An Alternative Blind Feedforward Symbol Timing Estimator Using Two Samples per Symbol

Yan Wang, Erchin Serpedin, and Philippe Ciblat

***Abstract*—Recently, Lee has proposed a blind feedforward symbol timing estimator that exhibits low computational complexity and requires only two samples per symbol. In this paper, Lee's estimator is analyzed rigorously by exploiting efficiently the cyclostationary statistics present in the received oversampled signal, and its asymptotic (large sample) bias and mean-square error (MSE) are derived in closed-form expression. A new blind feedforward timing estimator that requires only two samples per symbol and presents the same computational complexity as Lee's estimator is proposed. It is shown that the proposed new estimator is asymptotically unbiased and exhibits smaller MSE than Lee's estimator. Computer simulations are presented to illustrate the performance of the proposed new estimator with respect to Lee's estimator and the existing conventional estimators.**

***Index Terms*—Asymptotic performance analysis, blind feedforward estimation, cyclostationarity, symbol timing estimation.**

I. Introduction

During the last decade, nondata-aided (or blind) feedforward timing estimation architectures have received much attention in synchronization of bandwidth efficient and burst-mode transmissions (see, e.g., [2]–[7] and [10]). Most of the methods proposed in the literature require a sampling frequency of at least three times larger than the symbol rate [2], [5]–[7]. However, such high sampling rates are not desirable for high-rate transmissions, since the hardware cost of the receiver depends heavily on the required processing speed [10].

Recently, Lee proposed a new blind feedforward timing estimation algorithm that requires only two samples per symbol [3]. Compared with other two-samples-per-symbol-based timing estimators [4], [10], Lee's estimator has the advantage that it does not necessitate any low-pass filters. Lee's estimator exhibits a reduced computational complexity comparable with that of the second-law nonlinearity (SLN) estimator [6], which is known to be the simplest among the estimators using four samples per symbol and admits a very suitable digital implementation [3], [10]. However, Lee's estimator is asymptotically biased and its performance has not been analyzed thoroughly. The goal of this letter is to analyze and evaluate the performance of Lee's estimator and to propose a new unbiased timing estimator with improved mean-square error (MSE) performance. It is also shown that the proposed new estimator exhibits the same computational complexity as Lee's estimator, and significant MSE improvements are observable, especially in the case of pulse shapes with moderate and large excess bandwidth. The asymptotic (large sample) MSEs of these two estimators, together with the asymptotic bias of Lee's estimator, are established in closed form. Computer simulations illustrate the merits of the proposed new timing estimator.

II. System Model

Let us consider the same baseband model as used in [3]

$$x_c(t) = e^{j\phi} \sum_l w(l) h_c(t - \epsilon T - lt) + v_c(t) \tag{1}$$

where $\{w(l)\}$ is the sequence of independently and identically distributed (i.i.d.) phase-shift keying (PSK) symbols with $|w(l)| = 1$ [this assumption is not mandatory, in fact, $w(l)$ can be drawn from any linear memoryless modulation, e.g., quadrature amplitude modulation (QAM), pulse amplitude modulation (PAM)], $h_c(t)$ denotes the convolution of the transmitter's signaling pulse and the receiver filter, which is assumed to be a raised cosine pulse shape of bandwidth $[-(1+\rho)/2T, (1+\rho)/2T]$, where the parameter ρ represents the rolloff factor $(0 < \rho \le 1)$, $v_c(t)$ is the complex-valued additive Gaussian noise with variance N_0, T is the symbol period, ϕ denotes the received signal phase, $\epsilon \in (-1/2, 1/2]$ stands for the (normalized) symbol timing delay, and represents the parameter to be estimated.

To generate two samples per symbol, we oversample the received signal $x_c(t)$ (1) with the sampling period[1] $T_s := T/2$, and obtain the following discrete-time model:

$$\begin{aligned} x(n) &:= x_c(nT_s) \\ &= e^{j\phi} \sum_l w(l) h(n - 2l) + v(n), \quad n = 0, \ldots, N-1 \end{aligned} \tag{2}$$

with $v(n) := v_c(nT_s)$, and $h(n) := h_c(nT_s - \epsilon T)$.

Based on the above model, Lee proposed a blind feedforward symbol timing estimator, which with the notation adopted so far takes the following form (c.f. [3, Eq. (2)]):

$$\begin{aligned} \hat{\epsilon}_{\text{Lee}} := \frac{1}{2\pi} \arg \Bigg\{ &\sum_{n=0}^{N-1} |x(n)|^2 e^{-jn\pi} \\ &+ \sum_{n=0}^{N-2} \Re\{x^*(n)x(n+1)\} e^{-j(n-0.5)\pi} \Bigg\} \end{aligned} \tag{3}$$

where the notation "$\Re\{\cdot\}$" stands for the real part of the operand contained within the curly brackets.

[1]The notation ":=" stands for "is defined as".

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III. A New Blind Feedforward Symbol Timing Estimator

The time-varying correlation of the nonstationary process $x(n)$ is defined as $r_x(n;\tau) := \mathrm{E}\{x^*(n)x(n+\tau)\}$ and satisfies the relation $r_x(n;\tau) = r_x(n+2;\tau)$, where τ is an integer lag. Being periodic, $r_x(n;\tau)$ admits a Fourier series expansion, whose Fourier's coefficients, also termed cyclic correlations, are given for $k = 0, 1$, by the following expression [2]:

$$R_x(k;\tau) := \frac{1}{2}\sum_{n=0}^{1} r_x(n;\tau)e^{-jkn\pi}.$$

For $k = 1$, the following expression of $R_x(k;\tau)$ is obtained in [9]:

$$R_x(1;\tau) = e^{j\pi\tau}\cos\left[2\pi\left(\epsilon + \frac{\tau}{4}\right)\right]G(1;\tau) \qquad (4)$$

with

$$G(1;\tau) := \frac{2}{T}\int_{-\frac{1}{2T}}^{\frac{1}{2T}} H_c\left(F - \frac{1}{2T}\right)H_c\left(F + \frac{1}{2T}\right)e^{j\pi\tau TF}dF$$

where $H_c(F)$ stands for the Fourier transform (FT) of $h_c(t)$. Due to the symmetry property of the raised-cosine function $h_c(t)$, one can find that $H_c(F)$ is a real-valued even function [8, p. 546]. Note also that $G(1;\tau)$ and $R_x(1;\tau)$ are real-valued functions, since $e^{j\pi\tau} = (-1)^\tau$. Some straightforward calculations lead to the following more explicit expressions:

$$G(1;0) = \frac{\rho}{4} \quad \text{and} \quad G(1;1) = \frac{2\sin\frac{\pi\rho}{2}}{\pi(4-\rho^2)}.$$

In practice, the cyclic correlations $R_x(k;\tau)$ have to be estimated from a finite number of samples N, and the standard sample estimate of $R_x(k;\tau)$ is given by [1]

$$\hat{R}_x(k;\tau) = \frac{1}{N}\sum_{n=0}^{N-\tau-1} x^*(n)x(n+\tau)e^{-jkn\pi}, \quad \tau \geq 0$$

which is asymptotically unbiased and consistent in the mean-square sense. Thus, one can observe that Lee's estimator (3) can be expressed as

$$\hat{\epsilon}_{\text{Lee}} = \frac{1}{2\pi}\arg\left\{\hat{R}_x(1;0) + j\Re\left\{\hat{R}_x(1;1)\right\}\right\}$$

and its asymptotic mean is given by

$$\epsilon_0 := \lim_{N\to\infty} E\{\hat{\epsilon}_{\text{Lee}}\} = \frac{1}{2\pi}\arg\{R_x(1;0) + jR_x(1;1)\}. \qquad (5)$$

Based on (4) and (5), and for $\epsilon \in [0, 1/4]$, ϵ_0 can be expressed as

$$\begin{aligned}\epsilon_0 &= \frac{1}{2\pi}\arctan\left\{\frac{R_x(1;1)}{R_x(1;0)}\right\}\\ &= \frac{1}{2\pi}\arctan\left\{\frac{G(1;1)}{G(1;0)}\tan(2\pi\epsilon)\right\}\\ &= \frac{1}{2\pi}\arctan\{g(\rho)\tan(2\pi\epsilon)\}\end{aligned}$$

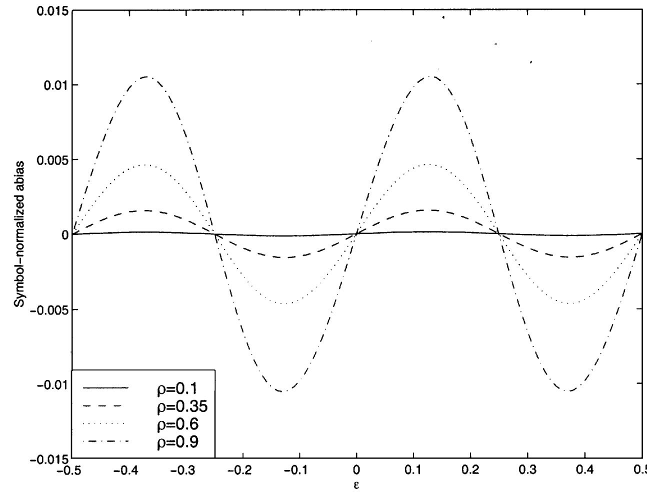

Fig. 1. Asymptotic bias of Lee estimator.

with $g(\rho) := G(1;1)/G(1;0)$. Obviously, ϵ_0 is not equal to the true value of the timing delay ϵ except for several special values of ϵ, since, in general, $g(\rho) \neq 1$ whenever $\rho \in (0,1]$. Now, it is not difficult to compute the asymptotic bias of Lee's estimator as

$$\begin{aligned}abias(\rho,\epsilon) &:= \epsilon - \epsilon_0\\ &= \frac{1}{2\pi}\left(\arctan\{\tan(2\pi\epsilon)\}\right.\\ &\qquad \left.- \arctan\{g(\rho)\tan(2\pi\epsilon)\}\right)\\ &= \frac{1}{2\pi}\arctan\left\{\frac{1-g(\rho)}{\cot(2\pi\epsilon) + \tan(2\pi\epsilon)g(\rho)}\right\}. \qquad (6)\end{aligned}$$

When ϵ assumes values other than [0, 1/4], the asymptotic bias of Lee's estimator can be obtained in a similar way and takes the same expression as (6). Fig. 1 plots $\mathrm{abias}(\rho,\epsilon)$ versus ϵ for several values of ρ, which is similar to the plot [3, Fig. 2], obtained by means of more laborious numerical calculations. From Fig. 1, it can be seen that the asymptotic bias is tolerable for small rolloff factors, but increases with ρ.

The above derivation suggests that by compensating the term $g(\rho)$, we can design a new blind asymptotically unbiased feedforward symbol timing estimator of the following form:

$$\begin{aligned}\hat{\epsilon} &= \frac{1}{2\pi}\arg\left\{g(\rho)\cdot\hat{R}_x(1;0) + j\Re\left\{\hat{R}_x(1;1)\right\}\right\}\\ &= \frac{1}{2\pi}\arg\left\{g(\rho)\cdot\sum_{n=0}^{N-1}|x(n)|^2e^{-jn\pi}\right.\\ &\qquad \left. + \sum_{n=0}^{N-2}\Re\{x^*(n)x(n+1)\}e^{-j(n-0.5)\pi}\right\}. \qquad (7)\end{aligned}$$

Note that this new estimator (7) has the same implementation complexity as that of Lee's estimator (3). In the next section, we establish in closed-form expressions the asymptotic MSEs of estimators (3) and (7), which are defined as follows:

$$\begin{aligned}\gamma_{\text{Lee}} &:= \lim_{N\to\infty} N\mathrm{E}\left\{(\hat{\epsilon}_{\text{Lee}} - \epsilon)^2\right\}\\ \gamma_{\text{new}} &:= \lim_{N\to\infty} N\mathrm{E}\left\{(\hat{\epsilon} - \epsilon)^2\right\}.\end{aligned}$$

IV. Performance Analysis for Estimators

In order to establish the asymptotic MSEs of estimators (3) and (7), it is necessary to evaluate the normalized asymptotic covariances of the cyclic correlations, which are defined as

$$\boldsymbol{\Gamma}_{u,v} = \lim_{N\to\infty} N\mathrm{E}\left\{\left(\hat{R}_x(1;u) - R_x(1;u)\right) \times \left(\hat{R}_x(1;v) - R_x(1;v)\right)^*\right\}.$$

The detailed expression for $\boldsymbol{\Gamma}$ is established in [9, Prop. 1], and will not be shown herein due to the space limitations. The interested reader is referred to [9].

The following theorem sums up the expressions of the asymptotic MSEs of the estimators (3) and (7), whose detailed derivation is presented in the Appendix.

Theorem 1: The asymptotic MSEs of the symbol timing delay estimators (7) and (3) are given by

$$\gamma\text{new} = \frac{1}{4\pi^2 G^2(1;1)} \times \left\{\frac{\cos^2(2\pi\epsilon)(\boldsymbol{\Gamma}_{1,1} - \Gamma_{1,-1})}{2} + g^2(\rho)\sin^2(2\pi\epsilon)\boldsymbol{\Gamma}_{0,0} - g(\rho)\sin(4\pi\epsilon)\boldsymbol{\Gamma}_{1,0}\right\}$$

$$\gamma_{\text{Lee}} = \frac{\sin^2(4\pi\epsilon_0)}{\sin^2(4\pi\epsilon)}\gamma\text{new} + N \cdot abias^2(\rho,\epsilon)$$

respectively.

Note that both estimators (3) and (7) assume that the frequency recovery has been achieved. If a symbol-normalized frequency offset f_e is present, it can be shown that the cyclic correlation (4) becomes [9]

$$R_x(1;\tau) = e^{j\pi\tau}e^{j\pi f_e\tau}\cos\left[2\pi\left(\epsilon + \frac{\tau}{4}\right)\right]G(1;\tau).$$

The additional f_e-related term can introduce the bias into the proposed estimator (7) and the resulting asymptotic bias can be obtained by following a similar procedure to that used in deriving (6)

$$\mathrm{abias}(f_e,\epsilon) = \frac{1}{2\pi}\arctan\left\{\frac{1-\cos(\pi f_e)}{\cot(2\pi\epsilon) + \tan(2\pi\epsilon)\cos(\pi f_e)}\right\}.$$

This bias can be counteracted by applying a blind feedforward frequency offset estimator, proposed in [2] and [9], which takes the form $\hat{f}_e = \arg\{\hat{R}_x(1;1)\}/\pi$, and then compensating the f_e-related term in the timing estimator (7).

A direct analytical comparison between γ_{Lee} and γnew seems intractable. Therefore, in the next section, we will resort to numerical illustrations.

V. Simulation Experiments

To corroborate the proposed asymptotic performance analysis, we conduct computer simulations to compare the theoretical bounds (The.) of estimators (3) and (7) (i.e., γ_{Lee} and γnew normalized with the number of samples N) with the experimental (Exp.) MSE results. The performance of conventional four-samples/symbol-based blind feedforward symbol timing delay estimators SLN [6], log nonlinearity (LOGN) [5], fourth-law nonlinearity (FLN) and absolute-value nonlinearity (AVN) [7], is also illustrated. The experimental results are obtained by performing 800 Monte Carlo trials assuming that the transmitted symbols are drawn from a quarternary phase-shift keying

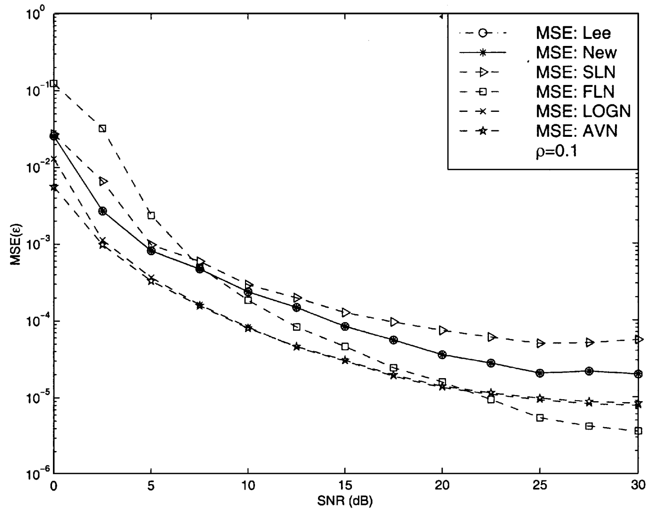

Fig. 2. MSEs of timing delay estimators versus SNR ($\rho = 0.1$).

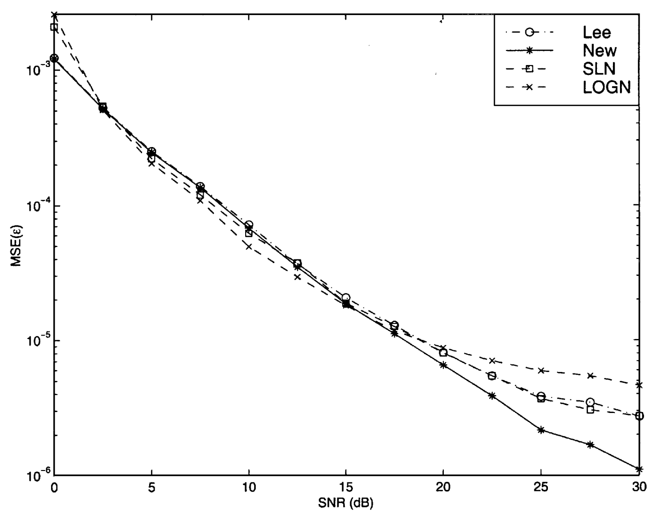

Fig. 3. MSEs of timing delay estimators versus SNR ($\rho = 0.35$).

(QPSK) constellation, the number of symbols $L = 512$, and the value of $\epsilon = 0.35$. The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} := 10\log_{10}(1/N_0)$. Figs. 2–5 show the simulation results for the rolloff factors $\rho = 0.1$, $\rho = 0.35$, and $\rho = 0.5$, respectively. From these figures, the following conclusions can be drawn.

- The experimental MSE of the estimators (3) and (7) are well predicted by the theoretical bounds derived in Section IV.
- The improvement of the proposed new estimator (7) over Lee's estimator (3) in medium and high SNR ranges is more and more significant when the rolloff factor ρ increases.
- At small rolloffs, both (3) and (7) outperform the SLN estimator, and are inferior to FLN, AVN, and LOGN estimators which, however, exhibit much higher computational load than estimators (3) and (7), which require only two samples per symbol.
- With ρ increasing, the difference of the estimation accuracy between the proposed algorithm (7) and FLN, AVN, and LOGN decreases, and further simulation results (not reported due to space limitations) show that at large

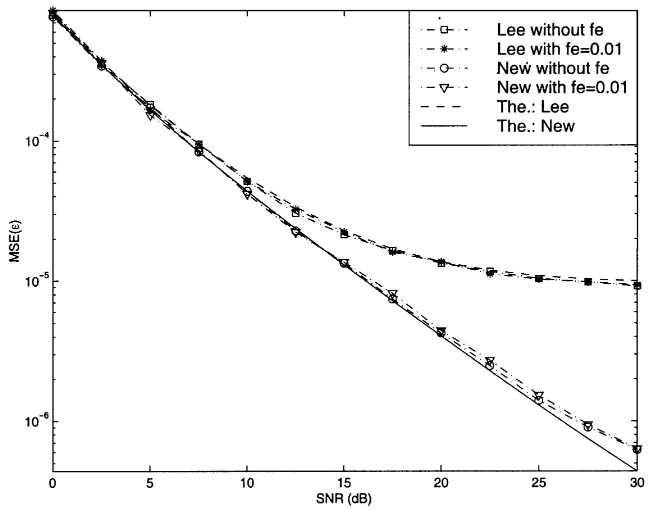

Fig. 4. MSEs of timing delay estimators versus SNR ($\rho = 0.5$).

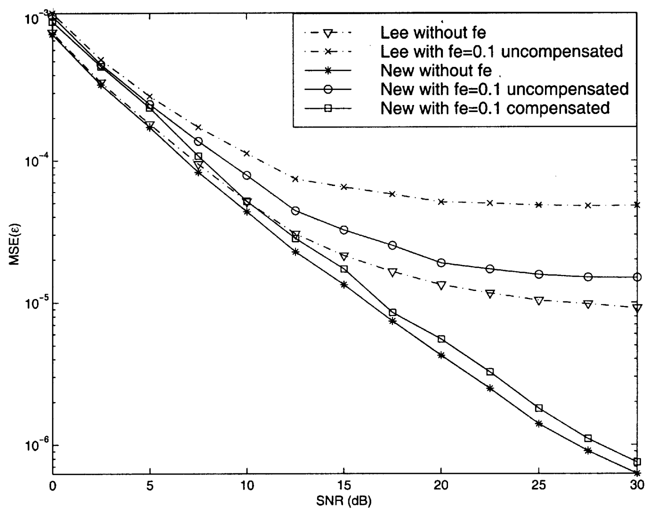

Fig. 5. MSEs of timing delay estimators versus SNR ($\rho = 0.5$).

rolloffs ($\rho > 0.5$), the estimator (7) outperforms FLN, AVN, and LOGN estimators.

- In the presence of frequency offset f_e, the proposed estimator (7) is robust against small frequency offsets. In the case of larger frequency offsets, f_e can be first estimated by adopting the blind frequency offset estimators proposed in [2] and [9], and then compensated in the timing estimator (7), which will result in an asymptotically unbiased timing estimator.

VI. Conclusions

In this letter, we have analyzed Lee's symbol timing delay estimator using a cyclostationary statistics framework. Although Lee's estimator presents the attractive property of a low computational load, it is asymptotically biased. To remedy this disadvantage, we have proposed a new unbiased estimator which outperforms significantly Lee's estimator at medium and high SNRs for large rolloff factors ($\rho > 0.5$), and which exhibits the same computational complexity as the latter. Moreover, the asymptotic MSEs of these two estimators, together with the asymptotic bias of Lee's estimator, are established in closed-form expressions. Computer simulations corroborate the theoretical performance analysis, evaluate the performance in the presence and absence of frequency offset, and illustrate the merits of the proposed new timing delay estimator.

Appendix
Derivation of *Theorem 1*

Equation (7) can be rewritten as

$$\hat{\epsilon} = \frac{1}{2\pi}\arg\left\{g(\rho)\cdot\hat{R}_x(1;0) + j\Re\left\{\hat{R}_x(1;1)\right\}\right\} = \frac{1}{2\pi}\arctan\left\{\frac{\hat{\alpha}}{\hat{\beta}}\right\} \tag{8}$$

where

$$\hat{\alpha} := \frac{1}{2}\left[\hat{R}_x(1;1) + \hat{R}_x^*(1;1)\right], \quad \hat{\beta} := g(\rho)\cdot\hat{R}_x(1;0).$$

For convenience, we define the following:

$$\alpha := \frac{1}{2}\left[R_x(1;1) + R_x^*(1;1)\right] = R_x(1;1),$$
$$\beta := g(\rho)\cdot R_x(1;0)$$

and $\Delta\alpha := \hat{\alpha} - \alpha$, $\Delta\beta := \hat{\beta} - \beta$. Equation (8) can be equivalently expressed as

$$\hat{\epsilon} = \frac{1}{2\pi}\arctan\left(\frac{\alpha}{\beta}\cdot\frac{1+\frac{\Delta\alpha}{\alpha}}{1+\frac{\Delta\beta}{\beta}}\right). \tag{9}$$

According to [9], $\Delta\alpha$ and $\Delta\beta$ are on the order of $o(1/\sqrt{N})$. Considering a Taylor series expansion of the right-hand side of (9), and neglecting the terms of magnitude higher than $o(1/\sqrt{N})$, it follows that:

$$\hat{\epsilon} = \frac{1}{2\pi}\left[\arctan\left(\frac{\alpha}{\beta}\right) + \frac{\alpha}{\beta}\frac{1}{1+\left(\frac{\alpha}{\beta}\right)^2}\left(\frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta}\right)\right] = \epsilon + \frac{\sin(4\pi\epsilon)}{4\pi}\left(\frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta}\right). \tag{10}$$

Simple manipulations of (10) lead to

$$\gamma_{\text{new}} = \frac{\sin^2(4\pi\epsilon)}{16\pi^2}\lim_{N\to\infty} N\mathrm{E}\left(\frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta}\right)^2 = \frac{\sin^2(4\pi\epsilon)}{16\pi^2 G^2(1;1)}\left(\frac{V_{11}}{\sin^2(2\pi\epsilon)} + \frac{V_{12}}{\cos^2(2\pi\epsilon)} - \frac{4V_{13}}{\sin(4\pi\epsilon)}\right)$$

where

$$V_{11} := \lim_{N\to\infty} N\mathrm{E}\left\{(\Delta\alpha)^2\right\}$$
$$V_{12} := \lim_{N\to\infty} N\mathrm{E}\left\{(\Delta\beta)^2\right\}$$
$$V_{13} := \lim_{N\to\infty} N\mathrm{E}\{\Delta\alpha\Delta\beta\}.$$

After some simple algebra manipulations of the above terms according to the definition of $\mathbf{\Gamma}_{u,v}$, the expression of γ_{new} follows. As for γ_{Lee}, we obtain

$$\begin{aligned}\gamma_{\text{Lee}} &= \lim_{N\to\infty} N\mathrm{E}\left\{(\hat{\epsilon}_{\text{Lee}} - \epsilon)^2\right\} \\ &= \lim_{N\to\infty} N\mathrm{E}\left\{(\hat{\epsilon}_{\text{Lee}} - \epsilon_0 + \epsilon_0 - \epsilon)^2\right\} \\ &= \lim_{N\to\infty} N\mathrm{E}\left\{(\hat{\epsilon}_{\text{Lee}} - \epsilon_0)^2\right\} + N\cdot(\epsilon_0 - \epsilon)^2 \\ &= \lim_{N\to\infty} N\mathrm{E}\left\{(\hat{\epsilon}_{\text{Lee}} - \epsilon_0)^2\right\} + N\cdot abias^2(\rho,\epsilon)\end{aligned}$$

where the ensuing derivation of the first term is similar to that of γ_{new}. This concludes the proof of *Theorem 1*.

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