

Problem 1. *Let us denote* $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_K]^T$ *and* $\boldsymbol{Q} = [Q_1, \cdots, Q_K]^T$. *The optimization problem boils down to*

$$(\boldsymbol{\gamma^*}, \boldsymbol{Q^*}) = \arg\min_{(\boldsymbol{\gamma}, \boldsymbol{Q})} \sum_{k=1}^{K} Q_k \tag{9}$$

subject to

(C1) $\eta_k(\gamma_k, Q_k) \geq \eta_k^{(0)}, \forall k,$ (C3) $\sum_{k=1}^{K} \gamma_k \leq 1,$

(C2) $\pi_k(G_k Q_k/\gamma_k) \leq \pi_k^{(0)}, \forall k,$ (C4) $\gamma_k \geq 0, Q_k \geq 0, \forall k.$

Problem 1 is feasible if, and only if, next condition holds

$$\sum_{k=1}^{K} \frac{\eta_k^{(0)}}{m_k R_k} < 1. \tag{10}$$

Indeed, assume Eq. (10) holds. Then, for some sufficiently small $\epsilon > 0$, the problem is feasible by considering Q_k large enough and $\gamma_k = (\eta_k^{(0)} + \epsilon)/(m_k R_k)$. The converse is straightforward. In the rest of the paper, Eq. (10) is assumed to be satisfied.

B. Optimal Algorithm

Before going further, we state the following Lemma. The proof is omitted due to the page limitation.

Lemma 1. *The constraint functions defined on* $[0,1] \times \mathbb{R}^+$ *by*

$$(\gamma_k, Q_k) \mapsto \eta_k(\gamma_k, Q_k) = \gamma_k m_k R_k \left(1 - \pi_k(G_k Q_k/\gamma_k)\right),$$
$$(\gamma_k, Q_k) \mapsto \pi_k(G_k Q_k/\gamma_k)$$

are respectively concave and quasi-convex [8], as long as $\pi_k : \mathbb{R}^+ \longrightarrow [0,1]$ *is a convex function.*

Problem 1 thus corresponds to the minimization of a convex function *over a convex constraint set*. Hence, as asserted in [9], under mild conditions, the Karush-Kuhn-Tucker (KKT) equations will provide a global optimal solution. Therefore, we now derive the associated KKT and find the optimal resource allocation algorithm.

In order to guarantee their minimum goodput, all the users have non-zero power and non-zero bandwidth. Then, after straightforward but tedious algebraic manipulations on the KKT equations, we are able to state the following Theorem.

Theorem 1. *Each point* $(\boldsymbol{\gamma^*}, \boldsymbol{Q^*})$ *is an optimal solution of Problem 1 iff (C1, C2, C3) hold as well as the following conditions:*

$$\eta_k^{(0)} - \gamma_k^* m_k R_k \left(1 - \pi_k(G_k \frac{Q_k^*}{\gamma_k^*})\right) = 0, \tag{11}$$

$$\left(\theta(G_k \frac{Q_k^*}{\gamma_k^*}) - \lambda^* G_k\right)\left(\pi_k(G_k \frac{Q_k^*}{\gamma_k^*}) - \pi_k^{(0)}\right) = 0, \tag{12}$$

$$\lambda^* \left(\sum_{k=1}^{K} \gamma_k^* - 1\right) = 0, \tag{13}$$

with

- λ *the non-negative Lagrange multiplier related to (C3),*
- $\theta(x) = x + (1 - \pi_k(x))/\pi_k'(x)$.

Notice that Eq. (11) implies that the goodput constraint (C1) is always active. To deduce an algorithm from Theorem 1, we have to know if the PER constraint (C2), related to Eq. (12), is active or not. Therefore, all the configurations on Eq. (12) are tested, *i.e.*, either the first factor is zero (then PER constraint is inactive) or not (then PER constraint is active), and we select eventually the best one with respect to the total transmit power. More precisely, in a first step, we consider that only $n \in \{0, \cdots, K\}$ user(s) have the PER constraint active. Then in a second step, we compute the total transmit power for all the tested configurations and select the best one. Let $\mathcal{U}_n$ be the set of all the sets of n users out of K. If $\boldsymbol{u}$ is a subset of K users, $\boldsymbol{u}^c$ is the associated complementary subset.

1. for $n = 0$ **to** K **do**
  for *each* $\boldsymbol{u} \in \mathcal{U}_n$ **do**
    $\forall k \in \boldsymbol{u}$, $\gamma_k = \eta_k^{(0)}/(m_k R_k(1 - \pi_k^{(0)}))$,
    $Q_k = \gamma_k \pi_k^{-1}(\pi_k^{(0)})/G_k$.
    $\forall k' \in \boldsymbol{u}^c$, $\gamma_{k'} = \eta_{k'}^{(0)}/(m_{k'} R_{k'}(1 - \pi_{k'}(\theta^{-1}(\lambda G_{k'}))))$,
    $Q_{k'} = \gamma_{k'} \theta^{-1}(\lambda G_{k'})/G_{k'}$.
    for $\lambda \in \mathbb{R}_*^+$ such that $\sum_{k \in \boldsymbol{u}} \gamma_k + \sum_{k' \in \boldsymbol{u}^c} \gamma_{k'} = 1$ (if no λ leads to equality, put first $\lambda = 0$ and test the condition $\sum_k \gamma_k < 1$. If the condition is not satisfied, then put $\lambda = \infty$)
    if $\exists k' \in \boldsymbol{u}^c$, *s.t.* $\pi_{k'}(G_{k'} Q_{k'}/\gamma_{k'}) > \pi_{k'}^{(0)}$ **or** $\lambda = \infty$ **then**
      $\mathcal{Q}_T(\boldsymbol{u}) = \infty$
    end
  end
end

2. Choose $\boldsymbol{u}$ minimizing $\mathcal{Q}_T(\boldsymbol{u})$.

For $n = 0$, the loop on $\boldsymbol{u}$ is implemented only once by considering $\boldsymbol{u} = \emptyset$ and $\boldsymbol{u}^c = \{1, \cdots, K\}$. As the number of combinations is huge ($\mathcal{O}(2^K)$), the previous algorithm has a high computational load and can only be used if the number of users is small enough. Therefore, we next propose two suboptimal but computationally tractable algorithms.

C. Suboptimal KKT based Algorithm (SKA)

In order to reduce the complexity of the previous optimal algorithm, we will force the algorithm to operate as if the condition on the PER were not active, *i.e.*, in Eq. (12), the left-hand term associated with the Lagrange multiplier vanishes. We remind that the constraint related to the goodput is always active (see Eq. (11)). As a consequence, the suboptimal KKT based algorithm can be described as follows:

Set $\lambda = 0$, $Q_k = \epsilon > 0$ and $\gamma_k = 1, \forall k$.
while $\sum_k \gamma_k > 1$ **or** $\exists k, \pi_k(G_k Q_k/\gamma_k) > \pi_k^{(0)}$ **do**
  1. $\forall k$, $\gamma_k = \eta_k^{(0)}/(m_k R_k(1 - \pi_k(\theta^{-1}(\lambda G_k))))$,
  2. $Q_k = \gamma_k \theta^{-1}(\lambda G_k)/G_k$.
  3. Increase λ.
end

D. Separate Linear Algorithm (SLA)

By remarking that Problem 1 can be rewritten into an equivalent form, we hereafter propose another way to exhibit a suboptimal algorithm. As $Q_k = \gamma_k E_k$, Problem 1 can be written only with respect to $(\boldsymbol{\gamma}, \boldsymbol{E})$ with $\boldsymbol{E} = [E_1, \cdots, E_K]^T$. As π_k is a non-increasing bijective function, the constraint

(C2) boils down to $E_k \geq \pi_k^{-1}(\pi_k^{(0)})/G_k$ and so leads to the following equivalent Problem.

Problem 2. *Problem 1 is equivalent to*

$$(\boldsymbol{\gamma}^*, \boldsymbol{E}^*) = \arg \min_{(\boldsymbol{\gamma}, \boldsymbol{E})} \sum_{k=1}^{K} \gamma_k E_k \tag{14}$$

subject to (C3) and

(C1') $\gamma_k \geq \eta_k^{(0)} / \left(m_k R_k \left(1 - \pi_k(G_k E_k)\right)\right), \forall k,$
(C2') $E_k \geq \pi_k^{-1}(\pi_k^{(0)})/G_k, \forall k,$
(C4') $\gamma_k \geq 0, E_k \geq 0, \forall k.$

Problem 2 is actually more complex than Problem 1 since the objective function is no longer convex, but only quasi-concave. Therefore, convex optimization tool is useless. However, we can use a suboptimal approach which consists in optimizing the objective function separately on each parameter. Therefore we propose to split Problem 2 into two subproblems.

Problem 2.a (on $\boldsymbol{E}$)**.** *For fixed* $\boldsymbol{\gamma}$*, the subproblem is*

$$\boldsymbol{E}^* = \arg \min_{\boldsymbol{E}} \sum_{k=1}^{K} \gamma_k E_k$$

subject to (C2', C4').

Problem 2.b (on $\boldsymbol{\gamma}$)**.** *For fixed* $\boldsymbol{E}$*, the subproblem is*

$$\boldsymbol{\gamma}^* = \arg \min_{\boldsymbol{\gamma}} \sum_{k=1}^{K} \gamma_k E_k$$

subject to (C1', C3, C4').

The solution of Problem 2.a is $E_k^* = \pi_k^{-1}(\pi_k^{(0)})/G_k, \forall k$. Constraint (C1') has been removed from Problem 2.a to avoid a deadlock issue. Indeed, if (C1') is added to Problem 2.a and is active for user k, then the solution of Problem 2.b actually is equal to the value of γ_k initializing Problem 2.a, and so the optimal γ_k is given by the initialization step!

The solution of Problem 2.b can be efficiently obtained by using linear programming tool, for instance, the Simplex method [10]. However, for some $\boldsymbol{E}$, Problem 2.b may not have a feasible solution since all the constraints may not be satisfied simultaneously. To overcome this issue, we suggest to increase $\boldsymbol{E}$ until a feasible solution to Problem 2.b is found as follows: if Problem 2.b is not feasible, add a small increment δ to each E_k as many times as necessary.

To sum up, we first fix $E_k = \pi_k^{-1}(\pi_k^{(0)})/G_k, \forall k$ (corresponding to the solution of Problem 2.a), and then we solve Problem 2.b until feasibility.

IV. Numerical results

An uncoded ARQ scheme with $L = 3$ is considered for $K = 4$ links. Each user sends a data packet composed of 32 uncoded BPSK symbols within a bandwidth $W = 1$ MHz. The path loss follows the free-space model $\ell(D) = 1/\left((4\pi f_0/c)^2 D^2\right)$ where c is the light celerity and f_0 is the carrier frequency. We put $f_0 = 400$ MHz and the noise density power is fixed to $N_0 = -170$ dBm/Hz. The distance D_k between both users associated with the kth link is uniformly selected in the interval $[D_m, D_M]$ with $D_m = 50$ m and $D_M = 1$ km. Each simulated point is obtained via 100 Monte-Carlo runs. For the sake of simplicity, each link has the same target data rate $\rho^{(0)}$ and PER constraints fixed to $\Pi^{(0)} = 10^{-2}$ or $\Pi^{(0)} = 10^{-4}$.

The total transmit power is displayed in Fig. 1 versus the sum data rate defined as $K\rho^{(0)}$. SLA performs very close to the optimal one, and SKA performs looser for medium sum data rates. For sum rates approaching 1 Mbps, *i.e.* when left-hand side of Eq. (10) approaches 1, the performance come close for all the algorithms, for any PER constraint value.

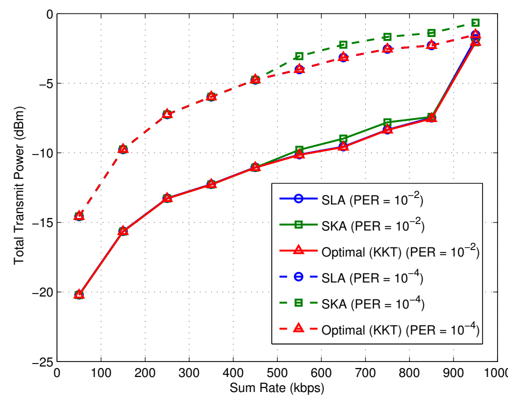

Figure 1. Total Transmit Power (in dBm) versus the sum data rate.

V. Conclusion

This paper provided new power and bandwidth resource allocation algorithms for HARQ/OFDMA based systems. The algorithms rely on the power minimization subject to goodput and PER constraints under statistical CSI knowledge. We observed that the proposed low-complexity algorithms perform well compared to the optimal solution which suffers from a high computational load.

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