Performance Analysis of Blind Carrier Phase Estimators for General QAM Constellations

Erchin Serpedin, Philippe Ciblat, Georgios B. Giannakis, *Fellow, IEEE*, and Philippe Loubaton, *Member, IEEE*

***Abstract*—Large quadrature amplitude modulation (QAM) constellations are currently used in throughput efficient high-speed communication applications such as digital TV. For such large signal constellations, carrier-phase synchronization is a crucial problem because for efficiency reasons, the carrier acquisition must often be performed blindly, without the use of training or pilot sequences. The goal of the present paper is to provide thorough performance analysis of the blind carrier phase estimators that have been proposed in the literature and to assess their relative merits.**

***Index Terms*—Asymptotic performance, blind estimation, carrier phase, Cramér-Rao bound, synchronization.**

I. Introduction

FAST acquisition of the carrier phase is a crucial issue in high-speed communication systems that employ large quadrature amplitude modulation (QAM) modulation schemes. One of the challenges associated with large QAM constellations is blind carrier acquisition, which is often required in large and heavily loaded multipoint networks for bandwidth efficiency and little effort involved in network monitoring. It is known that for large QAM constellations, the conventional carrier tracking schemes frequently fail to converge and result in "spinning" [10], [12]. Therefore, developing computationally simple blind carrier phase estimators with guaranteed convergence and good statistical properties is well motivated.

Recently, a number of blind carrier phase estimators have been proposed [1], [2], [5], [6], [13, p. 266–277], [14], but thorough performance analysis of all these algorithms has not been performed. In order to quantify the performance of these estimators, the large sample (asymptotic) performance analysis of these phase estimators will be established and compared with the stochastic (modified) Cramér–Rao bound [13, Sec. 2.4]. It is shown that the seemingly different estimators [1], [2], [5], [7], [13, p. 266–277], [14], are equivalent, whereas the estimator proposed in [6] has a larger asymptotic variance than the power-law estimator [5], [8], [14]. It is also shown that by exploiting the additional samples acquired through oversampling the received continuous-time waveform does not improve the performance of the power-law estimator in [5], [8], [14]. Finally, computer simulations are presented to corroborate the theoretical developments and to compare the performance of the investigated phase estimators. In the literature, two decision-directed (DD) phase estimators [8], [16] were reported to improve significantly the performance of the power-law estimators in the high signal-to-noise ratio (SNR) regimes. However, at low SNR, the reported DD estimators do not improve the performance of the power-law estimator. The performance analysis of DD algorithms is beyond the scope of this paper, and it will not be considered here.

Manuscript received April 3, 2000; revised April 4, 2001. The associate editor coordinating the review of this paper and approving it for publication was Prof. Gregori Vazquez.

E. Serpedin is with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128 USA (e-mail: serpedin@ee.tamu.edu).

P. Ciblat and P. Loubaton are with Laboratoire "Systèmes de Communication," Université de Marne-la-Vallée, Noisy le Grand, France.

G. B. Giannakis is with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA.

Publisher Item Identifier S 1053-587X(01)05856-1.

II. Problem Statement

We consider the baseband QAM communication system where the received signal $Y(n) = Y_r(n) + jY_i(n)$ is given by

$$Y(n) = e^{j\theta}X(n) + N(n) \tag{1}$$

where

- $Y_r(n)$ in-phase component of $Y(n)$;
- $Y_i(n)$ quadrature component of $Y(n)$;
- $X(n)$ independently and identically distributed (i.i.d.) input QAM symbol stream;
- $N(n)$ circularly distributed Gaussian noise, which is assumed to be independent of $X(n)$;
- θ unknown carrier phase offset.

The problem of blind carrier phase estimation consists of recovering the phase error θ only from knowledge of the received data $Y(n)$. Because the input QAM constellation has quadrant $(\pi/2)$ symmetry, it follows that it is possible to recover the unknown phase θ only modulo a $\pi/2$-phase ambiguity. This ambiguity can be further eliminated through the use of appropriate coding schemes. Therefore, without any loss of generality, we can assume that the unknown phase θ lies the interval $(-\pi/4, \pi/4)$. In the next section, we briefly outline the blind phase estimators [1], [2], [5]–[7], [13, p. 266–277], [14] and establish their exact large sample performance.

III. Blind Carrier Phase Estimators

A. Approximate Maximum Likelihood Estimator: Fourth-Power Estimator

The maximum likelihood (ML) estimator of θ can be theoretically derived by maximizing a stochastic likelihood function obtained by averaging the conditional probability density function of the received data with respect to the unknown data stream $X(n)$. However, for high-order QAM constellations, the computational complexity involved in calculating the likelihood function and, more importantly, the resulting nonlinear

optimization problem render the ML estimator impractical for most high-speed applications. The need for computationally simple estimators with guaranteed convergence calls for alternative (possibly suboptimal but computationally feasible) phase estimators.

Moeneclaey and de Jonghe have shown in [14] that for any arbitrary two-dimensional (2-D) rotationally symmetric constellations (such as square or cross QAM constellations), the fourth-power (or power-law) estimator can be obtained as an approximate ML estimator in the limit of small SNR [SNR $:= 10\log E|X(n)|^2/E|N(n)|^2$, where $:=$ stands for "is defined as"]. The power-law estimator and its sampled version are defined as

$$\theta := \tfrac{1}{4}\arg\left[\left(EX^{*4}(n)\right) EY^4(n)\right] \tag{2}$$

$$\hat{\theta} := \frac{1}{4}\arg\left[E\left(X^{*4}(n)\right)\frac{\sum_{n=1}^{N} Y^4(n)}{N}\right] \tag{3}$$

where the superscript $*$ stands for complex conjugation, and the operator $E(\cdot)$ denotes the expectation operator. The fourth-power estimator does not require any complex nonlinear optimizations, but it requires *a priori* knowledge of the input constellation $E(X^{*4}(n))$. However, this is not a restrictive assumption since for most QAM constellations, $EX^{*4}(n)$ is a negative real-valued number, whose effect can be easily accounted for. Using standard convergence results [3], [4], [11], it can be checked that asymptotically, (3) is[1] w.p. 1 a consistent estimator ($\hat{\theta} \to \theta$ as $N \to \infty$) for any SNR range. An explanation can be obtained by observing that in the presence of circularly and normally distributed noise $N(n)$, the following relation holds:

$$\frac{1}{N}\sum_{n=1}^{N} Y^4(n) \xrightarrow{\text{w.p.1}} EY^4(n) = e^{j4\theta}EX^4(n) \tag{4}$$

where the second equality in (4) is obtained by expanding $EY^4(n) = E(\exp(j\theta)X(n) + N(n))^4$, taking into account the independence between $X(n)$ and $N(n)$, and $EN^k(n) = 0$ for any positive integer k. Hence, (3) recovers the carrier phase from the phase of the fourth-order moment of the received data.

Cartwright has proposed estimating the unknown phase θ using a different set of fourth-order statistics [5]. Define the following fourth-order moments and cumulants

$$\begin{aligned}\gamma &:= E\left[Y_r^4(n)\right] + E\left[Y_i^4(n)\right] - 6E\left[Y_r^2(n)Y_i^2(n)\right] \\ &\qquad (E[Y_r(n)Y_i(n)] = 0) \qquad (5)\\ \gamma_a &:= \text{cum}(Y_r(n), Y_r(n), Y_r(n), Y_i(n)) \\ &= E\left[Y_r^3(n)Y_i(n)\right] - 3E\left[Y_r^2(n)\right]E[Y_r(n)Y_i(n)] \\ &= E\left[Y_r^3(n)Y_i(n)\right] \qquad (6)\\ \gamma_b &:= \text{cum}(Y_r(n), Y_i(n), Y_i(n), Y_i(n)) \\ &= E\left[Y_r(n)Y_i^3(n)\right] - 3E\left[Y_i^2(n)\right]E[Y_r(n)Y_i(n)] \\ &= E\left[Y_r(n)Y_i^3(n)\right]. \qquad (7)\end{aligned}$$

[1]The notation *w.p. 1* denotes convergence with probability one (almost surely).

Cartwright's estimator is defined by

$$\begin{aligned}\tan(4\theta) &= 4\left(\frac{\gamma_a - \gamma_b}{\gamma}\right) \Rightarrow \theta \\ &= \frac{1}{4}\,\text{atan}\left[4\left(\frac{\gamma_a - \gamma_b}{\gamma}\right)\right]. \qquad (8)\end{aligned}$$

To verify that Cartwright's estimator is the fourth-power estimator in (2), we equate the in-phase and quadrature components of

$$\begin{aligned}EY^4(n) &= e^{j4\theta}EX^4(n) \\ &= \cos(4\theta)EX^4(n) + j\sin(4\theta)EX^4(n) \qquad (9)\\ EY^4(n) &= E(Y_r(n) + jY_i(n))^4 \\ &= E\left[Y_r^4(n) + Y_i^4(n) - 6Y_r^2(n)Y_i^2(n)\right] \\ &\quad + 4jE\left[Y_r^3(n)Y_i(n) - Y_r(n)Y_i^3(n)\right] \\ &= \gamma + 4j(\gamma_a - \gamma_b). \qquad (10)\end{aligned}$$

It follows that $\gamma = \cos(4\theta)EX^4(n)$ and $4(\gamma_a - \gamma_b) = \sin(4\theta)EX^4(n)$, which implies the equivalence between estimators (2) and (8). Cartwright's (fourth-power) estimator requires only that $EX^4(n) \neq 0$ and the independence between $X(n)$ and additive circularly and normally distributed noise $N(n)$, and it can be applied to both square and cross-QAM constellations, as opposed to the estimator proposed in [6], which can be applied only to square-QAM constellations.

It is interesting to remark that three other phase estimators, which were derived using completely different arguments, are equivalent to the fourth-power estimator. An alternative robust phase estimator with guaranteed convergence has been proposed in [2] for square-QAM constellations. Herein, the carrier acquisition problem is reduced to the blind source separation problem of the linear mixture of the in-phase and quadrature-phase components of the received signal, and a cumulant-based source separation criterion is proposed to estimate the unknown phase-offset [2]. In [1] and [13, pp. 271–277], a low SNR approximation of the likelihood function, assuming PSK input constellations, is shown to have the same form as the estimator [2]. Furthermore, it is justified that this estimator can be used even for general QAM constellations [13, pp. 271–277]. By relying on Godard's quartic criterion [10], Foschini has shown an alternative derivation of this phase estimator in [7]. Next, we describe briefly the estimator proposed in [2], which relies on the observation that the in-phase and quadrature components of a square-QAM constellation are independent.

Let ϕ denote an estimate of the unknown phase offset θ, define the "rotated" output $\tilde{Y}(n) := \exp(-j\phi)\,Y(n)$, and assume that $X(n)$ belongs to a square-QAM constellation. In the absence of noise and if $\phi = \theta$, then the in-phase and quadrature components of $\tilde{Y}(n) = X(n)$ are independent. Thus, the joint cumulants of the in-phase $(\tilde{Y}_r(n))$ and quadrature $(\tilde{Y}_i(n))$ components of $\tilde{Y}(n)$ are equal to zero [3, p. 19]

$$\begin{aligned}\tilde{\gamma}_a &:= \text{cum}\left(\tilde{Y}_r(n), \tilde{Y}_r(n), \tilde{Y}_r(n), \tilde{Y}_i(n)\right) = 0 \\ \tilde{\gamma}_b &:= \text{cum}\left(\tilde{Y}_r(n), \tilde{Y}_i(n), \tilde{Y}_i(n), \tilde{Y}_i(n)\right) = 0 \qquad (11)\end{aligned}$$

and[2] $\tilde{\gamma}_a - \tilde{\gamma}_b = 0$. It is interesting to remark that (11) continues to hold true even in the presence of additive circularly and normally distributed noise $N(n)$ because the cumulants of the in-phase and quadrature components of $N(n)$ cancel out [3, p. 19]. By taking into account (10), it follows that $\tilde{\gamma}_a - \tilde{\gamma}_b = (E\tilde{Y}^4(n) - E\tilde{Y}^{*4}(n))/8j$. Thus, θ can be estimated from

$$\begin{aligned}\theta_a &:= \arg\min_{\phi} \left(E\tilde{Y}^4(n) - E\tilde{Y}^{*4}(n) \right) \\ &= \arg\min_{\phi} \left(e^{-j4\phi} EY^4(n) - e^{j4\phi} EY^{*4}(n) \right). \end{aligned} \quad (12)$$

If we consider the polar representation $EY^4(n) = \lambda^4 \exp(j4\theta)$, from (12), we obtain that $\theta_a = \arg\min_{\phi} \lambda^4(\exp(-j4(\phi - \theta)) - \exp(j4(\phi - \theta)))$, which implies that $\theta_a = \theta$ modulo a $\pi/4$-phase ambiguity. Hence, estimator (12) is the same as the fourth-power estimator (2). By taking advantage of the sign of $\tilde{\gamma} := (E\tilde{Y}^4(n) + E\tilde{Y}^{*4}(n))/2$ [see (5) and (10)], the $\pi/4$-phase ambiguity inherent in (12) can be reduced to a $\pi/2$-phase ambiguity [since if $\theta_a - \theta = \pi/4$ modulo $\pi/2$, then $\tilde{\gamma} = -EX^4(n) \neq EX^4(n)$].

In practice, many communication systems utilizing QAM constellations employ also coding, which implies that the SNR available at the synchronizer will be reduced by an amount proportional to the coding gain. In order to evaluate correctly the performance of these phase estimators at all SNR levels, we next provide an exact expression for the large sample variance of the power-law estimator, which is valid for any SNR level, and it is not restricted to the high SNR regime, as is the case with the approximate asymptotic expression presented in [14]. The next section will show that for 256-QAM, the expression of [14] is not valid for low and medium SNRs ($\leq$20 dB).

Theorem 1: Assuming that the i.i.d. symbol stream $X(n)$ is coming from a finite dimensional QAM-constellation and that the additive noise $N(n)$ is circularly and normally distributed and independent of $X(n)$, then the estimate (3) is asymptotically unbiased and presents the asymptotic variance

$$\lim_{N\to\infty} N\left(\hat{\theta} - \theta\right)^2 = \frac{\mu_{Y,44} - EX^8(n)}{32(EX^4(n))^2} \quad (13)$$

with[3] $\mu_{Y,40} := EY^4(n) = e^{j4\theta} EX^4(n)$, and

$$\begin{aligned}\mu_{Y,44} :=& E|X(n)|^8 + 16E|X(n)|^6 E|N(n)|^2 \\ &+ 36E|X(n)|^4 E|N(n)|^4 + 16E|X(n)|^2 E|N(n)|^6 \\ &+ E|N(n)|^8. \end{aligned} \quad (14)$$

Proof: Since $EX^{*4}(n)$ is real valued, it follows from (3) that

$$\hat{\theta} = \frac{1}{4} \operatorname{atan}\left[\frac{\operatorname{Im}\left\{ EX^{*4}(n)(1/N)\sum_{n=1}^{N} Y^4(n) \right\}}{\operatorname{Re}\left\{ EX^{*4}(n)(1/N)\sum_{n=1}^{N} Y^4(n) \right\}} \right]$$

$$= \frac{1}{4} \operatorname{atan}\left[\frac{\operatorname{Im}\left\{ (1/N)\sum_{n=1}^{N} Y^4(n) \right\}}{\operatorname{Re}\left\{ (1/N)\sum_{n=1}^{N} Y^4(n) \right\}} \right] \quad (15)$$

with Im(.) and Re(.) denoting the imaginary and real part operators, respectively. Using the notations $\rho := EY^4(n)$ and $\hat{\rho} := (1/N)\sum_{n=1}^{N} Y^4(n)$, (15) can be expressed as

$$\hat{\theta} = \frac{1}{4} \operatorname{atan}\left[\frac{\operatorname{Im}\{\rho\} + \operatorname{Im}\{\hat{\rho} - \rho\}}{\operatorname{Re}\{\rho\} + \operatorname{Re}\{\hat{\rho} - \rho\}} \right]. \quad (16)$$

By considering the first-order approximation of the argument in the right-hand side of (16), it follows that

$$\hat{\theta} \simeq \frac{1}{4} \operatorname{atan}\left[\frac{\operatorname{Im}\{\rho\}}{\operatorname{Re}\{\rho\}} + \epsilon \right], \quad (17)$$

$$\epsilon := \frac{\operatorname{Im}\{\hat{\rho} - \rho\}}{\operatorname{Re}\{\rho\}} - \tan(4\theta) \frac{\operatorname{Re}\{\hat{\rho} - \rho\}}{\operatorname{Re}\{\rho\}}. \quad (18)$$

The first-order Taylor expansion of (17) leads further to

$$\hat{\theta} \simeq \theta + \frac{\cos^2(4\theta)}{4} \epsilon \quad (19)$$

and hence

$$\lim_{N\to\infty} NE\left(\hat{\theta} - \theta\right)^2 = \frac{\cos^4(4\theta)}{16} \lim_{N\to\infty} NE\epsilon^2. \quad (20)$$

By defining

$$\begin{aligned} r_1 :=& NE(\hat{\rho} - \rho)^2 \\ =& NE\left(\frac{1}{N}\sum_{n=1}^{N} Y^4(n) - EY^4(n) \right)^2 \end{aligned} \quad (21)$$

$$\begin{aligned} r_2 :=& NE|\hat{\rho} - \rho|^2 \\ =& NE\left(\frac{1}{N}\sum_{n=1}^{N} Y^4(n) - EY^4(n) \right) \\ & \cdot \left(\frac{1}{N}\sum_{n=1}^{N} Y^{*4}(n) - EY^{*4}(n) \right) \end{aligned} \quad (22)$$

and using (18), simple manipulations show that

$$NE\epsilon^2 = \frac{r_2 - \cos(8\theta)\operatorname{Re}\{r_1\} - \sin(8\theta)\operatorname{Im}\{r_1\}}{2\cos^4(4\theta)(EX^4(n))^2}. \quad (23)$$

Expanding the right-hand side terms in (21) and (22), simple calculations lead to

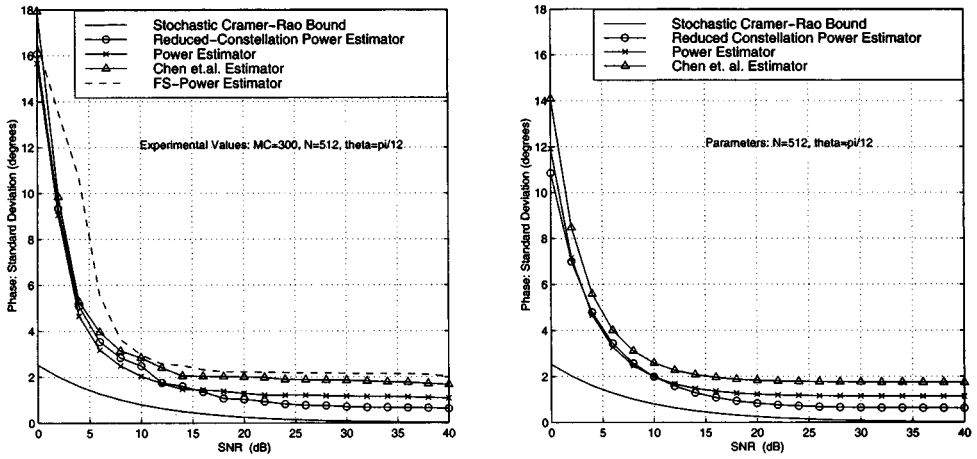
$$\begin{aligned} r_1 &= EY^8(n) - \left(EY^4(n)\right)^2 \\ r_2 &= E|Y(n)|^8 - \left|EY^4(n)\right|^2. \end{aligned} \quad (24)$$

Inserting (24) back into (23) and (20), we obtain the sought relation (13). The central limit theorem (CLT) and (17) and (18) imply that $\hat{\theta}$ is asymptotically normally distributed with zero mean. □.

The asymptotic variance (13) does not depend on the unknown phase θ but only on the input symbol constellation and the SNR. This confirms the conclusion drawn in [5] stating that the standard deviation of (8) appears to be constant with respect

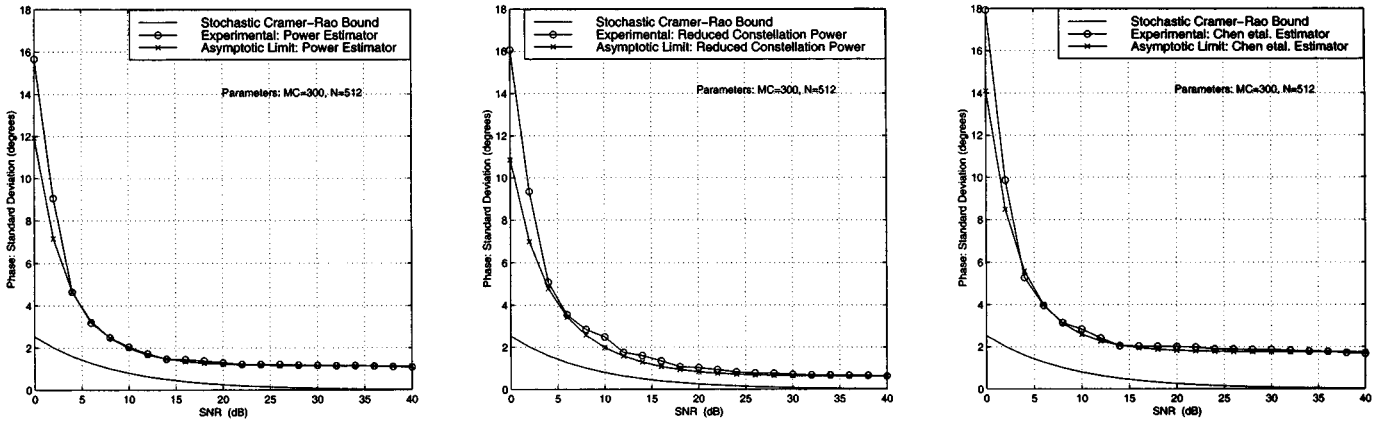
[2]We can easily check that $\tilde{\gamma}_a = -\tilde{\gamma}_b$ [6].

[3]The notation $\mu_{Y,kl} := EY^k(n)Y^{*l}(n)$ stands for the $(k+l)$th moment of $Y(n)$.



(a) (b)

Fig. 1. Standard deviation versus SNR. (a) Experimental values. (b) Asymptotic values (256 square-QAM).



(a) (b) (c)

Fig. 2. Standard deviation versus SNR. Experimental/theoretical values. (a) Power estimator. (b) Reduced-constellation power estimator. (c) Chen *et al.* estimator (256 square-QAM).

to the true value of θ. We evaluate next the asymptotic performance of a phase estimator based on an alternative set of statistics that was proposed in [6].

B. HOS-Based Phase Estimator of [6]

The phase estimator [6] extracts the unknown phase information $\theta \in (-\pi/4, \pi/4)$ using the relations

$$\cot(2\theta) = \frac{\gamma_a - \gamma_b}{2\gamma} \qquad \text{if } \left|\frac{\gamma}{\gamma_x}\right| \geq 0.125 \Leftrightarrow \theta \in \left(-\frac{\pi}{4}, -\frac{\pi}{8}\right] \cup \left[\frac{\pi}{8}, \frac{\pi}{4}\right) \tag{25}$$

$$\tan(2\theta) = \frac{2(\gamma_a - \gamma_b)}{\gamma_x - 4\gamma} \quad \text{if } \left|\frac{\gamma}{\gamma_x}\right| < 0.125 \Leftrightarrow \theta \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right) \tag{26}$$

with $\gamma_x := E[|X|^4] - 2\{E|X|^2\}^2$ and

$$\begin{aligned}\gamma &:= \text{cum}\{Y_r(n), Y_r(n), Y_i(n), Y_i(n)\} \\ &= E\left[Y_r^2(n)Y_i^2(n)\right] - E\left[Y_r^2(n)\right] E\left[Y_i^2(n)\right] \\ &= 0.25\sin^2(2\theta)\gamma_x. \end{aligned} \tag{27}$$

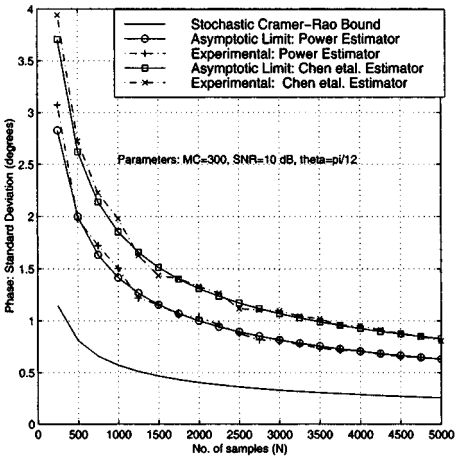
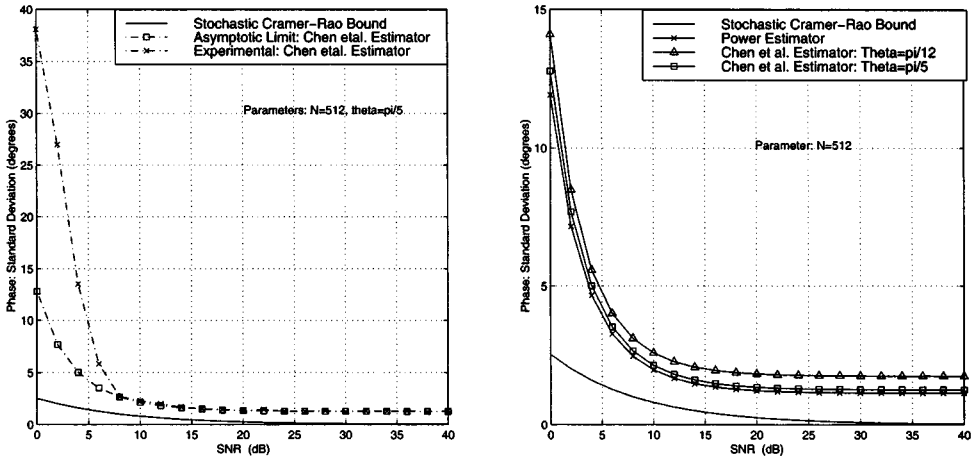


Fig. 3. Standard deviation versus number of samples. Power estimator versus Chen *et al.* estimator (256 square-QAM).

Let $\hat{\gamma}_a$, $\hat{\gamma}_b$, and $\hat{\gamma}$ denote sample estimates for γ_a, γ_b, and γ, respectively, and define by $\hat{\theta}_1$ and $\hat{\theta}_2$ the sample estimates corresponding to (25) and (26), respectively. Reasoning along the



(a) (b)

Fig. 4. Standard deviation versus SNR. (a) Chen *et al.* estimator ($\theta = \pi/5$). (b) Asymptotic limits (256 square-QAM).

lines of the proof presented for Theorem 1, the asymptotic performance of $\hat{\theta}_1$ and $\hat{\theta}_2$ can be established and is given by the following result.

Theorem 2: Assuming that the i.i.d. symbol stream $X(n)$ is coming from a finite-dimensional QAM-constellation and that the additive noise $N(n)$ is circularly and normally distributed and independent of $X(n)$, then the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ are asymptotically unbiased and present the asymptotic variances

$$\lim_{N\to\infty} N\left(\hat{\theta}_1 - \theta\right)^2 = \frac{\varrho_{11} + \cot^2(2\theta)\varrho_{22} - 2\cot(2\theta)\varrho_{12}}{\gamma_x^2} \qquad \theta \in \left(-\frac{\pi}{4}, -\frac{\pi}{8}\right] \cup \left[\frac{\pi}{8}, \frac{\pi}{4}\right) \tag{28}$$

$$\lim_{N\to\infty} N\left(\hat{\theta}_2 - \theta\right)^2 = \frac{\varrho_{11} + 4\tan^2(2\theta)\varrho_{22} + 4\tan(2\theta)\varrho_{12}}{\gamma_x^2} \qquad \theta \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right) \tag{29}$$

respectively, where we have (30)–(32), shown at the bottom of the next page, $\mu_{Y,44}$ is given by (14), and

$$\mu_{Y,62} := e^{j4\theta}\left[EX^6(n)X^{*2}(n) + 12EX^5(n)X^*(n)E|N(n)|^2 + 15EX^4(n)E|N(n)|^4\right] \tag{33}$$

$$\mu_{Y,51} := e^{j4\theta}\left[EX^5(n)X^*(n) + 5EX^4(n)E|N(n)|^2\right] \tag{34}$$

$$\mu_{Y,33} := E|X(n)|^6 + 9E|X(n)|^4E|N(n)|^2 + 9E|X(n)|^2E|N(n)|^4 + E|N(n)|^6 \tag{35}$$

$$\mu_{Y,22} := E|X(n)|^4 + 4E|X(n)|^2E|N(n)|^2 + E|N(n)|^4 \tag{36}$$

$$\mu_{Y,11} := E|X(n)|^2 + E|N(n)|^2. \tag{37}$$

As opposed to the power-law estimator, the asymptotic performance of the Chen *et al.* estimator [6] depends on the phase offset θ. As the simulation results will show (see Fig. 5), the asymptotic performance of this estimator deteriorates significantly whenever the *a priori* intervals (25) and (26) are missed,

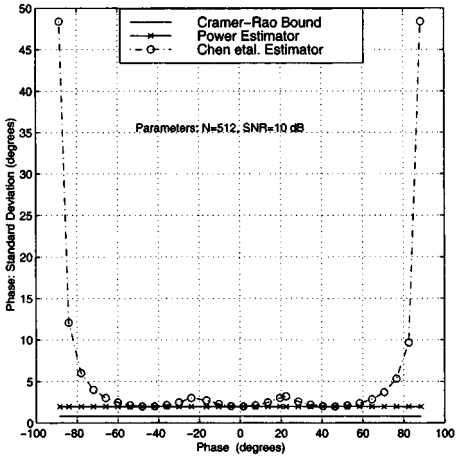
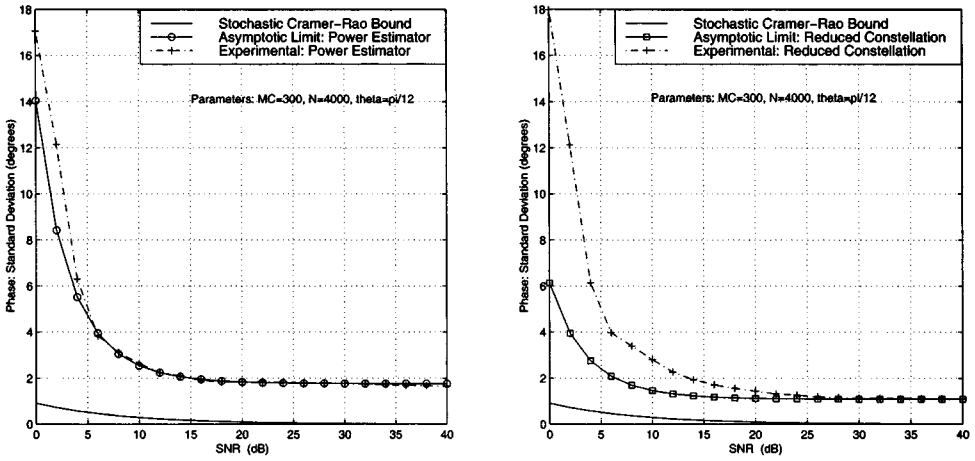


Fig. 5. Standard deviation versus phase offset. Asymptotic limit (256 square-QAM).

and for any SNR, it exhibits a larger variance than the power-law estimator.

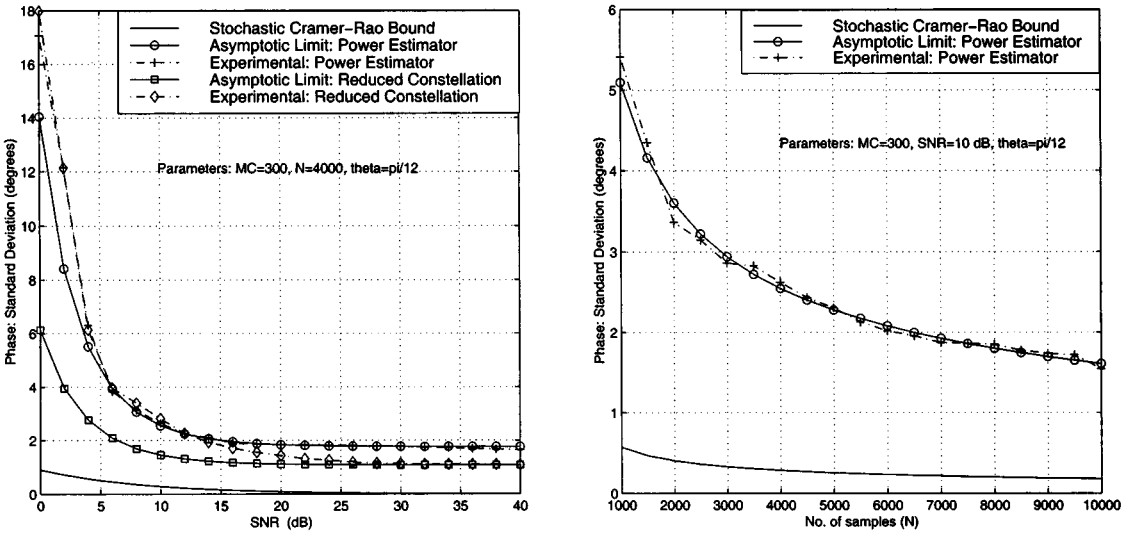
IV. Performance Comparisons

In this section, computer simulations are performed to assess the relative merits of the proposed phase estimators by comparing the theoretical (asymptotic) limits and the experimental standard deviations of the investigated estimators. Two additional estimators have been analyzed: the fractionally-sampled (FS) power-law estimator and the reduced-constellation power estimator. The FS-power estimator recovers the unknown phase offset θ by exploiting all the samples obtained by fractionally sampling (oversampling) the received continuous-time waveform in the estimator (3). A raised-cosine pulse shape with roll-off factor 0.3 and an oversampling factor $P = 3$ are assumed throughout the simulations. The reduced-constellation power estimator also relies on (3), but only the received samples that are larger in magnitude than a given threshold are processed [12, p. 1382], [8, p. 1482]. Thus, only the points closest to the four corners of the constellation are processed. The asymptotic performance of the reduced-constellation estimator is provided



(a) (b)

Fig. 6. Standard deviation versus SNR. (a) Power estimator. (b) Reduced-constellation power estimator (128 cross-QAM).



(a) (b)

Fig. 7. Standard deviation versus SNR/data. (a) Reduced-constellation power-law and power-law estimators. (b) Power estimator (128 cross-QAM).

by (13), with the higher order moments of the input sequence computed in accordance with the reduced constellation.

In Fig. 1(a) and (b), we have plotted the experimental and asymptotic standard deviations of all these estimators versus SNR, assuming a square 256-QAM constellation, $\theta = 15^0 (= \pi/12)$, $N = 512$ samples, $MC = 300$ Monte Carlo runs, and additive normally distributed noise. The threshold in the reduced-constellation power estimator has been set up so

$$\varrho_{11} := E[(\hat{\gamma}_a - \hat{\gamma}_b) - (\gamma_a - \gamma_b)]^2 = \frac{\cos(8\theta)\left[\left(EX^4(n)\right)^2 - EX^8(n)\right] - \left|EX^4(n)\right|^2 + \mu_{Y,44}}{32} \tag{30}$$

$$\varrho_{12} := NE\{(\hat{\gamma} - \gamma)[(\hat{\gamma}_a - \hat{\gamma}_b) - (\gamma_a - \gamma_b)]\} = \frac{-\sin(8\theta)\left[EX^8(n) - 2\left(EX^4(n)\right)^2\right] + 2\,\mathrm{Im}\{\mu_{Y,62}\}}{64} - \frac{4\sin(4\theta)EX^4(n)\left[\mu_{Y,22} - 3\mu_{Y,11}^2\right] + 8\left(E|X(n)|^2 + E|N(n)|^2\right)\mathrm{Im}\{\mu_{Y,51}\}}{64} \tag{31}$$

$$\varrho_{22} := NE(\hat{\gamma} - \gamma)^2 = \frac{\cos(8\theta)EX^8(n) + 3\mu_{Y,44} - 4\,\mathrm{Re}\{\mu_{Y,62}\} - 48\mu_{Y,11}^4 - 6\left[\cos(4\theta)EX^4(n) - \mu_{Y,22}\right]^2}{128} - \frac{32\mu_{Y,11}^2\left[\cos(4\theta)EX^4(n) - 2E|Y(n)|^4\right] - 16[\mathrm{Re}\{\mu_{Y,51}\} - \mu_{Y,33}]\mu_{Y,11}}{128} \tag{32}$$

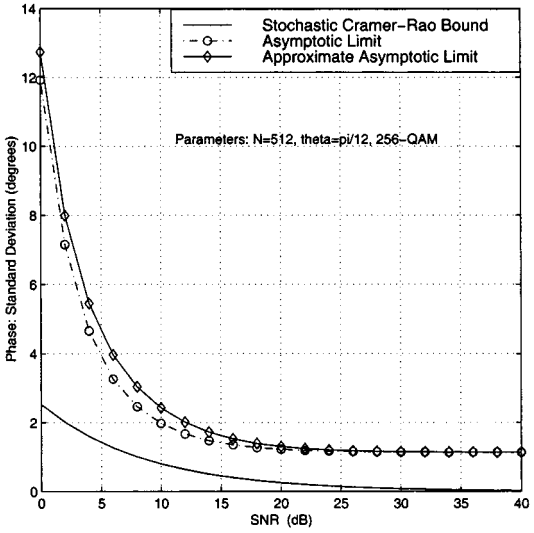


Fig. 8. Standard deviation versus SNR. Exact and approximate asymptotic limits (256 square-QAM).

that only the received samples corresponding to the 12 points of the input 256-QAM constellation with the largest radii are processed. The solid line denotes the stochastic Cramér–Rao bound $(\text{CRB} = 1/(N \cdot \text{SNR}))$ corresponding to the phase estimate. Fig. 1 shows that the power-law estimator performs better than the Chen *et al.* estimator [6] at all SNR levels but worse than the reduced-constellation power estimator at high SNRs (SNR $\geq$ 20 dB). The FS-based power estimator appears to have the worst performance. The reduced performance of the FS-power estimator is due to the increased "self-noise" generated by the residual intersymbol interference effects. For this reason, we have not pursued further the analysis of FS-based power-law estimators.

In Fig. 2, we have plotted separately the asymptotic and experimental standard deviations of the power-law, the reduced-constellation power-law, and the Chen *et al.* (26) estimators, assuming $MC = 300$ Monte Carlo simulation runs, $N = 512$ samples, $\theta = \pi/12$, and a 256-QAM input constellation. The experimental values are well predicted by the asymptotic limits for all three estimators, but the CRB seems to be a loose bound. In Fig. 3, the experimental and asymptotic standard deviations of the power-law and the Chen *et al.* estimators are plotted versus the number of samples (N), assuming SNR = 10 dB, $MC = 300$ Monte Carlo runs, and $\theta = \pi/12$. It turns out that both estimators achieve the asymptotic bound even when a reduced number of samples $N = 250 \div 500$ are used.

In Fig. 4(a), the asymptotic performance of the Chen *et al.* estimator (25) is analyzed, assuming $\theta = \pi/5$, $MC = 300$, and $N = 512$. Figs. 4(b) and 5 show that the performance of the Chen *et al.* estimator depends on the unknown phase θ and has a larger standard deviation than the power-law estimator for any phase offset θ (Fig. 5) and for any SNR-level [Fig. 4(b)]. In Fig. 5, the theoretical standard deviations (28) and (29) are plotted on the interval $(-\pi/4, \pi/4)$, assuming perfect *a priori* knowledge of the intervals (25) and (26), where θ lies. However, in the presence of an incorrect *a priori* knowledge on θ ($|\theta| \geq \pi/4$), the performance of estimator [6] deteriorates significantly. In Fig. 5, the set of values $\theta > \pi/4$ should be considered as reduced modulo $\pi/4$ to the interval $(0, \pi/4)$, whereas the set $\theta < -\pi/4$ is reduced modulo $-\pi/4$ to $(-\pi/4, 0)$.

In Figs. 6 and 7, we have analyzed the performance of the power-law and the reduced-constellation power-law estimators in the case of a cross 128-QAM constellation, assuming $\theta = \pi/12$, $MC = 300$, and $N = 4000$ samples. The threshold for the reduced-constellation power estimator is chosen such that in every quadrant only the two points with the largest radii are processed. We have selected the threshold to improve the performance of the reduced-constellation estimator. However, it turns out that for large cross-QAM constellations, the improvement provided by a reduced-constellation estimator relative to a full-constellation estimator is negligible. For such cross-QAM constellations, the Chen *et al.* estimator cannot be used since the in-phase and quadrature components of the input symbol stream are not independent. In Figs. 6 and 7(a), the experimental and asymptotic standard deviations of the full- and reduced-constellation power-law estimators are plotted for different SNR levels. Fig. 7(a) and (b) show that the asymptotic limit predicts well the experimental results for all SNR levels and number of samples $N \geq 1000$. It also appears that for cross-QAM constellations, the power-law estimator exhibits very slow convergence rate, and good estimates of the phase-offset can be obtained only by using a large number of samples ($N > 5000$). Finally, Fig. 8 reveals that for 256-QAM constellations, the approximate asymptotic limit derived in [14] does not predict well the exact asymptotic limit of the power-law estimator for small and medium SNRs.

REFERENCES

[1] A. N. D'Andrea, U. Mengali, and R. Reggiannini, "Carrier phase recovery for narrow-band polyphase shift keyed signals," *Alta Freq.*, vol. LVII, pp. 575–581, Dec. 1988.

[2] A. Belouchrani and W. Ren, "Blind carrier phase tracking with guaranteed global convergence," *IEEE Trans. Signal Processing*, vol. 45, pp. 1889–1894, July 1997.

[3] D. R. Brillinger, *Time Series Data Analysis and Theory*. San Francisco, CA: Holden Day, 1981.

[4] ——, "The comparison of least-squares and third-order periodogram procedures in the estimation of bifrequency," *J. Time Series Anal.*, vol. 1, no. 2, pp. 95–102, 1980.

[5] K. V. Cartwright, "Blind phase recovery in general QAM communication systems using alternative higher order statistics," *IEEE Signal Processing Lett.*, vol. 6, pp. 327–329, Dec. 1999.

[6] L. Chen, H. Kusaka, and M. Kominami, "Blind phase recovery in QAM communication systems using higher order statistics," *IEEE Signal Processing Lett.*, vol. 3, pp. 147–149, May 1996.

[7] G. J. Foschini, "Equalizing without altering or detecting the data," *Bell Syst. Tech. J.*, vol. 64, pp. 1885–1911, Oct. 1985.

[8] C. Georghiades, "Blind carrier phase acquisition for QAM constellations," *IEEE Trans. Commun.*, vol. 45, pp. 1477–1486, Nov. 1997.

[9] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: A cyclostationary approach," *IEEE Trans. Commun.*, vol. 46, pp. 400–411, Mar. 1998.

[10] D. Godard, "Self recovering equalization and carrier tracking in two dimensional data communication systems," *IEEE Trans. Commun.*, vol. COMM-28, pp. 1867–1875, Nov. 1980.

[11] T. Hasan, "Nonlinear time series regression for a class of amplitude modulated cosinusoids," *J. Time Series Anal.*, vol. 3, no. 2, pp. 109–122, 1982.

[12] N. Jablon, "Joint blind equalization, carrier recovery, and timing recovery for high-order QAM signal constellations," *IEEE Trans. Signal Processing*, vol. 40, pp. 1383–1397, June 1992.

[13] U. Mengali and A. N. D'Andrea, *Synchronization Techniques for Digital Receivers*, New York: Plenum, 1997.
[14] M. Moeneclaey and G. de Jonghe, "ML-oriented NDA carrier synchronization for general rotationally symmetric signal constellations," *IEEE Trans. Commun.*, vol. 42, pp. 2531–2533, Aug. 1994.
[15] A. J. Viterbi and A. M. Viterbi, "Nonlinear estimation of PSK-modulated carrier phase with application to burst digital transmission," *IEEE Trans. Inform. Theory*, vol. IT-29, pp. 543–551, July 1983.
[16] M. Morelli, A. N. D'Andrea, and U. Mengali, "Feedforward estimation techniques for carrier recovery in 16-QAM modulation," in *Broadband Wireless Communications*, M. Luise and S. Pupolin, Eds. New York: Springer-Verlag, 1998.

Erchin Serpedin received the Diploma of Electrical Engineering (with highest distinction) from the Polytechnic Institute of Bucharest, Bucharest, Romania, in 1991, the specialization degree in signal processing and transmission of information from Ecole Superiéure D'Electricité, Paris, France, in 1992, the M.Sc. degree from Georgia Institute of Technology, Atlanta, in 1992, and the Ph.D. degree from the University of Virginia, Charlottesville, in December 1998.

From 1993 to 1995, he was an instructor with the Polytechnic Institute of Bucharest, and between January and June 1999, he was a Lecturer at the University of Virginia. In July 1999, he joined the Wireless Communications Laboratory, Texas A&M University, College Station, as an Assistant Professor. His research interests lie in the areas of statistical signal processing and wireless communications.

Dr. Serpedin received the National Science Foundation Career Award in November 2000.

Philippe Ciblat was born in Paris, France, in 1973. He received the B.Sc. degree in mathematics from the University of Paris in 1994, the Engineer degree from Ecole Nationale Supérieure des Télécommunications, Paris, the M.Sc. in signal processing from Orsay University, Orsay, France, in 1996, and the Ph.D. degree from the University of Marne-la-Vallée, Noisy le Grand, France, in July 2000.

He is currently a postdoctoral researcher with the Department of Telecommunications, Université Catholique de Louvain, Louvain, Belgium. His research areas include statistical and digital signal processing, especially blind equalization, frequency estimation, and asymptotic performance analysis.

Georgios B. Giannakis (F'97) received the Diploma degree in electrical engineering from the National Technical University of Athens, Athens, Greece in 1981. From September 1982 to July 1986, he was with the University of Southern California (USC), Los Angeles, where he received the M.Sc. degree in electrical engineering in 1983, the M.Sc. degree in mathematics in 1986, and the Ph.D. degree in electrical engineering in 1986.

After lecturing for one year at USC, he joined the University of Virginia, Charlottesville, in 1987, where he became a Professor of electrical engineering in 1997. Since 1999, he has been with the University of Minnesota, Minneapolis, as a Professor of electrical and computer engineering. His general interests span the areas of communications and signal processing, estimation and detection theory, time-series analysis, and system identification—subjects on which he has published more than 120 journal papers, 250 conference papers, and two edited books. Current research topics focus on transmitter and receiver diversity techniques for single- and multiuser fading communication channels, redundant precoding and space-time coding for block transmissions, multicarriers, and wideband wireless communication systems. He is a frequent consultant for the telecommunications industry.

Dr. Giannakis is the (co-) recipient of three best paper awards from the IEEE Signal Processing (SP) Society (1992, 1998, 2000). He also received the Society's Technical Achievement Award in 2000. He co-organized three IEEE-SP Workshops (HOS in 1993, SSAP in 1996, and SPAWC in 1997) and guest (co-) edited four special issues. He has served as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING and the IEEE SIGNAL PROCESSING LETTERS, a secretary of the SP Conference Board, a member of the SP Publications Board, and a member and vice-chair of the Statistical Signal and Array Processing Committee. He is a member of the Editorial Board for the PROCEEDINGS OF THE IEEE, he chairs the SP for Communications Technical Committee, and serves as the Editor in Chief for the IEEE SIGNAL PROCESSING LETTERS. He is a member of the IEEE Fellows Election Committee and the IEEE-SP Society's Board of Governors.

Philippe Loubaton (M'91) was born in 1958 in Villers Semeuse, France. He received the M.Sc. and Ph.D. degrees from Ecole Nationale Supérieure des Télécommunications, Paris, France, in 1981 and 1988, respectively.

From 1982 to 1986, he was a Member of Technical Staff of Thomson CSF/RGS, where he worked in digital communications. From 1986 to 1988, he worked with the Institut National des Télécommunications as an Assistant Professor of electrical engineering. In 1988, he joined the Ecole Nationale Supérieure des Télécommunications, working in the Signal Processing Department. Since 1995, he has been Professor of electrical engineering at the University of Marne-la-Vallée, Noisy le Grand, France. His present research interests are in statistical signal processing and linear system theory, including connections with interpolation theory for matrix-valued holomorphic functions and system identification.

Dr. Loubaton is a member of the board of the GDR/PRC ISIS (the CNRS research group on signal and image processing), where he is in charge of the working group on blind identification.